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IN MEMORIAM

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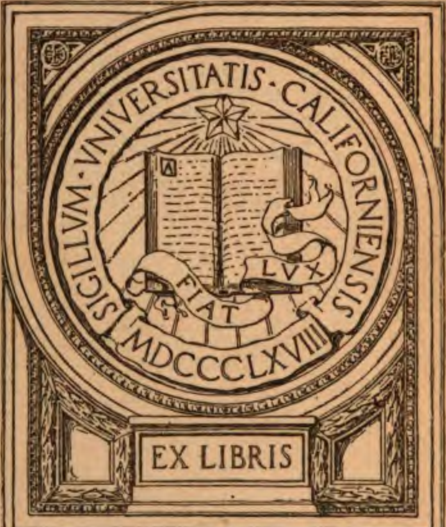

UNIV. OF CALIFORNIA

A TREATISE ON

NAVIGATION

AND

NAUTICAL ASTRONOMY

INCLUDING

THE THEORY OF COMPASS DEVIATIONS

PREPARED FOR USE AS A TEXT-BOOK

AT THE

U. S. NAVAL ACADEMY

BY

COMMANDER W. C. P. MUIR, U. S. NAVY
Head of the Department of Navigation
U. S. Naval Academy

FOURTH EDITION
Revised and Enlarged

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PREFACE.

In this volume, the endeavor has been made to place under one cover the allied subjects of Navigation, Theory of Compass Deviations, and Nautical Astronomy; and, though the book has been written primarily for the use of midshipmen, it is believed that the various subjects have been so presented that any zealous student with only a slight knowledge of trigonometry may be able to master any method given.

An effort has been made to include in this work not only the results of a large practical experience at sea, but information gleaned, during tours of duty in this department, from a study of the best English and American authorities. Much attention has been given to a description of the various navigational instruments, their uses, and errors; and to the principles involved in the construction of charts as well as to an account of the work usually performed on them.

The Theory of the Deviations of the Compass has been presented in the popular way for the practical man, and from a mathematical standpoint for more advanced students.

In Part II enough of Theoretical Astronomy has been incorporated to enable any one without a previous knowledge of that science to pursue the study of the practical part of Nautical Astronomy.

In the chapter on Time, I have gone much into detail and have illustrated the theory by the solutions of many examples, because an experience of years as an instructor has shown that beginners usually find it an intricate subject. In this chapter, as in all other parts of the book, practical illustrations follow immediately the theory on which they are based.

In a consideration of "lines of position," much space has been given not only to the theories and practice of Sumner, but also to the later adaptation of those theories by Johnson and Marcq Saint-Hilaire. All these methods are worthy of close examination, and each will be found to have its special advantages. However, if the student is pressed for time, he is advised to confine his attention to what will be described

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as the "chord method" which embodies the present practice of the United States Naval Service.

A chapter on "Tides" and one on "The Identification of Heavenly Bodies" have been included; a knowledge of both these subjects is essential to the modern navigator.

The text contains no reference to "lunars," that method of finding longitude being regarded as obsolete in these days of excellent chronometers. It is believed that the time is not far distant when "lunar distances" will disappear from the Nautical Almanac.

Acknowledgments are due to Lt. Comdr. W. V. Pratt, U. S. N., and to Lieut. C. P. Snyder, U. S. N., for assistance in proof-reading; to Midshipman H. G. Knox, U. S. N., for work on Tables II and III; and to the following-named firms for the loan of certain electrotypes: E. S. Ritchie & Sons of Boston; Keuffel & Esser, T. S. & J. D. Negus, and John Bliss & Co. of New York.

In conclusion, I desire to express my gratitude to Lt. Comdr. B. W. Wells, U. S. N., and to Lt. Comdr. G. R. Marvell, U. S. N., for valuable criticism of the original manuscript and for assistance in eliminating errors from the finished book.

W. C. P. MUIR.

DEPARTMENT OF NAVIGATION,
U. S. NAVAL ACADEMY, MAY 1, 1906.

NOTE TO SECOND EDITION.

Chapter XX has been rewritten and enlarged, and, besides a few minor changes which have been made in certain parts of the text where deemed desirable for greater clearness, the following additions have been made: a new method of equal altitudes suggested by Mr. G. W. Littlehales, Hydrographic Engineer, U. S. Navy Department; Plates X to XVI; and four appendices.

This opportunity is taken of thanking those officers of the service and friends outside the service who have kindly expressed appreciation of my efforts, or who may have offered suggestions with reference to this edition.

W. C. P. M.

JULY 1, 1908.

NOTE TO THIRD EDITION.

A complete revision of this book has been made wherever changes have been found necessary to make the text and examples correspond with the new method of estimating course and azimuth, an innovation due to the recently adopted graduation of the compass. Some new matter has also been added.

For suggestions as to this revision, my thanks are due Commanders S. S. Robison and J. H. Sypher, U. S. N., and to Commander Geo. R. Marvell, U. S. N., who relieved me as Head of Department of Navigation, U. S. Naval Academy, on my detachment from active duty, August, 1909.

W. C. P. M.

SHELBYVILLE, KY., MARCH 15, 1911.

NOTE TO FOURTH EDITION.

The work has been brought into conformity with the most recent issues of the Navy Department and related to the year 1918 by Mr. G. W. Littlehales, Hydrographic Engineer, U. S. Navy Department, under the supervision of the Navigation Department, U. S. Naval Academy.

The statement in the preface that "the chord method embodies the present practice of the United States Naval Service" no longer holds true, as the method of Marcq Saint-Hilaire has almost entirely supplanted the time sight.

An appendix on the gyro-compass and a small amount of other new matter have been added to bring the book into accord with the latest practice.

J. W. GREENSLADE,
Secretary and Treasurer,
U. S. Naval Institute.

ANNAPOLIS, MD., OCTOBER 15, 1917.

LIST OF WORKS CONSULTED.

Spherical and Practical Astronomy, Chauvenet.
General Astronomy, Young.
Elements of Astronomy, White.
The Heavens, Guillemin.
Navigation and Nautical Astronomy, Coffin.
Navigation, Asa Walker.
American Practical Navigator, Bowditch.
Navigation and Nautical Astronomy, Martin.
Navigation, Merrifield & Evers.
Nouvelle Navigation Astronomique, Villarceau.
Modern Navigation, Hall.
Wrinkles in Practical Navigation, Lecky.
Finding a Ship's Position at Sea, Sumner.
Finding Latitude and Longitude in Cloudy Weather, A. C. Johnson.
How to Find the Stars, Rosser.
Handy Book of the Stars, Whall.
Marine Surveying, U. S. N. A.
Notes on Navigation, U. S. N. A.
Practical Problems and Compensation of the Compass, Diehl.
British Admiralty Manual of Deviations.
Mathematical Theory of the Deviations of the Compass, Howell.
Manual of Deviatons, F. J. Evans.
Instructions for Care of Chronometers and Watches, Bureau of Equipment, Navy Department.
Proceedings U. S Naval Institute.
Method of Least Squares, Merriman.

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PART I.

NAVIGATION AND THEORY OF THE DEVIATIONS OF THE COMPASS.

NAVIGATION, THEORY OF THE DEVIATIONS OF THE COMPASS, AND NAUTICAL ASTRONOMY.

CHAPTER I.

DEFINITIONS AND GENERAL OBSERVATIONS.

Article 1. Navigation is the science of determining the position of a ship at sea, and of conducting a ship from one position on the earth to another.

2. There are three general methods of locating a ship: (1) When near the coast by bearings, or bearings and distances, of known objects on charts constructed by various methods or projections to represent the earth's surface; (2) by course and distance made good from a known position, involving the principles of plane trigonometry; (3) by observations of heavenly bodies, involving the principles of spherical trigonometry. The first may be called pilotage, the second dead reckoning, the third nautical astronomy—all independent in theory, but all used practically in the course of a voyage from one port to another distant port.

1) Pilotage
2) D. R.
3) Nautical A.

3. As a ship is located by the latitude and longitude of her position, it is proper to begin here with the elementary geographical definitions.

The earth is regarded as an ellipsoid of revolution having elliptical meridians and circular parallels of latitude. Computed on the ellipsoid of reference officially adopted in the United States, the equatorial radius is 3963.307 statute miles and the polar radius 3949.871 statute miles. For the general purposes of navigation the earth is assumed to be a sphere.

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The axis of the earth is that diameter passing through the poles of the earth and about which the earth daily revolves from west to east.

The earth's equator is a great circle of the earth whose plane is perpendicular to the axis at its middle point. The plane of the equator divides the earth into two hemispheres, the one containing the north pole being called the northern hemisphere, the one containing the south pole being called the southern hemisphere. Every point of the equator is equidistant from the poles.

Terrestrial meridians are great circles of the earth passing through the poles.

The meridian of a place on the earth is that meridian passing through the place.

The prime meridian is that meridian from which the longitude of places on the earth is measured. The meridian of Greenwich is almost universally accepted as the prime meridian.

Parallels of latitude are small circles of the earth whose planes are perpendicular to the axis.

The latitude of any place on the earth's surface is its angular distance from the plane of the equator north or south, measured from 0° to 90°, on the meridian passing through the place.

The middle latitude of two places in the same hemisphere is half the sum of their latitudes. The term is not strictly applicable where the places are situated on opposite sides of the equator.

The longitude of any place is the inclination of its meridian to the meridian of some fixed station known as the prime meridian, and is measured by the arc of the equator included between these two meridians. Longitude is usually reckoned from 0° to 180° east or west of the prime meridian (usually that of Greenwich). It is thus apparent that any point whose latitude and longitude are known can be located on the globe or chart representing the earth's surface.

The difference of latitude of any two places is the portion of a meridian included between the two parallels of latitude passing through the places. When both places are on the same side of the equator, their difference of latitude is found by subtracting the smaller from the larger latitude, and when the two places are on opposite sides of the equator, the difference of latitude is found by adding the two latitudes; when a ship in any latitude sails towards the pole of that hemisphere she increases her latitude, when she sails away from the pole she decreases her latitude; the difference of latitude being called N. or S. to indicate the direction of the change.

The difference of longitude of any two places is the angle at the pole, or the corresponding arc of the equator, between the meridians passing through the two places. When the two places are in longitudes of a different name, their difference of longitude is found by taking the sum, or 360° — the sum. The difference of longitude is called E. or W. to denote the direction of change. In other words, in combining latitude and difference of latitude, also longitude and difference of longitude, the operation must be performed algebraically, the terms N. and S. being considered as opposite signs, likewise the terms E. and W.

The geographical and nautical miles.—The geographical mile is the length of a minute of arc of the equator; the nautical or sea mile is the length of a minute of arc on the great circle of a sphere whose surface is equal to the surface of the earth. The former contains 6087.15 feet; and the latter 6080.27 feet, which corresponds to the Admiralty knot whose length is 6080 feet.

The meridians being ellipses flattened at the poles, the linear length of 1′ of the meridian is slightly different for different latitudes. It is least at the equator and greatest at the poles, and has a mean value of 6076.82 feet. For navigational purposes, the minute of latitude and the geographical and nautical miles may be considered the same.

The rhumb line or loxodromic curve is a line on the surface of the earth which makes a constant angle with each successive meridian.

If a ship sails on a loxodromic curve, the constant angle made by this line with the meridian is called the "true course." *For trigonometric computations,* the course is measured in degrees from North or South toward East or West, according to the data; though, *in practice,* navigators consider it as estimated, in both hemispheres, from the North point, around to the right, from 0° to 360°.

The distance between two places, or the distance run by the ship on a course, is the length of the loxodrome joining the two places measured in nautical miles.

4. Sailing a certain distance on a given true course, the distance North or South from the place left, measured on a meridian, is the difference of latitude, and the distance East or West on a parallel is the departure for that latitude, both expressed in sea miles. Should the course be due East or West on the equator, the distance would be difference of longitude.

Later on the relation between departure and difference of longitude will be shown to be that departure equals the difference of longitude multiplied by the cosine of the latitude.

5. The bearing of an object or place is the angle which the great circle passing through the object (or place) and observer makes with the meridian. It may be expressed as true, magnetic, or per compass, according as the meridian is true, magnetic, or per compass. Bearings, like courses, are expressed *practically* by modern navigators, from the North point, around to the right, from 0° to 360°.

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CHAPTER II.

NAVIGATIONAL INSTRUMENTS.

Description and Use of Logs, Leads, Sounding Machines, Charts and Protractors.

6. Besides being provided with the usual book outfit consisting of a log book, a treatise on navigation, one on deviation of the compass, useful tables, azimuth tables, nautical almanac for the current year, also tide and sunset tables, corrected buoy lists, light lists and sailing directions, a file up to date of notices to mariners, and an outfit of charts for the regions to be sailed over, a navigator must be provided with a compass, azimuth circle, pelorus, sextant, protractor, parallel rulers, dividers, chronometers, artificial horizon, mercurial barometer, a wet and dry bulb thermometer, a log and line (preferably a patent log), hand and deep-sea leads and lines, and, if possible, Sir Wm. Thomson's or Captain Tanner's sounding machine, a good binocular and long glass.

The more important of the instruments used in applying the principles of navigation will be considered in this and the succeeding chapter; whilst the sextant and chronometer, belonging properly to the subject of nautical astronomy, will be considered in Part II.

7. **Speed measurers.**—The distance traversed by a ship on any course being dependent on her speed, the accurate determination of this speed is a matter of great importance to the navigator.

Revolution of screw.—The author has found in his experience on ships of various classes that the revolutions of the screws furnish a most convenient and accurate log. Having made runs of known distances, in given times, under favorable conditions, the speed being uninfluenced by currents, and revolutions carefully noted, it is easy to find the coefficient of revolutions per minute for one knot and to tabulate the revolutions per minute to make any desired speed. A little experience should teach the navigator what allowance to make for adverse winds and seas, and for any unusual trim of the ship.

Patent logs.—There are many mechanical contrivances, of as many various forms but embodying the same general principles, called patent logs, which, under normal conditions, are very fair registers of speed. However, they are far from accurate and need careful watching, even when in good working order. If correct at one speed they are not liable to be so at a faster or slower speed, and they register differently in a head or following sea. The error of each patent log should be ascertained under varying conditions of wind and sea, at different speeds and draft for every run between well-determined points, provided the speed is not affected by tide; each rotator and register should be lettered and a record kept of their errors.

8. General description.—The most successful type may be said to consist of (1) the rotator, a hollow but enclosed conical shaped piece of brass with small vanes, towed astern by a specially made line, and caused to revolve more or less rapidly according to the ship's speed, by the pressure of water; (2) the register, located on the rail aft, in which cyclometer gear is worked by the rotator through the agency of the line, and the miles and tenths of a mile run thus indicated on the dial plate; (3) the specially made line, the length of which is an important factor in correct registering; experience can best decide this length for different speeds; a high speed requires a greater length than a low speed; under ordinary conditions it is advisable to use the length of line issued with the log by the manufacturer, about 400 feet.

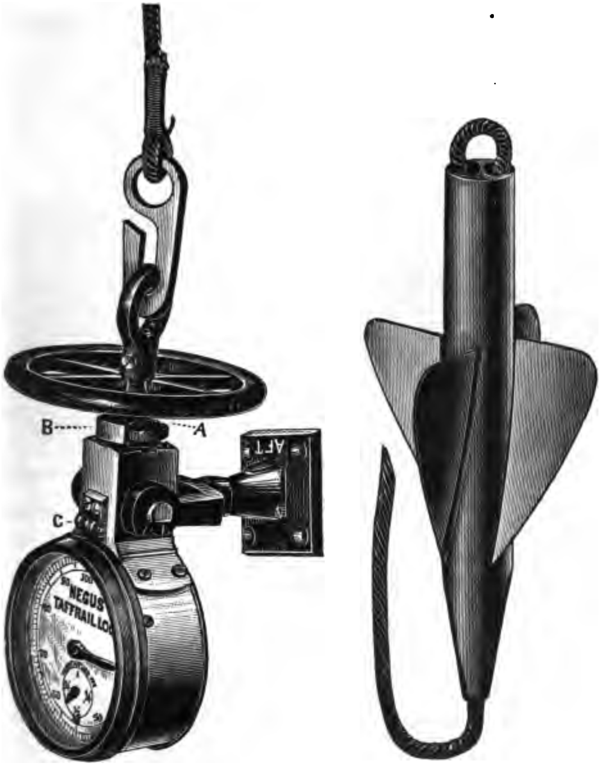

FIG. 1.—Negus Taffrail Log.

It is well to have two patent logs, each of a different manufacture, one on each quarter, and the error of each should be carefully determined by readings taken at times when the position of the ship has been accurately found, and on runs unaffected by currents.

The Bliss and Negus logs are perhaps the most reliable ones on the market. They are shown in Figs. 1, 2, and 3.

The mechanism of the patent log requires care and frequent oiling. In use, the lines must be watched to prevent being fouled by each other, by seaweed, cleaning rags, barrels, or debris carelessly thrown overboard.

Instruments usually accompany patent logs for changing the pitch of the rotator blades to correct an error in registering; but, if the error is small, it is better to leave it uncorrected, and apply it to the record of speed.

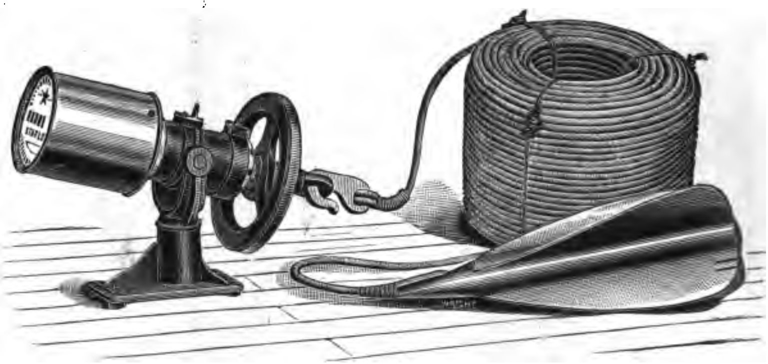

FIG. 2.—Bliss Star Log.

9. Patent electric log.—This log is the same as the ordinary patent log except that the gearing registering the knots and tenths of a knot closes an electric circuit every time a tenth is turned. The circuit thus closed magnetizes a solenoid, which in turn attracts a bar. This bar, by means of suitable levers, moves a train of gearing which registers the tenths of a knot and knots. This electrically controlled register is placed on the bridge, or in the pilot house, where it can be easily read by the officer of the deck.

[illegible]

10. Log chip, line and sand glass.—The speed of sailing

NOTE.—The Nicholson Ship Log includes a clock, speed dial, counter, record drum and chart. It is operated by floats in the load level and speed pipes, the height of water in speed pipe depending on the vessel's speed ahead. It shows the speed and mileage sailed, and records the speed on a chart for every minute of run.

ships, before the patent log came into vogue, was determined by the use of the log chip, log line, and sand glass. The log chip was a wooden quadrant about 5 inches in diameter, weighted with lead on the circular edge to make it float upright, joined by a three-legged bridle to the log line wound on the reel. The two legs of the bridle to the lower corners were joined to a pin which fitted into a socket secured to the leg attached to the upper corner.

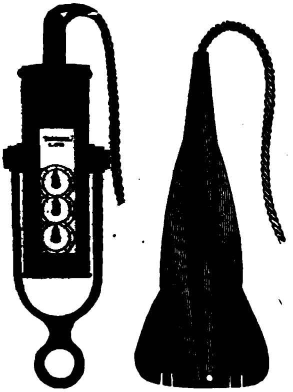

FIG. 3.—**Bliss Taffrail Log.**

The first 15 or 20 fathoms, called the stray line, was indicated by a piece of red bunting, and as this bunting went over the rail (the chip being well clear of the ship), the sand glass was turned at the order "Turn," and at the order "Up" the line was held, and by a sharp jerk the chip was untoggled. The order "Up" was given when the sand had run through, the length of line out at that instant, indicated by knots and tenths of a knot (marks on the line), gave the speed of the ship.

The line was subdivided into lengths of 47 feet and 3 inches called knots, and marked by short pieces of fish line, thrust through the strands, and having one, two, or three knots, etc., tied in them according to the number of lengths from the stray line mark. Each knot was subdivided by pieces of white rag into lengths of two-tenths of a knot each. In marking the line, the distance between knots was gotten from the proportion, "length of knot in feet is to one sea mile in feet as 28 seconds are to the number of seconds in an hour."

The glass itself, being a 28-seconds glass, was for speeds of four knots and under; for higher speeds a 14-seconds glass was used, and to get the ship's speed per hour, using this glass, the knots and tenths, as shown by the line run out, were doubled. Much depended on the manner of heaving the log, and errors of line and glass were hard to guard against. It did not afford a continuous record of the speed, and was not to be depended on for speeds over 10 knots.

Obj: Depth + Bottom.

11. Sounding has for its object the measuring of the depth of water and ascertainment of the character of the bottom; the former being shown on charts in fathoms or feet, according to the depth, and the latter noted, where known, as mud, sand, ooze, coral, etc.

12. Sounding apparatus.—The depth of water is ascertained by the sounding machine or the lead. There are several kinds of leads used, according to the depth of water.

Hand lead.—On entering or leaving port and in shallow water, generally speaking in less than 20 fathoms, casts are taken by the hand lead, a cylindrical lead, weighing from 7 to 14 pounds, attached to a line of from 20 to 30 fathoms in length, properly marked, and that too when wet.

[illegible]

The coasting and deep-sea leads.—The coasting lead, weighing from 25 to 50 pounds, is used in depths from 25 to 100 fathoms, beyond which depth the deep-sea lead, weighing from 80 to 100 pounds, becomes necessary. The coasting and deep-sea leads are hollowed out at the bottom to receive an arming of tallow to bring up specimens of the bottom. The lines are marked at 10 fathoms with one knot, at 20 fathoms with two knots, ~~and~~ so on, and at every intermediate ~~five fathoms with small strands~~; at 100 fathoms it is marked ~~with a piece of red bunting~~.

[illegible]

It is necessary to reduce the speed of the ship when getting casts with either the coasting or deep-sea lead, and a loss of time ensues; however, those disadvantages are obviated if the ship is supplied, as every ship should be, with a sounding machine, of which class of instruments Sir Wm. Thomson's and Commander Tanner's are certainly the best. Very accurate results can be gotten, without loss of time or reduction of speed, in depths up to 100 fathoms of water with these machines.

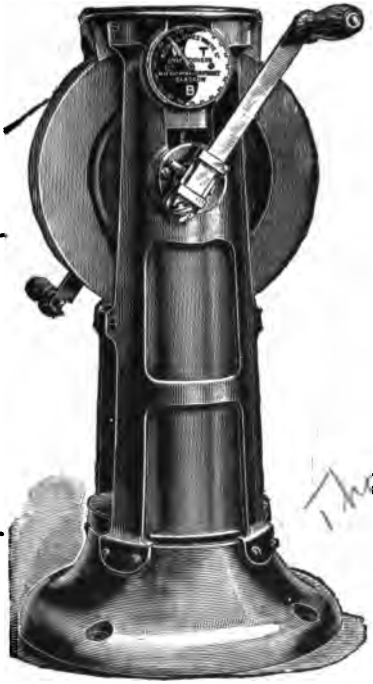

FIG. 4a.

Thomson's

13. Sir Wm. Thomson's sounding machine consists of a wooden or metal frame bolted to the deck, and carrying a drum of about one foot in diameter, on which is wound about 300 fathoms of seven-stranded flexible galvanized steel wire rope; the drum is controlled by a brake, and is provided with handles for heaving in, and a dial plate to record the amount of wire run out. To the wire is attached about 9 feet of log line, and an elongated sinker; between the wire and sinker, fast to the line, is a small copper tube closed by a cap with a bayonet joint at the top though perforated at the lower end. This tube is fitted to carry, when sounding, a glass tube hermetically closed at its upper end but open at its lower end. The glass tube is coated on the inside with chromate of silver. As the lead sinks, the sea water is forced up the tube in obedience to well-known physical laws, and chemical action of the salt water changes the coating into chloride of silver and its color from light salmon to a milky white, and this

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change takes place as far up as the water ascends in the tube. A graduated boxwood scale to which the glass tube is applied shows the depth to which it descended, and the arming on the sinker shows the character of the bottom. In heaving in, after sounding, care must be taken to keep the glass tube upright and prevent water from running into the upper part of the tube. *Chemically coated tubes which have been used in shallow water may be used again in water known to be deeper, and hence tubes discolored for only a fraction of their length should be saved for future use.*

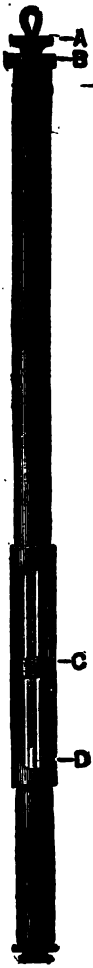

FIG. 4b.—Depth Recorder.

The depth recorder.—Instead of using the glass tubes the depth recorder may be used. This is a metallic tube in which a piston is forced up by water pressure against the tension of a spring; as the sinker descends the piston is forced up, carrying with it a small marker; on being hauled up after sounding the piston descends, but the marker remains and indicates on a graduated scale the depth to which the recorder had been. The marker must be set at zero and the valve screwed up just before use; after each cast unscrew the nut, slacken the valve, and turn the recorder upside down to drain out the tube. Fig. 4b.

Directions for use.—A pamphlet containing full directions for using the machine is issued by Messrs. John Bliss & Co., of New York, the American agents, and accompanies each machine.

The Thomson machine is shown in Fig. 4a.

14. Error of machine.—The sounding machine is not reliable within the field of action of the hand lead, and, always after a short use, has an error which the navigator should determine. This can be easily done when a ship is stationary in fairly deep water

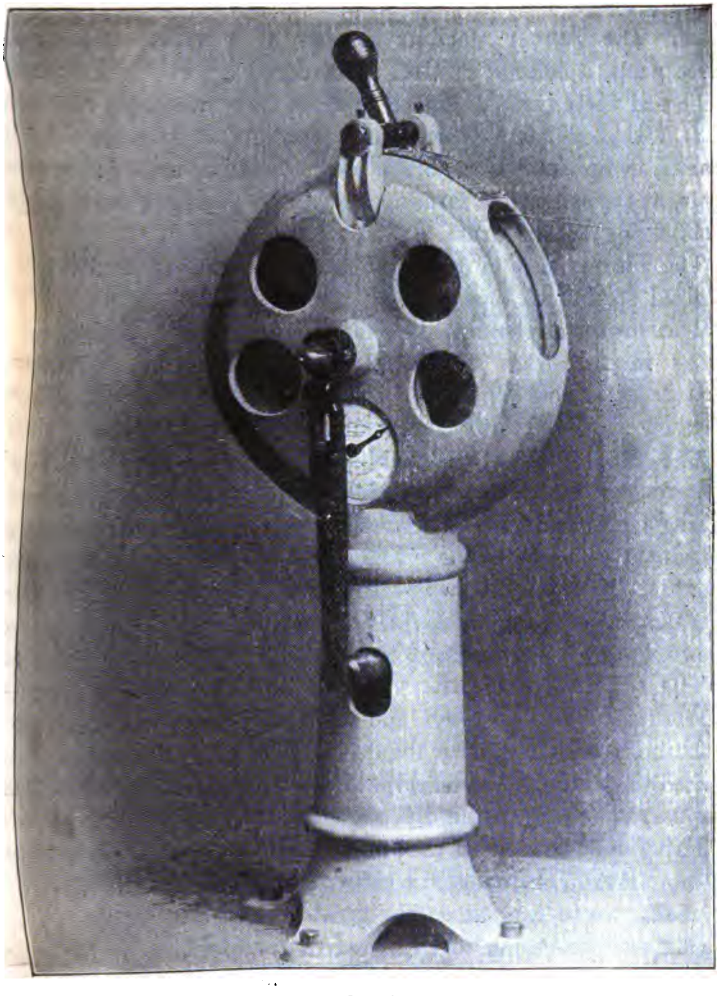

FIG. 5.

Correcting sounding machine

by comparing depths gotten by the machine and by use of the coasting lead.

15. The Tanner machine consists of a metal frame in three parts, a column of steel surmounted by two brass discs joined at their peripheries. The discs carry a shaft, a drum with V-shaped flanges on which is wound the sounding wire, cranks, compressor arms, brake lever, register, and correction table, at the same time forming a guard to prevent slack turns from flying off the drum.

The wire consists of 300 fathoms of 7-stranded flexible galvanized steel wire rope.

The brake mechanism is simple and direct acting, and being in full view of the operator is easily controlled. The drum is held by moving the lever in one direction, and released by the reverse movement.

The cranks are not to be unshipped from the machine. They are provided with automatic locking bolts which act when preparing for action; when these bolts are withdrawn, the cranks fall down each side of the column, and the handles are thrust into the friction scores where they are securely held, and at the same time exert a slight friction on the shaft, almost counteracting the inertia of the drum while sounding.

The register, which shows approximately the amount of wire out, is directly beneath the shaft, on the port side of the machine as it is set up on the deck. The correction table attached to the top of the machine shows the number of fathoms out corresponding to the dial register.

Ratio of wire to actual depth.

Experience has shown that at 10 knots speed with a depth of more than 50 fathoms, the ratio of wire out to actual depth is about two to one; at depths greater than 50 fathoms, and with speed increasing to 15 knots, the proportion is three to one; and in a heavy sea from four to one.

The sinker, which weighs about 18 pounds, has the appearance of an ordinary coasting lead with an iron rod projecting

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from its upper end. The sounding wire secures to an eye in the end of the rod. However, a length of stray line made of cotton cod line or signal halliard stuff, long enough to reach the machine when the sinker is up, is often preferred on account of its flexibility. The glass tube, which may be either the Tanner-Blish or Thomson tube, when in use with this machine, is carried in a brass shield seized to the wire or stray line above the sinker.

The Tanner combination lead is sometimes used instead of the sinker; it weighs 30 pounds, carries a shield and sounding tube within a central tube in the lead itself. On account of the delicate sounding tube, care must be taken not to let the lead strike the ship's side when reeling in.

16. The Tanner-Blish tube.—Commanders Tanner and Blish, U. S. N. have patented a tube designed to be used continuously if not accidentally broken. It is a glass tube 24 inches long of small uniform bore, the walls of which are ground or clouded. They are translucent when dry, but clear when wet, and the line of demarcation between the clear and the translucent part is used to determine the depth in the same manner as with the chemical tube. The bore is ground spirally to counteract the effect of capillary attraction. In the act of sounding, the upper end of the tube being closed by a rubber cap and the lower end remaining open, the column of air will be compressed by the water which will enter it in proportion to the depth to which the tube descends, the dividing line between the clear and translucent glass indicating the height to which the water entered the tube. One end of the tube is left unground, and the rubber cap should be placed always on the other end.

When its interior is dry the tube will indicate the depth without fail; if not dry, no results will follow the cast. Free circulation of air is essential in drying the bore, and to this end the cap must be removed. In rainy or foggy weather the

Tube must [illegible]

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tube may be dried in the engine room. The bore may be cleaned at any time by allowing a few drops of alcohol to run through it, back and forth, several times, and then rinsing the bore in fresh water. The tube should not be used more than a dozen times without being rinsed in fresh water.

Before sounding see the tube dry and translucent: If any part of it is clear, put that end to the lips and draw dry air through it with long inhalations filling the lungs, repeating the process until the whole glass is translucent; then put on the cap and proceed with the sounding.

Among the articles furnished with the machine is a small air pump for drying the tubes.

17. The machine in action.—The quartermaster of the watch and two men are required for the efficient operation of the machine. The sounding tube is inserted in the shield, open end down, and the shield is seized to the wire or stray line; the sinker is armed, bent to the wire or stray line, and lowered over the stern, the wire or line passing over the roller of the stern leader.

At the machine the slack turns are reeled in by one man, brake applied, and cranks thrown out of action, the handles thrust into friction scores, and the pointer noted at zero.

When ready, the quartermaster, with hand on the brake, eases down the sinker till it is near the water, then allows the wire to run freely till the sinker reaches the bottom, or the designated amount of wire has run out; as the wire runs out it is checked only as required to prevent slack turns, and a bent metal rod or hardwood stick, called the finger pin, is pressed lightly on the running wire, and indicates that the sinker has reached bottom by suddenly approaching the deck as the wire momentarily slackens; at this moment the brake is applied, both cranks thrown into action, and the wire reeled in by the two assistants. The quartermaster, watching over the stern, regulates the speed of reeling in, and signals

Finger pin

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"stop" when the sinker is up, hauls it on board, examines the arming, notes the character of the bottom, the brakes having been applied, and handles thrown out of action. The sounding tube is removed from the shield, applied to the scale and depth read, after which the arming of the lead is renewed and the tube dried preparatory to another cast.

If the Tanner lead is used instead of a sinker, the shield (with tube) is inserted in the central tube of the lead before or after it is suspended over the stern. The Tanner machine is shown in Fig. 5.

18. Use of sounding data.—It is often possible to ascertain, and frequently possible to verify, a ship's position by soundings.

In thick and foggy weather, soundings may prove the only safeguard. The U. S. Naval Regulations are very stringent in their requirements of a navigator on this subject. However, great care must be exercised in trusting to soundings alone. Several taken at random will seldom locate a ship; in fact may, by misleading, invite disaster.

When approaching land and on soundings, with no known marks in sight from which the position of the ship can be gotten, keep the lead going; note the nature of the bottom as evidenced by the arming of the lead, the depth and time of each sounding, and the course and distance to the next one. Take a piece of tracing paper or muslin sufficiently large to cover the area likely to include the ship's position during the runs considered; rule it with the meridians of the chart in use. Mark an assumed position in such a part of the tracing as to have sufficient room for the courses and distances from that position, write near it the depth and character of bottom at the first sounding. Lay down to the scale of chart the course and distance from the first position to that of the second cast, note as before the depth and character of bottom at that cast. Having a traverse of several positions and sound-

Tracing paper [illegible]

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ings, move the tracing up and down, and from side to side, keeping the ruled meridians parallel to those of the chart, until the soundings and character of bottom on the tracing correspond in close agreement with those on the chart. In this way a fair location of the ship on the chart may be gotten, even though exact correspondence of data is not found.

1) General
2) Sheet
3) Harbor

19. Charts are representations of certain parts of the earth's surface upon a plane surface, in accordance with some one of several definite systems of projection, an effort being made to satisfy the conditions that any two distances from the center of the chart shall have the same ratio as the corresponding distances on the earth's surface, that the ratio of the areas of definite limits on the chart and the ratio of the same limits on the earth shall be the same, and that the ratio of all corresponding angles shall be unity. However, since the earth is an ellipsoid, and the representation of it is on a flat surface, it is evident there must be distortion, and the effort should be to make this a minimum.

Charts are made primarily for the use of seagoing men, and show meridians and parallels of latitude, the details of the coast, light houses, life saving stations, mountains and prominent hills near the coast, soundings, dangers and shoals, nature of bottom, light ships, fog signals, buoys, beacons, tidal and current data, variation, etc., all of which may assist the navigator in making a successful voyage.

Charts are divided into general, sheet, and harbor charts; the elaborateness of detail depending on the scale, and the character depending on the purposes to be served.

General charts comprise an entire ocean, or a large part of it, or a considerable extent of coast line with adjacent waters. In addition to information referred to above, general charts should show the principal sailing routes.

A sheet chart is a detached portion of a general chart and is made on a larger scale. It usually gives the information first referred to, and enables a navigator to use the channels for entering the bays and large harbors.

A harbor chart is one of a harbor or a harbor and its approaches; the curvature of the earth is not considered, but owing to the small extent included in the chart, there is no distortion. It is made by assuming an observation spot, the latitude and longitude of which are determined, measuring a base line, cutting in signals by sextant or theodolite, filling in the detail, and running cross lines of soundings.

When the approaches to a harbor are of any extent, as for the U. S. harbors, the charts made by the Coast Survey are on the polyconic projection.

20. Systems of projection.—There are three principal systems of projection used in chart making, (a) Polyconic, (b) Gnomonic, (c) Mercator, a so-called projection by which rhumb lines appear as right lines on plane surfaces.

Omit

21. The polyconic projection.—In this the earth's surface is developed on a series of cones tangent to the earth, a different one for each parallel, the parallel forming the base of the cone, the vertex of which is on the axis of the earth produced. The parallels of latitude are developed as arcs of circles, but being from different centers and with different radii they are not parallel; the meridians, except the middle one, are curved and converge toward the pole.

The degrees of latitude and longitude are projected, practically in their true length, in consequence there is no distortion at the middle meridian, and very little anywhere, if the limits of the chart in longitude are narrow. As the minutes of latitude are practically of the same length, one scale of distances may be used for any part of the chart.

The geodesic line between two places (the shortest distance on the spheroid) will be projected practically as a straight line in its true length, but the loxodrome will be projected as rhumb line

a curved line and the true course will change from the beginning to the end of the voyage. For this reason practical sea-going people find it an inconvenient projection. Fortunately, however, the course may be taken as a straight line on a chart of large scale.

Certain meridians and parallels are subdivided in different parts of the chart; and whenever it is desired to plot the ship's position, the subdivisions nearest to the position must

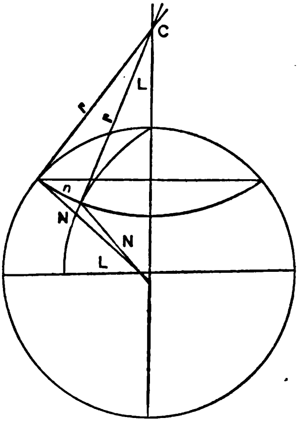


FIG. 6.

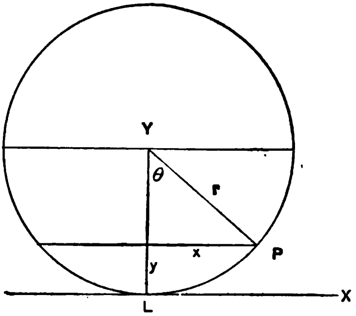


FIG. 7.

be used, and the same rule must be followed in taking off the latitude and longitude of a position.

22. Equations for the coordinates of a polyconic chart.—If, as in Fig. 6, N is the normal terminating in the minor axis, and L the angle it makes with the major axis; a, the equatorial radius; e, the eccentricity; then $N=\dfrac{a}{(1-e^2\sin^2 L)^{\frac{1}{2}}}$ and $r=N\cot L$ is the slant height of the tangent cone and the radius of the developed parallel, the developed parallel being a circle. Since in practice it would be inconvenient to describe the arcs with radii, they can better be drawn by constructing them from their equations, and it will be found convenient to have x and y, the rectangular coordinates of a point, whose latitude is $L°$ and whose longitude differs from that of the middle meridian by $n°$, expressed as functions of the radius of the developed parallel and the angle the radius makes with the middle meridian. Let θ be this angle (Fig. 7), the origin being taken at L, the point of intersection of any parallel with the middle meridian; the middle meridian as axis of Y, the perpendicular through L as axis of X, then the coordinates of any point P whose latitude is $L°$ and longitude is $n°$ from the middle meridian, will be

$$x = r \sin \theta = N \cot L \sin \theta \qquad (1)$$

$$y = r\,(1 - \cos \theta) = N \cot L \operatorname{versin} \theta \qquad (2)$$

where θ is some function of $n°$.

To determine the relation of n and θ, it is only necessary to remember that the parallels are projected with their true length, in other words, the distance LP, Fig. 7, equals the distance between L and P on the spheroid, measured on the parallel passing through L and P, therefore angles at the centers of the two arcs will be in inverse proportion to the radii, or $N \cot L \times \theta = N \cos L \times n°$; therefore,

$$\theta = n° \sin L. \qquad (3)$$

These three equations are sufficient to project any point of the spheroid given by its latitude and the number of degrees of longitude from the middle meridian.

In tables prepared by the U. S. Hydrographic Office, the elements of the terrestrial spheroid, and the coordinates of curvature, x and y, are tabulated in meters.

23. Construction of a polyconic chart (Fig. 8).—Draw a straight line LL' for the middle meridian; using Table IV of the projection tables referred to, take out for Lat. L from column D_m (degree of Lat.) the distance, which is laid off from L according to the scale of the chart. It is equal to Lm_1 and locates the parallel for Lat. m_1 at the middle meridian; lay off m_1m_2, taken out of the tables for Lat m_1, and locate Lat. m_2, continue this till the rectified arc of the meridian LL' is completed. Through the points thus found, draw the perpendiculars KLK, $K_1m_1K_1$, etc., to represent the axis of X in each case.

DIAGRAMMATIC DRAWING OF A POLYCONIC CHART.
Distorted for illustration.

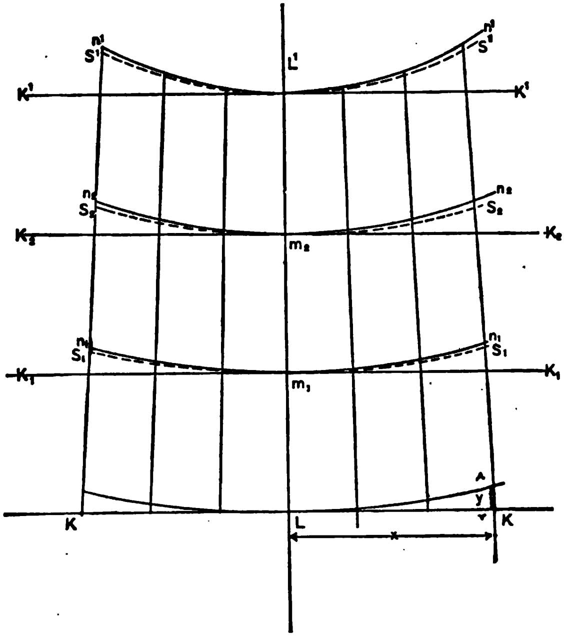


FIG. 8.

x and y the coordinates for latitude $L°$ and for $n°$ of longitude each side of the middle meridian.

On these perpendiculars, set off to E. and W. of the middle meridian the abscissæ x, and on lines at right angles towards the pole the ordinates y. The coordinates are taken from Table II, or Table III, of "The Projection Tables," according to the detail required, and laid down, according to the given scale, for each parallel of latitude and each required longitude.

The diagrammatic sketch of a polyconic chart, Fig. 8, will serve to illustrate the distortion at meridians removed from the middle one. The series of cones divides the surface to be projected into a series of zones, each zone tangent to those adjacent to it above and below only at the middle meridian, and separating from them to the eastward and westward. To complete the tangency and make the chart continuous, $s'n_2$, s_2n_1, etc., should be stretched, so that the lower edge of the zone $n_2m_2n_2$ will coincide with the upper edge $s_2m_2s_2$ of the lower zone along a middle curve, even then producing slight distortion which would increase with the longitude of points from the middle meridian.

24. Gnomonic projection.—In this system, the earth's surface is projected by rays from the center upon a plane tangent to the earth's surface at a given point, so it is apparent that all great circles will be projected as straight lines. The great circle track is represented as a straight line and for this reason such charts are often called great circle charts. Except when the point of tangency is at the pole, the parallels will be conics. The U. S. Hydrographic Office issues a series of charts to cover the various cruising grounds of the world, and on these are diagrams with full explanations for their use in finding the great circle course and distance between two points.

The polar chart.—The simplest form of the gnomonic chart is the polar chart, Fig. 10, in which the tangent plane is tangent at the pole; on such a chart great circles are

gnomonic chart [illegible]
upon a plane [illegible]
earth's center [illegible]
[illegible] to a given pt.

straight lines, the meridians are right lines radiating from the pole, whilst the parallels are projected as circles whose center is the pole; though accurate in high latitudes, this projection would give a distorted chart for low latitudes.

25. Construction of a polar chart.—In Fig. 9, let AB be the plane tangent at the pole S, pp' the parallel to be projected, $p_1p'_1$ will be the diameter of the projected parallel which will be a circle. Let R = earth's radius, x the radius of projected parallel; therefore, $x = R \cot L$. The radius of the parallel of 45° Lat., after projection, is R since cot 45° is unity, and this radius is called the radius of the chart; and to find the radius of any other parallel on the chart, we have $x = R \cot L$, where R is the number of units of the scale in the radius of the projected 45th parallel. To find the length of a degree of longitude on any parallel we have $\frac{2\pi x}{360}$.

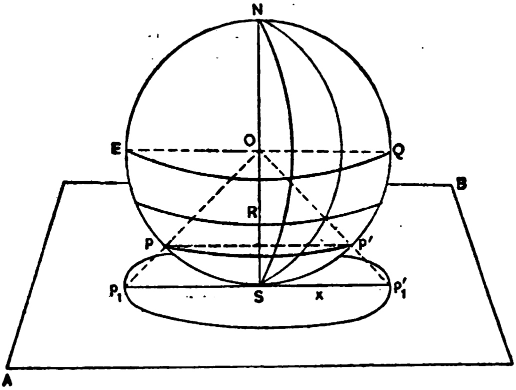


FIG. 9.

Referring to Fig. 10, let us construct a polar chart to comprise the earth's surface from 45° N. Lat. to the pole. Parallels at intervals of 5°, meridians at intervals of 15°, $R = 36$

millimetres; taking cotangents of latitude to nearest third decimal place. we have the following values of x:

A POLAR CHART.

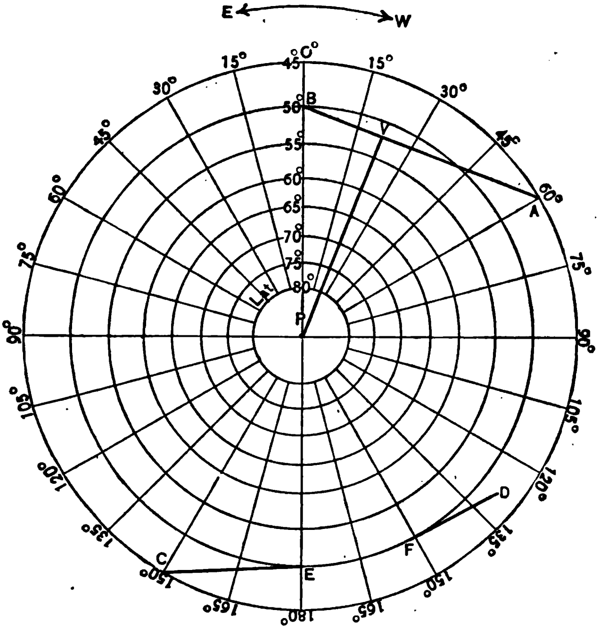


FIG. 10.

Scale: Radius lat. 45° = 36 millimetres.

For radius of 50°, $x = 36 \times .839 = 30.204$ millimetres.
“ “ “ 55°, $x = 36 \times .700 = 25.200$ “
“ “ “ 60°, $x = 36 \times .577 = 20.772$ “
“ “ “ 65°, $x = 36 \times .466 = 16.776$ “
“ “ “ 70°, $x = 36 \times .364 = 13.104$ “
“ “ “ 75°, $x = 36 \times .268 = 9.648$ “
“ “ “ 80°, $x = 36 \times .176 = 6.336$ “

With these different radii and also $R = 36$ mm., draw the concentric circles, number the parallels properly, and with a protractor divide the outer circumference into 15° divisions, drawing radii through the points. Mark one of these meridians 0° and the others as indicated on the chart; W. longitude to right, E. longitude to left.

Use of the polar chart.—The navigator, by drawing a straight line between the two required points, can see at a glance whether it is practicable to follow the great circle route; take off, if desirable so to do, the latitude and longitude of the vertex, and of other points along the track, transfer them to the mercator chart, and then lay courses on the mercator chart from point to point of this transferred track. For instance, suppose it is desired to go by great circle route from a point off Sable Island, Lat. 45° N., Long. 60° W. to a point in the English Channel, Lat. 50° N., Long. 0°. Use the polar chart, Fig. 10. Draw the straight line *AB* and then *PV* perpendicular to it from *P*. The position of *V* which is the vertex, the point nearest the pole, is readily seen by inspection; or measure the distance *PV* in millimetres, divide it by 36, the scale of the chart, and the result is the natural cotangent of the Lat. of *V*. For the longitude measure with a protractor the angle between the meridian of *V* and the one adjacent to it, applying the angle properly.

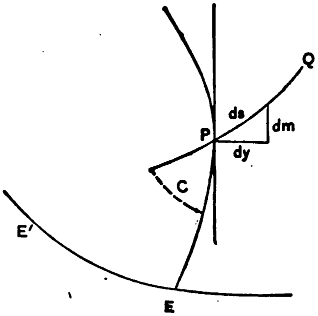


FIG. 11.

26. The loxodrome.—Before taking up the subject of the mercator chart and its construction, it is desirable to consider mathematically the loxodromic curve and to find the equation expressive of the difference of longitude, reckoned from the point at which this curve intersects the equator in terms of any given latitude and the constant course C. The deduction following is taken principally from Walker's Navigation by permission of the author of that excellent treatise.

In Fig. 11, let P be any point on the earth's surface, situated on the meridian PE and on the loxodrome PQ, and let EE' be the equator. Denote the equatorial radius by a, the radius of the parallel of P by x, and the radius of curvature of the meridian PE at point P by ρ. Denote the latitude of P by L, its longitude by λ, the course by C, the earth's meridional eccentricity by e, the longitude at which the loxodrome crosses the equator by λ_0. If we let y represent the number of sea miles along the parallel of P included between the prime meridian and the meridian of P, and as the longitude of P is λ, we shall have, when λ is in minutes of arc representing sea miles, $\frac{y}{x} = \frac{\lambda}{a} =$ the circular measure of the angle between the planes of the two meridians. Let ds denote the rate of the point P along the tangent to the loxodrome, and dm and dy be its rectangular components in the tangent plane at P. Then,

$$dy = \frac{xd\lambda}{a}. \tag{4}$$

Now, since the element of the terrestrial meridian at its intersection with any parallel of latitude is equal to the product of the radius of curvature and element of latitude at that point, in accordance with the principle that the radius of curvature varies inversely as the angle between consecutive normals, or, in this case, as the element of latitude, we have $dm = \rho dL$, therefore, $\tan C = \frac{dy}{dm} = \frac{xd\lambda}{a\rho dL}$, (5)

$$d\lambda = \frac{\tan C a\rho dL}{x}. \tag{6}$$

But from differential calculus we have the following well-known expressions for the properties ρ, x, and e, of the terrestrial spheroid considered as an ellipsoid of revolution:

$$\rho = \frac{a(1-e^2)}{(1-e^2\sin^2 L)^{\frac{3}{2}}}, \quad x = \frac{a\cos L}{(1-e^2\sin^2 L)^{\frac{1}{2}}}, \quad e = \sqrt{2c-c^2},$$

$c = 0.003407562$ the compression of the earth,
hence by substitution in (6),

$$d\lambda = \frac{a\tan C\,(1-e^2)\,dL}{(1-e^2\sin^2 L)\cos L}. \tag{7}$$

By integrating (7) between the limits λ_0 and λ, 0 and L, we shall have $\lambda - \lambda_0$, the difference of longitude required.

$$\lambda - \lambda_0 = a\tan C\int_0^L \frac{(1-e^2)\,dL}{\cos L\,(1-e^2\sin^2 L)}$$

$$= a\tan C\int_0^L \frac{(1-e^2)\cos L dL}{\cos^2 L\,(1-e^2\sin^2 L)} \tag{8}$$

$$= a\tan C\left[\int_0^L \frac{\cos L dL}{\cos^2 L} - e\int_0^L \frac{e\cos L dL}{1-e^2\sin^2 L}\right] \tag{9}$$

but $\displaystyle\int_0^L \frac{\cos L dL}{\cos^2 L} = \int_0^L \frac{\cos L dL}{1-\sin^2 L}$

$$= \int_0^L \tfrac{1}{2}\left(\frac{\cos L}{1+\sin L} + \frac{\cos L}{1-\sin L}\right)dL = \tfrac{1}{2}\log\frac{1+\sin L}{1-\sin L}$$

$$\int_0^L \frac{e\cos L dL}{1-e^2\sin^2 L} = \int_0^L \tfrac{1}{2}\left(\frac{e\cos L}{1+e\sin L} + \frac{e\cos L}{1-e\sin L}\right)dL$$

$$= \tfrac{1}{2}\log\frac{1+e\sin L}{1-e\sin L}$$

and $\displaystyle\left(\frac{1+\sin L}{1-\sin L}\right)^{\frac{1}{2}} = \tan\left[\frac{\pi}{4} + \frac{L}{2}\right],$

hence $\displaystyle\tfrac{1}{2}\log\frac{1+\sin L}{1-\sin L} = \log\tan\left[\frac{\pi}{4} + \frac{L}{2}\right],$

also

$$\tfrac{1}{2}\log\frac{1+e\sin L}{1-e\sin L} = e\sin L + \frac{e^3\sin^3 L}{3} + \frac{e^5\sin^5 L}{5} + \text{etc.},$$

whence by substitution in (9),

$$\lambda - \lambda_0 = a\tan C\left[\log\tan\left(\frac{\pi}{4} + \frac{L}{2}\right)\right.$$
$$\left. - e^2\left(\sin L + \frac{e^2\sin^3 L}{3} + \frac{e^4\sin^5 L}{5} + \text{etc.}\right)\right] \tag{10}$$

or if we put $e\sin L = \sin\phi$, we shall have,

$$\lambda - \lambda_0 = a \tan C \left[\log \tan \left(\frac{\pi}{4} + \frac{L}{2} \right) - e \log \tan \left(\frac{\pi}{4} + \frac{\varphi}{2} \right) \right]. \quad (11)$$

In this the logarithm is Naperian, so to reduce to common logarithms divide by the modulus $m = .434294482$; then introducing the value of a, the equatorial radius, 3437.74677 minutes of equatorial arc, or nautical miles, we have,

$$\lambda - \lambda_0 = D = \tan C \left[7915.704 \left(\log_{10} \tan \left(\frac{\pi}{4} + \frac{L}{2} \right) - e \log_{10} \tan \left(\frac{\pi}{4} + \frac{\varphi}{2} \right) \right) \right]$$

$$\text{or } D = 7915.'704 \tan C \log_{10} \frac{\tan \left(\frac{\pi}{4} + \frac{L}{2} \right)}{\left[\tan \left(\frac{\pi}{4} + \frac{\varphi}{2} \right) \right]^{e}} \quad (12)$$

27. The mercator chart.—At the equator on the spheroid a degree of longitude slightly exceeds a degree of latitude, but as the poles are approached the length of a degree of longitude becomes less, and finally zero at the poles, while the degrees of latitude undergo but slight change. On the mercator chart, which owes its origin to one Gerard Mercator, who lived in Flanders from 1512 to 1574, the meridians are drawn parallel to each other and perpendicular to a straight line representing the earth's equator, the distance apart on the chart being the distance between them on the spheroid, in minutes of arc on the equator, multiplied by the scale of the chart; thus the departure on the various parallels of latitude is increased and made equal to the difference of longitude.

As a compensation, and in order to preserve the proportion that exists between degrees of latitude and longitude at different parts of the earth's surface, and to maintain the relative position and direction of objects charted, the infinitesimal divisions of a meridian in the latitude of any parallel must be increased in the same ratio as the departure on that parallel. Regarding the earth as a sphere this ratio would be as sec L to 1, though allowance is usually made for the meridional eccentricity. The series of parallels will, therefore, appear as a series of right lines parallel to, and at such increasing distance from the equator as to maintain the required equality of angles and make the loxodromic curve a straight line.

Let D denote the difference of longitude between the meridians marking the intersections of the loxodrome, first with the equator, and second with any parallel of latitude L; and let M denote the augmented latitude for latitude L on the chart, we then have,

$$D = M \tan C;$$

but this D and this C are the same ones that appear in equation (12) above, therefore,

$$M = 7915.'704 \log_{10} \frac{\tan\left[\frac{\pi}{4} + \frac{L}{2}\right]}{\left[\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\right]^{e}}. \qquad (13)$$

The value of M in nautical miles for any latitude L is known as the meridional parts for that latitude. The meridional parts have been computed and are found tabulated in various works; in Bowditch's Navigator and in "The Useful Tables" they are found in Table 3 from 0° to 79° 59′, at intervals of 1′. The value of the compression used in computing Table 3 was $\frac{1}{293.465}$.

Since the meridional parts for any latitude L are the number of nautical miles, or 1′ of longitude, in the meridional distance from the equator to the parallel of latitude L on the Mercator chart, the meridional difference of latitude for Lats. L and L' is the difference of meridional parts for those latitudes if of the same name, or the sum of the meridional parts for Lats. L and L' if of a different name; in other words, it is the algebraic difference in either case, represented by m, or $m = M_2 \sim M_1$.

If in equation (12) we regard e as zero, in other words, if the earth be considered as a sphere instead of as a spheroid, we shall have,

$$D = 7915'.704 \tan C \log_{10} \tan\left(45^\circ + \frac{L}{2}\right). \qquad (14)$$

28. Construction of a Mercator chart.—The method of construction depends on whether the chart is to include the equator, and if so, the position of equator on the chart; and also whether the scale is to depend on the extent of paper in the direction of the meridians, or at right angles to them. By the term scale is meant the actual length on the chart of 1′ of arc of longitude on the earth's surface. If the chart includes the equator, the values of M as taken from Table 3 are to be measured off directly from the equator in the proper direction.

If the chart does not include the equator, then the lowest parallel to be represented on the chart is taken as the origin of parallels, and the distance from it to any other parallel is the meridional difference of latitude, as explained in the preceding paragraph.

If the extent of paper between the upper and lower parallel is limited, this distance is measured and divided by the meridional difference of latitude for the two parallels, and the result is the length of 1′ of arc of longitude or the scale of the chart. Multiply this by 60 for the length of one degree, and the length of one degree by the number of degrees of longitude to be charted, to obtain the distance between the Eastern and Western neat lines or bounding meridians of the chart.

In case the paper is limited in an East and West direction, draw a line near the bottom of the paper to represent the lowest parallel; divide this line into as many equal parts as there are degrees of longitude to be represented on the chart; then the length of one of these divisions divided by 60 gives the scale of the chart. This scale multiplied by the meridional difference of latitude for the parallel representing the origin and any other parallel *L′* will give the actual distance between the two parallels.

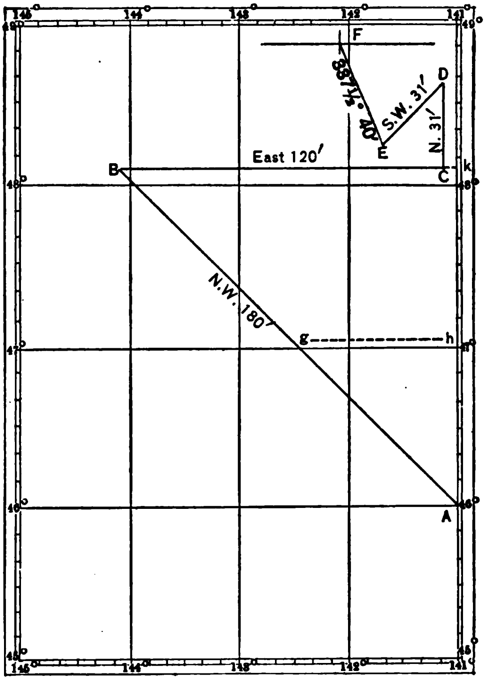


FIG. 12.

As an illustration, construct a Mercator chart to include latitudes 45° N. to 49° N. and longitudes 141° W. to 145° W., scale 14.4 mm. = 1° longitude, and subdivide each degree of both latitude and longitude into six divisions of 10′ each (see Fig. 12).

In the center of the paper draw a vertical line to represent the middle meridian 143° W. Near the lower edge of the sheet erect a perpendicular to this line which is the southern inner neat line of the chart, or 45th parallel of latitude. From the intersection of these two lay off distances of 14.4 mm. and 28.8 mm. on the parallel, both Eastward and Westward, and through these points draw lines parallel to the middle meridian; the two outer lines will be the extreme meridional neat lines of the chart, or the meridians of 141° W. for the Eastern limit, and 145° W. for the Western limit. It now remains to locate and draw in the parallels; their distances from the origin of parallels is determined from the following self-explanatory table:

Latitude.	Mer. Pts., *M.*	Mer. Diff. Lat. or *m.*	Multiplier.	Distance to parallels in Millimetres.	The Multiplier is $\frac{14.4}{60}$ because 1° = 14.4 mm. ∴ 1′ = $\frac{14.4}{60}$ mm. Always check the work as indicated.
45°	3013.4		$\frac{14.4}{60}$ =.24		
46	3098.7	85.3		20.472	
47	3185.6	172.2		41.328	
48	3274.1	260.7		62.568	
49	3364.4	351.		84.240	

Check 869.2 × .24 = 208.608

On the right or Eastern neat line lay off the distances in millimetres, as shown in the table above, from the lowest parallel; 20.472 mm. to the 46th, 41.328 mm. to the 47th, 62.568 mm. to the 48th, and 84.240 mm. to the 49th parallel. Through the points thus determined, rule right lines perpendicular to the meridians and these will be the various parallels required. Check the rectangularity of the construction by measuring the diagonals which should be equal. Draw the outer neat lines of the chart at distances desired, extend to them the meridians and parallels. Subdivide the degrees of latitude and longitude between the inner and outer neat lines by using proportional dividers or by a geometrical process.

29. Advantages and disadvantages of the different projections.—The polyconic chart has practically no distortion along the middle meridian, is well adapted to all latitudes, shows areas in their proper relation as to magnitudes, and permits the use of a single scale of distance anywhere. However, the meridians and parallels are curved, the rhumb line is curved, and there is distortion as the longitude departs from the middle meridian. The gnomonic chart is useful simply for finding the great circle course and distance; for navigational purposes it is useful in high latitudes where the Mercator projection fails. It gives a distorted idea of the earth's surface at points some distance from point of tangency of plane of projection, and on it the rhumb line is curved. On the special form known as the polar chart the rhumb line is spiral.

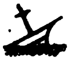
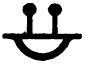
For navigational purposes the Mercator chart is by far the most convenient. The shapes of small areas are but little distorted; latitudes and longitudes may be laid down easily and accurately. The ship's track is a straight line, and the angle this line makes with any meridian is the course. However, it cannot be used in very high latitudes advantageously, the expansion being too great. The relative areas of land or bodies of water in different latitudes cannot be compared by the eye. The first objection is obviated by using a polar chart for those regions, the second is unimportant to mariners.

30. Conventional notation, and hydrographic signs.—Soundings are in feet or fathoms, as indicated under the title, and refer to the plane of mean low water for Atlantic Coast charts, that of the mean of lower low water of each tidal day for Pacific Coast charts, issued by U. S. Coast Survey. On British Admiralty charts the plane of reference is mean low water of ordinary spring tides.

Upon harbor and bay charts of the United States, the contour lines, or lines of equal depth, are traced for every fathom up to five fathoms. Within the three-fathom mark the chart is shaded, the shading being lighter for each fathom; beyond the three-fathom line there is no shading. On section charts of the coast, contours of 5, 10, 20, 30, and 100 fathoms are shown. Only the latter curve is given on large general charts. No bottom, for instance, at 50 fathoms, $\frac{0}{50}$.

Nature of the bottom.—The material of the bottom is expressed by capital letters, M for mud, G for gravel, S for sand, etc.; colors or shades by two small letters, yl., yellow, gy., gray, etc.; other qualities by three small letters, as brk., broken, sml., small, etc. A combination of these placed by a sounding shows at once the material, color, and nature.

Buoys.—These are indicated thus: B., black; R., red; H. S., horizontal stripes, black and red, danger buoy; V. S., vertical stripes, black and white, channel buoy. N means a nun buoy, C a can buoy, S a spar buoy. On entering a harbor, black buoys are left on the port hand, red on the starboard hand. Black buoys have odd, red buoys even numbers. Buoys with perch and square, or with perch and ball, are often found at turning points. There are also bell and whistling buoys, lighted (gas or electric) buoys, and white anchorage buoys. Yellow buoys are used to mark quarantine grounds or stations.

Dangers.—Rock awash at low water, *; sunken rock, +. Dangers of doubtful existence, marked E. D.; if known, but of doubtful position, marked P. D. Anchorage, ⚓; a wreck,  or +++; light ship, .

Lights.—Light houses are indicated by a yellow spot with a red or black dot, or as shown in Plates X and XI, end of book. Visibility is for a height of eye of 15 feet above the sea level.

NOTE.—Symbols on charts vary according to the origin of the charts. See Plates X and XI.

Character of light.—Indicated by abbreviations:

Lt. F. W.—A fixed steady light, white.

Lt. Fl. R.—Short flashes, longer intervals, color red.

Lt. Occ. R.—Long flashes, short intervals, color red.

Lt. Rev. W.—Intensity gradually increasing and decreasing, color white.

Lt. F. and Fl.—Combined fixed and flashing.

Currents.—These are indicated by feathered arrows pointing in general direction of set, with figures to indicate drift in knots per hour; current flood, by a half-feathered arrow with one, two, or three cross marks for 1st, 2d, or 3d quarter of flow, with figures to indicate velocity in knots per hour; current ebb, as for flood, using an unfeathered arrow.

31. Use of charts.—Spread the chart out before you on the chart board with the North direction away from you; in this way no readings will be upside down. In connection with the chart a navigator requires the use of a pair of parallel rulers, a pair of dividers, a sharp pencil, a reading glass, and sometimes a course protractor. The parallel rulers are used to transfer a course or bearing from the compass rose so as to pass it through a given point, or to transfer a line passing through a given point to the compass rose in order to ascertain the true or magnetic bearing or course; the dividers are used for taking off and measuring distances, whilst both are used in plotting or taking off the latitude and longitude of a point.

To find the latitude of a place on a Mercator chart.—Bring the edge of the parallel rulers to pass through the place parallel to a parallel of latitude; where it cuts the graduated meridian on the chart's side is the latitude.

To find the longitude.—Bring the edge of ruler to pass through the place parallel to a meridian; where it cuts the graduated parallel at top or bottom of chart is the longitude.

To plot a given latitude and longitude on a Mercator chart.—Place edge of ruler along the parallel of latitude nearest given latitude, move ruler parallel to itself till edge passes through given latitude on the graduated meridian, hold it firmly to prevent slipping; with dividers take from upper or lower graduated margin the distance of given longitude from nearest meridian, and lay it off from the same meridian along the edge of the parallel rulers. Or, in the absence of dividers, with a pencil point draw a light line along edge of ruler across approximate longitude; then lay the ruler parallel to the meridian, the edge cutting the longitude scale at the proper longitude, and cross the above line along the ruler's edge; the intersection is the plotted position.

On a polyconic chart, positions are plotted, or taken off, less accurately; the graduated parallel and meridian of that graduated subdivision nearest the position being used.

To measure a distance between two points on a Mercator chart.—In whatever way the distance may run, take off the distance with a pair of dividers and measure it along the graduated meridian or latitude scale, so that the middle of the line will be in the middle latitude between the two points; for instance, on chart, Fig. 12, the line *AB* should be measured so that its middle point *g* will be over *h*. In case the distance runs E. and W. on a parallel, then the distance should be measured equally each side of the parallel; for instance, on the same chart as above (Fig. 12), the distance *BC* should be so applied to the latitude scale that its middle point would be over *k*. In case the distance is too great to be conveniently included between the points of the dividers, take with the dividers a convenient unit from the latitude scale so that the middle latitude will be about midway between the points of the dividers, then step off this unit along the distance to be measured, turning the dividers alternately to right and to left, counting the number of times the unit is contained in the distance. The unit, which may be 5, 10, or any number of miles, multiplied by the number of times it is stepped off, plus any fraction of the unit (measured in its own middle latitude) to the end of the line, will give the required distance. In making the above measurements the middle parallel is never drawn but is assumed by inspection.

In measuring distances on a polyconic chart, reference is made not to the margins of the chart but to the single scale of distance under the title of the chart.

To find the course from one point to another on the Mercator chart.—Lay down the ruler so its edge passes through the two points, and draw a line if desired. Now move the ruler parallel to itself till the same edge passes through the nearest compass rose and read the course or bearing from the diagram. Or, having drawn the line in the first place, measure with a protractor the angle it makes with any meridian. When the diagram is constructed with reference to the true meridian, its readings indicate the true course, otherwise the magnetic course.

Use ‖ rulers & Compass rose [illegible]

Exactly the same method of procedure is followed in finding the course on a polyconic chart, but, from the nature of the projection, it is evident that this straight line is not a rhumb line, and that the course must be changed after a time, on account of the angle between the meridians; the length of the time depending on the general bearing between the points, on the distance, on the latitude, and on the scale of the chart.

The course and distance run by a ship on the rhumb line from a given point being known, to find the ship's position on the mercator or polyconic chart.—Place the edge of the parallel rulers so as to pass through the center of the compass rose and the reading of its circumference representing the course (true or magnetic). Move the ruler parallel to itself till the same edge passes through the given point. Draw a light line in the desired direction and lay off the distance run from the given point on this line, or, along the edge of the ruler, if the line is not drawn, and the ship's place is determined. The distance is taken from the proper scale as explained in previous paragraphs for both the mercator and polyconic charts.

To plot the ship's position by cross bearings.—Correct each bearing for the deviation of the compass due to the direction of the ship's head when bearing was taken; the magnetic bearings are thus obtained. Place the edge of the parallel rulers over a magnetic compass rose, the edge passing through the center and reading of the circumference representing the magnetic bearing. Move the ruler parallel to itself till the same edge passes through the proper object, draw a light line through the approximate position of the ship. This line is a line of bearing and the position of the ship is somewhere on it. In the same way draw the line of bearing corresponding to the second object. The ship being on both lines will be at their intersection on the chart. To obtain good cuts, these lines should make angles not less than 30°, the best cuts, of course, being given when the lines are at right angles to each other. If the compass rose is a true and not a magnetic rose, the bearings must be corrected for the variation as well as the deviation.

32. Correction of charts.[1]—Charts, to be of any service, should be reliable, and to be reliable they must be kept corrected to date. The information for this purpose can be gotten from "Notices to Mariners," bulletins published weekly by the Hydrographic Office of the Navy Department, and the Light-House Board; also from the branch hydrographic offices at our important sea ports.

33. Arrangement and stowage of charts.—The U. S. Hydrographic Office issues to ships of the navy Hydrographic Office (H. O.), Coast Survey (C. S.), and British Admiralty (B. A.) charts. Regardless of the publication or chart number, all charts issued are arranged as far as practicable in geographical sequence, numbered consecutively, and divided into portfolios, each portfolio containing about 100 charts. The consecutive numbers in each portfolio begin with the even hundred; a chart whose consecutive number is, for instance, 520, will be found in portfolio No. 5.

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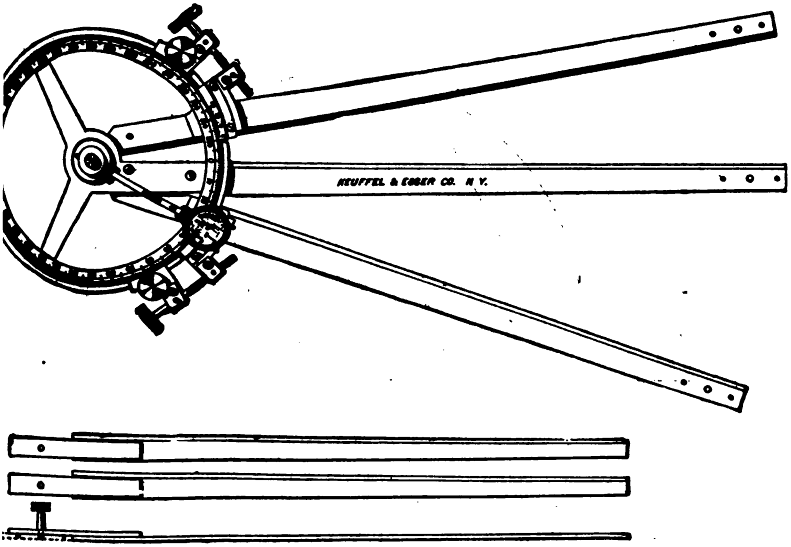
[1] The U. S. Naval Wireless Telegraph Stations on the seaboard transmit daily at 6 a. m., 2 p. m., and 10 p. m., standard time, from the Hydrographic Office to vessels at sea, information as to obstructions that are dangerous to navigation.

General charts of the ocean will be found in portfolio No. 1. General charts of the station for the use of the commander-in-chief will be found in portfolio No. 43 for the Atlantic and European stations, No. 45 for the Pacific station, and No. 47 for the Asiatic station; those for the use of the wardroom officers in portfolio No. 44 for the Atlantic and European stations, No. 46 for the Pacific station, and No. 48 for the Asiatic station. Each portfolio should have a separate drawer, in a nest of drawers, built in the pilot house and convenient to the chart table.

34. The three-arm protractor.—In determining the position of a ship by sextant angles between known objects along a coast, the three-arm protractor will prove itself an invaluable instrument. It consists of a graduated brass circle having three arms, the straight edges of which all pass through the center of the circle. The center arm is fixed and the zero of graduation is coincident with its straight edge. The other two arms are movable and both are fitted with clamp screws and tangent screws. As the movable arms turn away from the central arm, the angles gradually increase, and when the arms are clamped, a vernier, with reading microscope, gives the angle to the least count of the vernier. Extension pieces are provided for each arm. It is impossible to shut the right arm close home, as the beveled straight edge of the fixed arm is on its left side, so if the right arm cannot be set for a small right observed angle, set the left arm for it; then swing the right arm around and set it for the sum of the two observed angles, reading from zero to the left.

To plot a vessel's position with a three-arm protractor.— Select three objects that can be seen and reflected, that are well located on the chart, and so situated with reference to each other that the observer's position will be well determined. Get simultaneously the angle between the middle object and the right one (called the right angle), and the angle between the middle object and the left one (called the left angle). The lateral arms of the protractor having been set to their proper angles, and the same verified, the instrument is placed on the chart, the edge of the central arm passing through the middle object and kept there whilst the instrument is moved around till the edges of the lateral arms also pass through their respective objects. The center of the instrument is at



The Three-Arm Protractor.

the point of observation which is lightly marked upon the chart by pencil or the spring point of the center punch.

Tracing paper or linen with angles laid off and properly numbered may be used as a substitute.

The diagram (Fig. 13) will illustrate the different cases

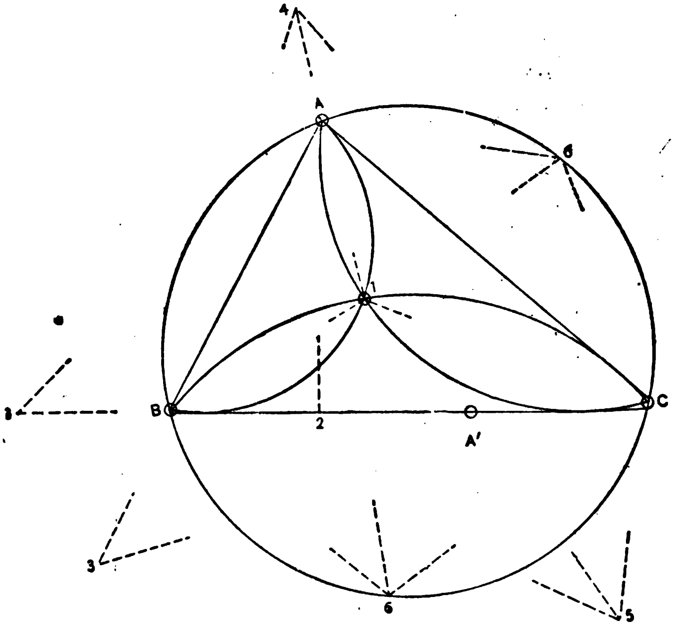


FIG. 13.

that may be met with in practice. *A, B, C* are the three objects forming the triangle called the great triangle, the circle through which is called the great circle. The position of the observer is at the intersection of the circles of which the sides of the great triangle are chords, the position of the centers of these circles, and hence of their intersection, depends on the observed ~~angles~~. The nearer these secondary circles intersect at 90°, the better the "fix." In cases in which the centers of the circles are near each other, and near the center of the great circle, the position is more or less indeterminate, and such angles are called "revolvers."

CASE 1.—The two angles observed are $>180°$; the position of observer is within the triangle and is well determined.

CASE 2.—The sum of the two angles $=180°$; the observer is on one side of the great triangle, and the position is well determined.

CASE 3.—A range and one angle; a good determination of position.

CASE 4.—The middle object is nearer than the other two; the position can be determined very well, but *A* should not be so close as to make angles too small, small angles making position uncertain.

CASE 5.—Using three objects in line or nearly so, as in the case of objects *B*, *A′*, and *C*. An excellent arrangement; the larger the angles, the more reliable the "fix."

CASE 6.—Where the sum of the observed angles is the supplement of *BAC*; the position is indeterminate as it may be anywhere on the great circle.

CHAPTER III.

NAVIGATIONAL INSTRUMENTS.

The Compass and Pelorus.—Compass Error.—Theory of Deviations.

SECTION I.

35. The mariner's compass is one of his most important and essential instruments, showing him how he is steering, enabling him to direct his ship on a desired course, or to get bearings of objects in sight from which to determine his position.

It consists essentially of a needle, or a series of needles, of strong and powerful magnetism, attached to a properly graduated card ~~which is mounted~~ at its center on a pivot in the center of the compass bowl, and has free movement in the horizontal plane. The bowl is made of copper, hemispherical in shape, is heavy as well as ballasted, and swings on knife edges in gimbals, thus enabling the card to maintain a horizontal position even in a seaway.

Inside the bowl are painted two vertical black lines 180° apart, the one towards the head of ship being called the lubber's line. The bowl is so mounted that a line through the pivot and the lubber's line is parallel to the keel line of the ship, so that this lubber's line indicates the **course, or the direction of the ship's head per compass.**

The compass card is divided into 360°; the graduation beginning with 0° at North runs around to the right and is numbered at every fifth degree. The card is also divided into points and quarter points (see Appendix E).

3950
3963

The two general classes.—There are two general classes of magnetic compasses in use, the dry and the liquid. In the latter, the bowl is filled with liquid which, together with the hollow card, gives a certain amount of buoyancy to the card and hence regulates its pressure on the pivot and ease of movement, and also, through its inertia, tends to prevent or reduce vibrations due to the ship's motion.

Dry & liquid.

The liquid compass is used in the U. S. Navy, and, according to the purposes it serves, a compass is designated as service, conning tower, or boat compass. Though made on the same general principles, they embody different degrees of excellence and have cards of different sizes. The service compass has a card 7½ inches, conning tower 5 inches, boat compass 4 inches in diameter.*

S 7½" C
C 5" C
B 4" C

The service compass is further designated, according to its use and location on board, as standard, steering, manœuvring, battle, auxiliary battle, top and check compass.

Location of standard.—The standard is the compass by which the ship should be navigated, all others being regarded as auxiliaries, as for the use of helmsmen, etc.

It should be placed in the midship line of the ship, at a position where the mean directive force is a maximum, if possible; as far removed as practicable from considerable masses of iron, especially if vertical, the influences of the dynamos or electrical currents, stands of arms, or other iron or steel subject to occasional removal. It should be mounted at least five feet from an iron deck or beams, in a compensating binnacle, easily accessible at all times, conveniently near the steering compass, and so located that all around bearings of land or heavenly bodies can be observed.

1 Stan[illegible]
[illegible]

36. The service or 7½″ liquid compass.—This compass consists of a tinned brass skeleton card 7½ inches in diameter. It is of a curved annular type, the outer ring convex on the upper and inner side, graduated to read to quarter points,

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* The compasses in submarines are of special types, usually furnished by the contractors to suit the special conditions. As a rule they are transparent and set in the deck so as to be read either from inside or outside of the boat, reflecting prisms and lenses being used where necessary.

V

with the outer edge divided to half degrees, and figured at each fifth degree from 0° at North, numbering to the right through 360°. The card has a concentric spheroidal air vessel, to assist in giving buoyancy to the card and magnets, so that the pressure on the pivot at 60° F. will vary between 60 and 90 grains. The air vessel has a hollow cone, open at the lower end, carrying a sapphire cap at the apex, by which the card is supported on the pivot.

Magnets

The magnets, four in number, consist of cylindrical bundles of steel wires, each .06 of an inch in diameter, strongly magnetized, put into a sealed cylindrical case and secured to the card parallel to its North and South diameter. The cases of two of the magnets, each magnet 5¼" long, pass through the air vessel to which they are soldered, and have their ends secured to the bottom of the card ring, like ends on chords of nearly 15° passing through their extremities. The other two cases containing magnets, each 4⅜ inches long, are placed parallel to the longer magnets, on chords of nearly 45° of a circle through the extremities, and the ends are secured to the bottom of the card ring.

The card is mounted in a bowl, made of cast bronze, on a bell-metal pivot fastened to the center of the bottom of the bowl by a flanged plate and screws. Through this plate and the bottom of the bowl are two small holes which communicate with a metallic self-adjusting expansion chamber located just beneath the bowl. These holes permit a circulation of liquid between the bowl and expansion chamber, and it is the function of the latter to keep the bowl full of liquid without show of bubbles, or undue pressure that might be caused by change in the volume of the liquid due to changes of temperature.

The liquid used is composed of 45 per cent pure alcohol and 55 per cent distilled water, and remains liquid at a temperature lower than —10° F. The inside of the bowl is painted white with a paint insoluble in the above liquid. An enameled plate is secured on the inside of the bowl, and on this plate a lubber's line is drawn.

The bowl is fitted with a glass cover, the edge of which is closely packed with rubber, completely preventing leakage or evaporation of the liquid, which at all times fills the bowl. The rim of the compass bowl is made rigid and its outer and upper edges turned accurately, that the service azimuth circle when in use may be properly seated. The bowl has a false bottom containing a leaden weight as ballast to keep the bowl horizontal.

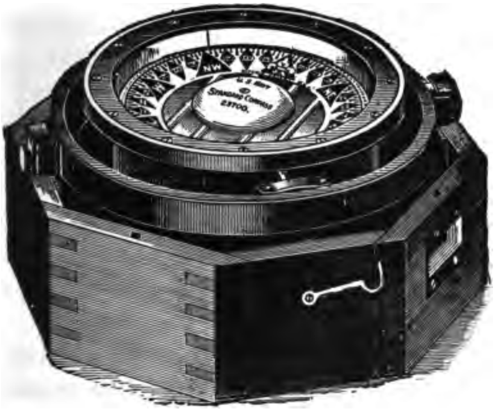

FIG. 14.—United States Navy Standard Compass.
As made by E. S. Ritchie & Sons.

The compass is mounted in gimbals with knife-edge bearings in its binnacle.

In some of the instances of more recent manufacture, the compass card and the bottom of the compass bowl are made transparent, in order that the card may be illuminated by electric light from beneath the bowl.

37. The U. S. Navy standard compensating binnacle.— The binnacle stand includes, in a single brass casting, the circular base, cylindrical pedestal, conical magnet chamber, cylindrical compass chamber, and the graduated arms for the quadrantal correctors at right angles to the keel line of the binnacle (see Fig. 15).

The hood is spun of stout polished brass, has a hinged plate-glass front opening upwards, and a sliding door opposite the glass to permit bearings to be taken in wet weather without removal of the hood. Clips on the binnacle take over the rounded edge of the binnacle hood and hold it on. The hood carries in its center a lamp provided with a prism for reflecting on the compass dial the light of a 5 c. p. electric light. The plug for this light is in the pedestal below.

The rectangular method * of compensation is used in correcting the semicircular deviation. The correcting magnets are mounted on trays which can be raised or lowered, independently of each other, by a screw moved by beveled gears, and so constructed that they will pass each other in any position, the mechanism permitting an extent of travel of 12 inches.

The semicircular magnets are held in their receptacles by a spring-closing device, each carrier or tray having a group of three magnets each side of the vertical axis, making six in all, the tubes being horizontal, one over the other.

The quadrantal correctors are removable soft iron spheres, secured to the brackets by screw bolts, the centers of the spheres lying in the same horizontal plane as the compass needles. They are capable of motion towards or from the compass, the distance from whose center is indicated by a scale in inches and quarter inches on each arm.

The heeling corrector consists of a cylindrical magnet having a hook in each end to which is attached a chain. Centrally, in the vertical axis of the binnacle, there is a hollow brass tube extending the entire depth of stand from the bottom of compass chamber. By removing the compass, the heeling corrector attached to its chain may be lowered into this tube, and held at the proper height by the chain which passes over a roller at the top of the tube and secures to a cleat, or by a set screw in the magnet chamber.

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* By fore and-aft and athwartship magnets (Art. 81, 82.)

7½"

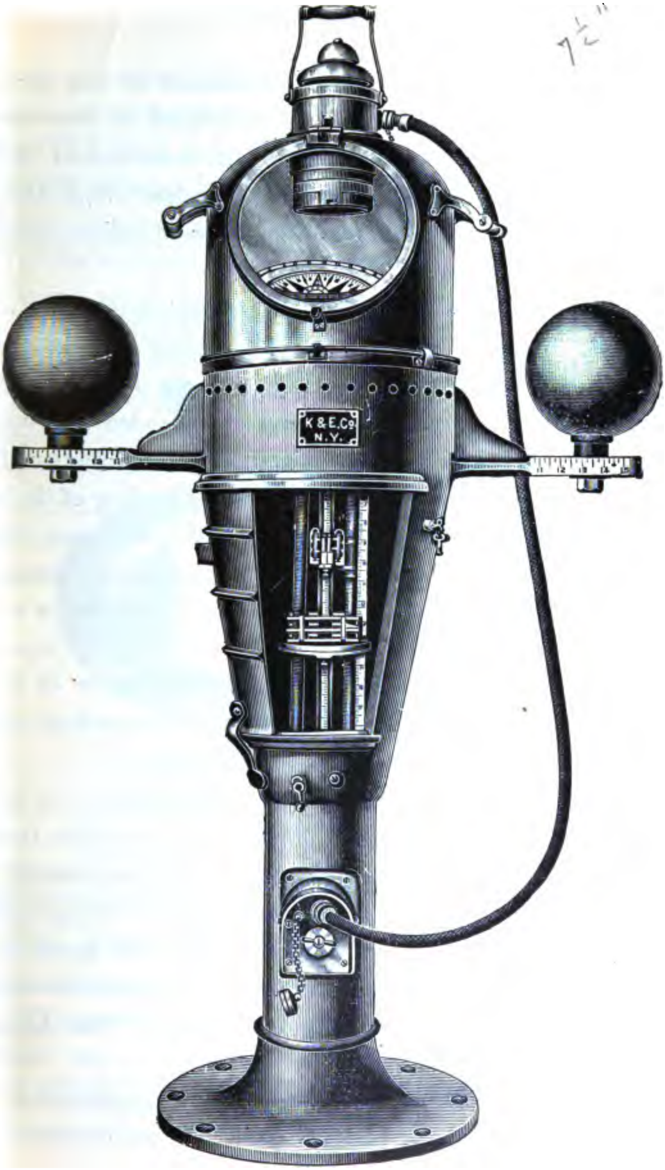


Fig. 15.—U. S. Navy Standard Compensating Binnacle.
As made by Keuffel & Esser Co.

38. The gyro-compass.—The diminution of the directive force of the magnetic compass when inclosed in structures of magnetic materials has necessitated the development, for use in battleships and submarines, of a compass whose directive force is derived from dynamical principles of action instead of magnetic influences.

This compass consists essentially of a rapidly spinning rotor, usually driven by a three-phase alternating current of electricity at a rate varying, according to the type, from 8000 to 21,000 revolutions per minute, and so suspended that it automatically places its axis approximately in the direction of the geographical meridian, and permits of the reading of the heading of the ship, unaffected by any magnetic influence, from a graduated compass card like that in use on magnetic compasses. From the "master compass," which may be located in a compartment below, electrical connections are made to "repeating compasses" on the bridge, in the conning-tower or in the steering engine room, so that the ship's true heading may be transmitted to any desired part of the vessel.

The action of the gyro-compass, affected as it is by the earth's rotation, conforms to Foucault's general law that "a spinning body tends to swing around so as to place its axis parallel to the axis of any impressed forces." Small corrections, depending upon the latitude, course, and speed, can be readily computed for application to the gyro-compass readings either mechanically or by reference to tables. (See Appendix F.)

39. Azimuth circle.—This consists of a composition ring turned true to fit the compass bowl. At one extremity of a diameter is a curved mirror hinged to move around a horizontal axis and facing at the other extremity a prism completely encased in a brass case except for a narrow vertical slit. The sun's rays reflected by the mirror upon the slit appear as a thin pencil of light on the graduations of the card circle. A level on the rim serves to show when it is horizontal.

A second set of vanes, or sometimes a telescope, fitted in the direction of a diameter at right angles to the first, is used for direct bearings of distant objects, of stars, or of the sun when partially obscured. At one end is a mirror reflecting the image of the sun or star, and a prism reflecting the card circle and vane simultaneously to the eye at the other end (Fig. 16).

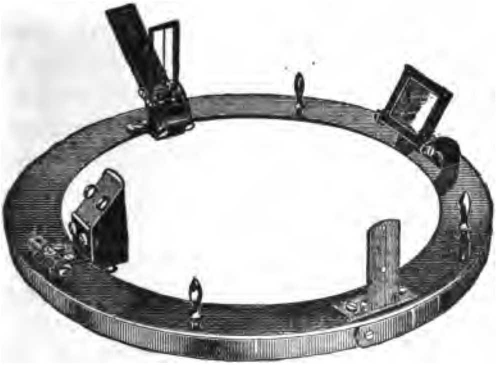


FIG. 16.—U. S. Navy Azimuth Circle.
(To fit Navy Standard Compass No. 1.)
As made by E. S. Ritchie & Sons.

40. The pelorus (Fig. 17).—This is an instrument located on board ship at some point where a clear view can be obtained for taking bearings. It is most convenient to have one at each end of the flying bridge. It consists of a circular plate mounted in gimbals whose knife edges rest in the Y's of a vertical standard rigidly secured in place.

The circular plate has a raised flange on its periphery with heavy marks 90° apart to correspond with the fore-and-aft and athwartship lines of the ship.

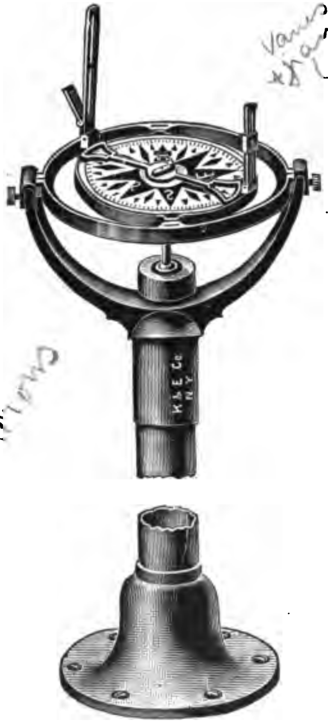


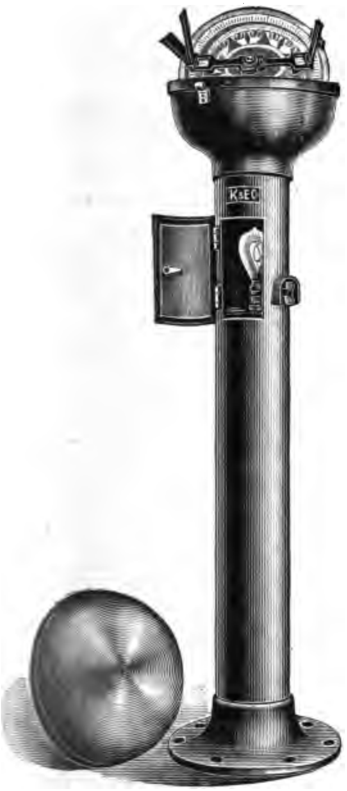
FIG. 17.

Vanes

Pelorus

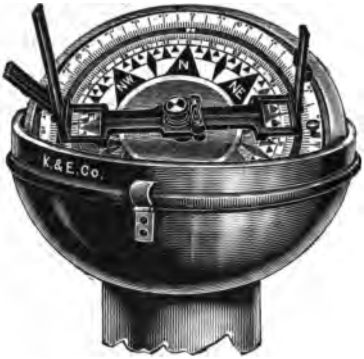
Concentric with the plate and each other are a dial plate and an alidade, each capable of independent movement in azimuth. The dial plate is simply a dumb compass card of metal, graduated to quarter points and single degrees, whose upper surface is flush with the raised periphery, and is provided with a clamp. The alidade is fitted with a level, folding sight vanes, hinged reflector, and a sliding peep sight with neutral glass. The line of sight through the vanes passes through the vertical axis of the instrument and is indicated on the dial plate by an index at each end of the alidade. The alidade is also fitted with a clamp. A heavy balance weight is attached to the lower center of the plate. It may be used to eliminate the compass error from observed bearings by setting the alidade to a reading which is the compass course corrected for the error, and, as long as the ship is on that particular heading for which the dial is set, all bearings by the pelorus will be true; if correction is made for deviation only, then the bearings will be magnetic.

PLATE I.



The Illuminated Dial Pelorus.

PLATE II.



The Illuminated Dial Pelorus-Chamber and Bowl.

√

Column

Base
supporting
chamber

41. Illuminated dial pelorus (Plates I and II).—The pelorus standard consists of three parts, the base, supporting column, and pelorus chamber; on the top edge of the latter are scored the fore-and-aft and athwartship marks. The pelorus bowl is mounted on two trunnions in a gimbal ring which pivots on the athwartship diameter of pelorus chamber. The pelorus bowl is built up of a top bowl ring and a shell of sheet brass. A seat is turned in the top bowl ring to receive the assembled pelorus card, the principal part of which consists of a disk of clear plate glass 9 inches in diameter by $\frac{3}{16}$ of an inch thick. The card is graduated in degrees near its outer edge, every fifth degree accentuated, and every tenth degree is marked in figures. Graduations run from 0° at N. to 360° around to right, and, inside these degree marks, the card is graduated to quarter points. An azimuth circle marked in degrees is permanently secured to the bowl ring. The pelorus card has unobstructed rotation except when clamped by a clamping screw, which is at the 90° graduation of the azimuth circle. There is an alidade capable of free revolution either way, or of being securely clamped; it is provided with a level, folding sights, hinged reflector, and peep sight. This pelorus is designed to be used at night without a navigator's lantern, the transparent dial being illuminated by an electric light placed beneath it in the standard.

42. The use of pelorus to determine a magnetic heading.—To place the ship's head on any magnetic point by the pelorus:

(1) With the known latitude of place and declination of body—say the sun—find from the azimuth tables the sun's true bearing for certain selected local apparent times. From these true bearings find the sun's magnetic bearings for the same times by applying the variation of the locality, easterly variation being applied to the left, westerly variation to the right of the true azimuth.

(2) Shortly before the earliest time selected, set that point

of the pelorus corresponding to the magnetic heading desired on the forward keel line or indicator and clamp the plate; set the sight vanes to correspond with the sun's magnetic bearing at the selected time and clamp the vanes to the plate.

(3) By the use of engines and helm, bring the sight vanes on the sun and keep them there, being careful not to disturb the clamps of plate or vanes, noting at the instant of the selected local apparent time the heading per compass. That heading per compass corresponds to the magnetic direction desired.

SECTION II.

43. Compass corrections.—The compass needle seldom points to the true North, so in order to obtain a true course or a true bearing, certain corrections must be applied to the compass course or the compass bearing, as the case may be. They consist, according to circumstances, of one or more of the following; *i. e.,* variation, deviation, leeway. Each of these terms will be explained at the proper time.

44. The earth's magnetism.—The earth acts like a huge, natural but irregular magnet, having in each hemisphere a resultant pole which, however, is not coincident with the geographical pole, and a magnetic equator which crosses and recrosses the geographical equator. The North magnetic pole is approximately in Lat. 70° N, Long. 96¾° W, the South magnetic pole in Lat. 73½° S, Long. 147½° E. Recognizing the laws of attraction and repulsion between two bar magnets, and the analogy that exists between the magnetic character of the earth and a bar magnet, it is evident that the magnetism of the North magnetic pole is of an opposite kind to that of the North-seeking end of a magnetized needle; therefore, if we regard the magnetism of the North seeking end of the needle as North magnetism, we must consider the North magnetic pole as having South polarity and the South magnetic pole as having North polarity.

However, physicists do not agree as to which shall be called North magnetism, that of the North-seeking end of the needle or that of the North magnetic pole, so it is convenient to distinguish them by colors, calling the first red and the second blue.

The general effect of the earth's magnetism is to draw the North end of the needle towards the North and the South end towards the South; but, with the exceptions noted further on, a freely suspended magnetized needle, affected only by terrestrial influences, generally speaking, neither points in the direction of the true meridian, nor lies in a horizontal plane, nor occupies the same relative position in two different places.

Line of force and dip.—The direction in which the needle does point at any given place is the "line of total magnetic force" at that place, the inclination of which below the horizontal plane is called the "magnetic dip." The line of total force is spoken of as the "line of force."

The magnetic poles.—Those two positions at which the line of total force is vertical are known as the magnetic poles; a freely suspended magnetized needle would be vertical at the poles, with the North end down at the North magnetic pole and the South end down at the South magnetic pole.

The magnetic equator.—The line joining all those positions on the earth's surface at which the "line of force" is horizontal is known as the magnetic equator, which is to the northward of the geographical equator in the Indian Ocean and the western half of the Pacific Ocean, and crosses to the southward of it in the Atlantic and eastern half of the Pacific Ocean.

As we have magnetic poles and a magnetic equator, analogously we have magnetic latitude; all points of the earth in North magnetic latitude have South or blue magnetism, and all points in South magnetic latitude have North or red magnetism.

The dip increases from 0° at the magnetic equator to 90° at the magnetic poles; the "total force" increases from a minimum at the magnetic equator to a maximum at the "magnetic foci," of which there are two in each hemisphere, located about in 52° N., 92° W.; and 70° N., 120° E.; 70° S., 145° E.; and 50° S., 130° E. The total force at the foci is between two and three times that at the magnetic equator.

The magnetic meridian.—The magnetic meridian of any place is that great vertical circle in the plane of which the "line of force" lies. The so-called "magnetic meridians" are the lines along which one would travel were he to set out at any place on the earth and always follow the direction of the compass needle, and hence they exhibit at every point the actual direction of the compass needle, not by numbers, but by angles.

The variation.—Excepting at points along three lines, called "lines of no variation," one at present passing through Brazil and the eastern part of the United States, a second through Australia, the Arabian Sea, and the Black Sea, and a third passing in an elliptical course over eastern China and Siberia and the northern part of the Pacific Ocean, the magnetic meridian nowhere corresponds with the true meridian, but inclines to the East or West of it, making with it at a given place an angle called the "variation" at that place. As a compass needle is constrained by its mode of suspension to move only in a horizontal plane, variation may be defined as the angle through which the compass needle is deflected from true North by terrestrial magnetism alone.

45. Elements of the earth's magnetism.—The distribution of the earth's magnetism at any place may be indicated by its three elements:

(a) The variation.
(b) The dip.
(c) The total force or magnetic intensity.

(a) and (c) are found by means of the magnetometer and (b) by the dip circle, but for the purpose of representing the amount and direction of the earth's force on the needle, instead of considering the total force on the needle, it is more convenient to consider the components of that force, viz.:

(1) The horizontal force, or that component in the direction of a tangent to the earth's surface, and in the plane of the magnetic meridian.

✓

(2) The vertical force, or that component acting downwards and at right angles to the above.

Relation of dip and the forces of the earth's magnetism.—

Letting $\theta =$ the magnetic dip,
$T =$ the total force,
$H =$ the horizontal force,
$Z =$ the vertical force, we shall have (Fig. 18),

$$\left.\begin{aligned} H &= T\cos\theta. \\ Z &= T\sin\theta. \\ Z &= H\tan\theta. \\ T &= \sqrt{H^2+Z^2}. \end{aligned}\right\} \qquad (15)$$

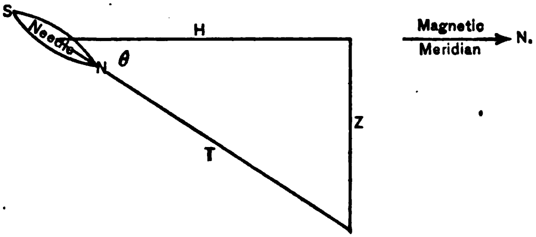


FIG. 18.

46. Charts of the earth's magnetic elements.—The U. S. Hydrographic Office (also corresponding offices in foreign countries) issue variation charts, charts of magnetic dip and also of the horizontal intensity of the earth's magnetism. The tide tables, issued annually by the U. S. C. and G. Survey, give the variation at most of the world's seaports.

The variation chart.—This shows by lines of equal value, called isogonic lines, drawn at convenient intervals, the amount and direction of the variation over the surface of the globe. Generally speaking the variation is westerly over the Atlantic Ocean, the Mediterranean, and the Indian Ocean excepting the Bay of Bengal; easterly over the Bay of Bengal, the Pacific Ocean, the Gulf of Mexico, and the Caribbean Sea.

The chart of magnetic dip.—This shows by lines of equal value, called isoclinal lines, drawn at intervals of one degree, the magnetic dip over the surface of the globe. In the regions of northerly dip, where the North end of the needle is drawn downward, these lines are full; in the regions of southerly dip, where the South end of the needle is drawn downwards, they are broken lines. The annual rates of change of dip are expressed in minutes of arc by numbers in the regions where they are placed; a plus sign indicating increasing, a minus sign decreasing dip.

Taking the horizontal force at the magnetic equator as unity, the increase of dip with magnetic latitude, as shown by this chart, is approximately in accordance with the formula $\tan \text{dip} = 2 \tan \text{mag. Lat.}$

The chart of horizontal force.—This shows the horizontal intensity expressed in C. G. S. units by lines of equal value.

An inspection of this chart will show that the horizontal force is a maximum near the magnetic equator and diminishes as we approach the magnetic poles where it is zero. Since the horizontal intensity is the directive force on the needle, it is plain that a disturbing influence would have greater effect on the needle when the value of H is less and vice versa; and, therefore, at places in high magnetic latitudes, the needle is less reliable than at places near the magnetic equator, when subjected to the same influences antagonistic to that of the earth.

As a knowledge of the value of H in the locality of the ship is often necessary in compass work, the navigator will find this chart a useful one. With the value of H from this chart and the value of θ from the chart of magnetic dip, the value of Z, the vertical component of the earth's total force at a place, may be found. Charts of Z may also be used.

47. Relation of true and magnetic meridians.—From what has preceded, it is plain that a compass needle, constrained by mechanical arrangements to move in a horizontal plane and installed on board a perfectly stable and non-magnetic ship, will lie in the magnetic meridian, at an angle called the variation with the true meridian, to the eastward or westward of it, depending on the geographical position, and will possess a directive force depending on the magnetic latitude. The variation is not only different in different localities, except at those places on the same isogonic line, but it is different in the same locality at different times, owing to a small but gradual and continual change in the earth's magnetic state.

The "line of no variation," which now passes through the Arabian and Black seas, was a little to the westward of the Azores in 1492, and it is recorded that Columbus, on his westward voyage, noted the change in the compass bearing of the pole star; the needle at first pointing to the eastward of the pole star, then directly at it, and finally to the westward of it as the voyage progressed.

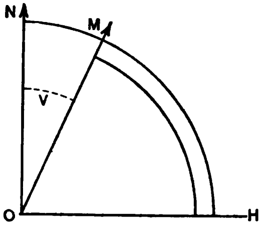


FIG. 19.

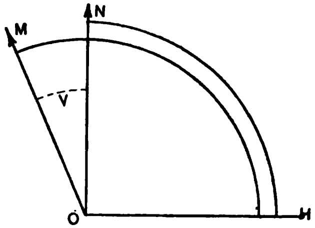


FIG. 20.

Besides the regular and periodic annual change, the needle is subject to a slight diurnal change, moving gradually back and forth through a very small arc.

Variation is shown on all navigational charts at the compass rose and also by isogonic lines.

All magnetic courses and bearings are estimated from the magnetic meridian, but it is often desirable and necessary to find the corresponding angles from the true meridian, which may be found by applying the variation of the place to the magnetic course or bearing in the proper way.

In Figs. 19 and 20, let ON be the true meridian; OM the magnetic meridian; $V = NOM$ the variation of the place; OH the keel line or direction of the ship's head; then NOH is the true course and MOH the magnetic course, considered positive to the right. In Fig. 19 the North end of the needle is drawn to the eastward and

The true course = the magnetic course + variation.

In Fig. 20 the North end of the needle is drawn to the westward and

The true course = the magnetic course — variation.

48. Rule for naming variation.—Mark it East (E) or +, if the North end of the needle is drawn to the right, the observer considered as at the center of the compass and looking in the direction of that end of the needle; mark it West (W) or —, if the North end of the needle is drawn to the left, observer as before.

When the North point of the needle is drawn to the right, the magnetic meridian is to the right of the true meridian, and the magnetic bearing of a fixed object is to the left of its true bearing by the amount of the variation; similarly when the North point of the needle is drawn to the left, the magnetic meridian is to the left of the true meridian, and the magnetic bearing of a fixed object is to the right of its true bearing by the amount of the variation.

Rule for applying variation.—Hence, when applying variation to magnetic courses or bearings to obtain the corresponding true courses or bearings, looking from the center of the compass toward the compass rhumb, apply variation to the right when E. or +; to the left when W. or —. Or, if the magnetic course or bearing is in degrees, add the variation if E. or +, subtract it if W. or —. To find the magnetic courses (or bearings) from the true courses (or bearings) do the reverse.

EXAMPLES, VARIATION EAST.

49. Given the following magnetic courses, to find the corresponding true courses:

Magnetic Course	**85°**	**137°**	**230°**	**353°**
Variation	**+15**	**+15**	**+15**	**+15**
True Course	**100°**	**152°**	**245°**	**8°**

EXAMPLES, VARIATION WEST.

Given magnetic courses, to find the true courses:

Magnetic Course	**85°**	**137°**	**230°**	**3°**
Variation	**—15**	**—15**	**—15**	**—15**
True Course	**70°**	**122°**	**215°**	**348°**

50. Local attraction.—There is cause of disturbance of the compass needle, when the ship is in certain localities, due to the fact that the mineral substances in the land under the water possess magnetic properties, especially in shallow waters of volcanic regions. Well-authenticated observations show that the navigator must be on his guard against the dangers of this attraction in many parts of the world, and that prominent instances of its occurrence are experienced on Lake Superior and Lake Ontario, in southeastern Alaska, on the coasts of Iceland, off Cape St. Francis, in Odessa Bay, off the coast of Madagascar, off the volcanic islands near Java, at the Isles de Los, and especially near Cossack in North Australia.

51. Deviation.—So far we have considered the compass as if installed on board an absolutely non-magnetic ship, and as affected only by terrestrial magnetic influences, with variation as its only correction or error. However, when that same compass is mounted on board an iron or steel ship, it is subject to further error in its indications. Besides having a directive force in the magnetic meridian given it by *H*, the earth's horizontal intensity, the North end of the needle is acted upon by the general magnetism induced in the iron or steel of the ship by the earth's inducing forces, with the result that the needle assumes a resultant direction, the angle between which and the magnetic meridian is called "deviation."

When the deviation is zero, the ship's force acts in the magnetic meridian, increasing or diminishing the earth's directive force.

The deviation of a compass varies with the ship in which it is mounted; varies according to position in the same ship; and for a given position, under like circumstances, varies in amount and direction according to the heading of the ship. It also varies with change of ship's position on the earth's surface. For these reasons the deviations of every compass, mounted on board and used for navigating or steering the ship, should be determined for every 15° compass rhumb at a time when the vessel is on an even keel, in her normal seagoing condition, with projectiles, guns, davits, cranes, removable masses of iron, etc., secured as if for sea.

The deviations of the standard compass, which alone must be used for navigating the ship, should be tabulated and a corrected copy of the table should be kept on deck for the use of the officer of the deck and the navigator. Such a table is needed for finding the magnetic course from the compass course steered; for correcting the compass bearings of fixed objects on shore; or for obtaining the compass course to be steered to make good a certain magnetic course.

52. Rule for naming deviation.—Mark it East (E) or + if, under the influence of the ship's magnetism, the North end of the needle is drawn to the eastward, or to the right of the magnetic meridian; mark it West (W) or — if the North end of the needle is drawn to the westward, or to the left of the magnetic meridian.

When the North point of the needle is drawn to the right, the observer at the center and looking in the direction of that point, the compass meridian is to the right of the magnetic meridian and the compass bearing of a fixed object is to the left of its magnetic bearing by the amount of the deviation; similarly when the North point of the needle is drawn to the left, the compass meridian is to the left of the magnetic meridian, and the compass bearing of a fixed object is to the right of its magnetic bearing by the amount of the deviation.

Rule for applying deviation.—When applying deviation to compass courses, or bearings, to obtain magnetic courses, or bearings, looking as if from the center of the compass toward the compass rhumb, apply the deviation, due to the ship's heading at the time, to the right when E. or +; to the left when W. or —. Or, if the compass course or bearing is in degrees, add the deviation if E. or +, subtract it if W. or —. To find the compass course from a given magnetic course do the reverse.

In this connection, attention is particularly called to the fact that **all bearings** are to be corrected for the deviation due **to the direction of the ship's head at the moment they were taken.**

53. Compass error.—When variation and deviation are to be allowed for at one time, add them algebraically, giving the name of the greater to the result which is known as "compass error," generally written C. E. To obtain the true course or bearing from a given compass course or bearing, apply the C. E. to right if E. or +; to the left if W. or —, looking from the center of the compass toward the compass rhumb. Or, if the compass course or bearing is in degrees, add the compass error if E. or +, subtract it if W. or —. To obtain a compass course from a given true course do the reverse.

For office work and in examples similar to 4, 5, 6, and 8 the signs + and — are preferable to the terms E. and W. The use of the latter, however, will be illustrated in examples 1, 2, 3, and 7.

Example 1.—Given Var. = 13° W, Dev. on North (p. c.)

2° E, on NE (p. c.) 1° W, and on East (p. c.) 4° W; find the true courses corresponding to the above compass courses.

Var. 13° W	**Course (p. c.)** **North**	**Var. 13° W**	**Course (p. c.)** **45°**	**Var. 13° W**	**Course (p. c).** **90°**
Dev. 2° E	**C. E. 11° W**	**Dev. 1° W**	**C. E. 14° W**	**Dev. 4° W**	**C. E. 17° W**
C. E. 11° W		**C. E. 14° W**		**C. E. 17° W**	
True Course	**349°**	**True Course**	**31°**	**True Course**	**73°**

54. Leeway.—With sailing ships, the wind, besides driving the ship in the direction of her keel, frequently forces her bodily to leeward, so that the course through the water is to leeward of the one steered.

This angle between the course and the direction the ship is actually moving, as indicated by the ship's wake, is the leeway. Being always from the wind, as a correction it is marked East when the ship is on the port tack, West when the ship is on the starboard tack. Here also, East is + and West is —.

Given a course (p. c.), to find the true course.—

Ex. 2.—A schooner sails 28° (p. s. c.), Dev. 6° E, Var. from chart 21° E, wind SE, leeway 11°. Find the true course.

Var.	21° E.	Course (p. c.)	28°
Dev.	6 E.	Correction	16 E.
Leeway	11 W.	True course	44°
Correction	16° E.		

Given the true course, to find the compass course.—

Ex. 3.—The true course to destination from the ship's position is 22° 30′, Var. 15° W, Dev. 6° E. The ship will be on the port tack, probable leeway 6°. Find the course to be steered.

Var.	15° W.	True course	22° 30′
Dev.	6 E.	Reversed correction	3 E.
Leeway	6 E.	Compass course	25° 30′
Correction	3° W.		

The correction in this example being 3° W is applied the reverse way, or, as if it is easterly.

The word correction is used here because leeway is not an error of the compass. Strictly speaking variation is not an error and cannot be compensated for; deviation only is an error.

SECTION III.

55. Finding the deviation.—For reasons that are now apparent it is essential that a table of deviations should be obtained for all compasses mounted on board as soon as possible after a vessel is commissioned, that the table for the standard compass should be checked from day to day, and a new one made out after any marked change of magnetic latitude.

For a new vessel built of iron or steel, observations should be made on the 24 equidistant 15° rhumbs before compensation; after compensation the residual deviations may be found by observing on 12 equidistant headings, though in both cases, if possible to do so, it would be better to swing with both helms and to take the mean of the two deviations on each heading as the correct deviation for that heading.

As the ship is steadied on each heading and observations for deviation are made at the standard, the ship's head should be noted by observers at the steering and pilot-house compasses; then from the headings and deviations of the standard, the magnetic heading of the ship at each observation may be found. A comparison of each magnetic heading with the corresponding heading by each of the compasses will give the deviation for the heading of the compass compared. Before an observation is taken on any heading, the ship should be steadied on it for three or four minutes, in order that the needle may be at rest and under magnetic influences normal for that heading at the time of observation.

The ship itself should be steady, or its motion a minimum, when the observer takes his observations.

The deviation may be obtained by any one of four methods:

(1) By reciprocal bearings;

(2) By bearings of a distant object;

(3) By azimuths or amplitudes of a celestial body;

(4) By ranges of known magnetic bearing; or, by two or more of the above combined.

56. (1) By reciprocal bearings.—This method is available when the ship is in a basin or a smooth harbor, and the compasses are free from all disturbing influences except the ship's own magnetism and that of the earth; and when there is a suitable position on shore for mounting a compass where there are no local magnetic influences, above or below ground, to disturb its readings.

A careful observer is sent ashore with a spare compass on a tripod which is placed where it can be seen distinctly from the ship with the naked eye, in a spot absolutely free from all local magnetism.

The requisite warps having been prepared, the ship is swung around so as to bring her head, per standard compass, upon each heading on which observations for deviation are to be taken; of course, if circumstances permit, it is advisable to observe on each of the 24 equidistant 15° rhumbs.

Then, by means of prearranged signals, the mutual bearings of this shore compass and the standard compass on board are observed at the moment when the ship's head is steady, and has been steady at least three minutes, on each of the required compass headings. To guard against mistakes, the time of each bearing should be observed, both on board and ashore, by compared watches; and it is advisable for the shore observer to mark the time and bearing of the standard from the shore compass at each observation on a blackboard provided for the purpose, so that in case of an apparent inconsistency, the observations can be immediately repeated and the necessity obviated for again swinging the ship.

NOTE.—Whenever bearings are taken with the azimuth circle, it should be horizontal with the bubble of the level centered. Celestial bodies should be observed for deviation when on or near the P. V. and at a low altitude (see Art. 222).

The bearing of the standard from the shore compass at a given instant, reversed, is the correct magnetic bearing of the shore compass from the standard at that instant, and the difference between this magnetic bearing and the bearing taken at the standard on board at the same time will be the deviation due to the particular heading of the ship at the moment of observation.

This deviation is marked according to the rule given in Article 52.

The results of the swinging are recorded as in the form used in the following example solved on page 69.

Ex. 4.—Having swung a Monitor for deviations of the standard and battle compasses by method of reciprocal bearings, find the deviations of standard on 24 rhumbs, and of battle compass for the magnetic headings. Data as in form.

57. (2) **By bearings of a distant terrestrial object.**—This method is convenient when the ship is at anchor in a harbor, or roadstead, with the object so far distant that the magnetic bearing will not alter sensibly as the ship heads on the various headings—say about eight to ten miles for a ship swinging, anchored at short stay. This method may be used at sea, the ship steaming around an entire circle, provided the object is so far distant that the parallax does not exceed 30′, the parallax being the angle whose tangent equals the radius of the circle in which the ship is swinging divided by the mean distance of the object. At sea, even under the most favorable conditions, it involves more or less error; and, if the ship is in the locality of tides or currents, this method should not be used with the ship underway.

By this method, a distant but distinctly visible object, as a clearly defined point of a distant peak, a light-house, or other mark, is observed as the ship at anchor swings slowly to tide, is steamed around, or swung at her moorings, but steadied sufficiently long on each heading to allow the magnetism of the ship to settle down.

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SOLUTION OF EX. 4, PAGE 68, AND FORM FOR WORK.

Times.	Ship's Head by Standard.	Simultaneous Bearings.		Deviation of Standard Compass.	Ship's Head Magnetic.	Ship's Head by Battle Compass.	Deviation of Battle Compass.
		Of Shore Compass from Standard Compass.	Of Standard Compass from Shore Compass.				
2h 31m 05s	0°	217° 30′	31° 15′	— 6° 15′	353° 45′	359° 00′	— 5° 15′
2 35 20	15	207 00	26 10	— 0 50	14 10	6 50	+ 7 20
2 40 30	30	200 00	23 50	+ 3 50	33 50	15 10	+18 40
2 45 00	45	193 20	21 45	+ 8 25	53 25	24 30	+28 55
2 51 30	60	188 50	18 40	+ 9 50	69 50	36 00	+33 50
2 56 00	75	186 30	17 50	+11 20	86 20	49 20	+37 00
3 01 10	90	185 40	17 40	+12 00	102 00	68 00	+34 00
3 06 20	105	185 40	17 00	+11 20	116 20	90 00	+26 20
3 10 20	120	187 10	17 30	+10 20	130 20	112 30	+17 50
3 15 30	135	189 15	18 10	+ 8 55	143 55	131 45	+12 10
3 20 05	150	191 10	18 40	+ 7 30	157 30	147 20	+10 10
3 24 15	165	193 00	19 15	+ 6 15	171 15	162 00	+ 9 15
3 30 10	180	196 05	20 40	+ 4 35	184 35	177 00	+ 7 35
3 35 00	195	199 20	21 50	+ 2 30	197 30	193 00	+ 4 30
3 40 30	210	203 10	23 10	0 00	210 00	206 15	+ 3 45
3 45 40	225	207 45	24 20	— 3 25	221 35	226 00	— 4 25
3 51 00	240	212 50	26 20	— 6 30	233 30	252 15	—18 45
3 57 10	255	218 40	29 20	— 9 20	245 40	281 40	—36 00
4 02 30	270	222 10	29 50	—12 20	257 40	299 30	—41 50
4 08 00	285	224 40	30 20	—14 20	270 40	315 00	—44 20
4 12 10	300	228 40	32 40	—16 00	284 00	326 00	—42 00
4 16 15	315	229 20	32 45	—16 35	298 25	335 00	—36 35
4 21 10	330	227 40	34 10	—13 30	316 30	344 15	—27 45
4 25 50	345	223 45	33 20	—10 25	334 35	352 00	—17 25

The difference between these bearings and the magnetic bearing of the distant object, or, in other words, the compass bearing it would have from on board if the compass were not disturbed by the attraction of the iron of the ship, will be the deviation of the compass for the headings of the ship at the time the bearings were taken, marked as per rule in Article 52.

The magnetic bearing of the distant object may be found:

(a) From a chart. if it is one of a recent ~~and reliable~~ survey.

(b) From the mean of four or more compass bearings on equidistant compass headings.

(c) From a true or astronomical bearing, applying the known variation of the place.

Ex. 5.—As a ship swung to tide, at anchor, the following bearings were taken of the sharp peak of a distant mountain by the standard compass. Required the deviations on the headings indicated.

Head (p. s. c.).	Bearings of object (p. s. c.).	Deviations of Standard.	Head (p. s. c.).	Bearings of object (p. s. c.).	Deviations of Standard.
0°	222°	+ 3° 22½′	180°	227°	− 1° 37½′
45	221	+ 4 22½	225	230	− 4 37½
90	219	+ 6 22½	270	232	− 6 37½
135	223	+ 2 22½	315	229	− 3 37½

The correct magnetic bearing, taken as the mean of all eight bearings (p. c.), } = 225° 22½′.

58. (3) **By bearings of a celestial body.**—This method may be used whenever either of the previous methods is available, provided the body observed is not too high in altitude. It is particularly important as it is the only one available at sea.

In this method the ship is steadied for the required time on each heading per standard compass. By this compass the

NOTE.—The magnetic bearing as obtained above under (*b*) will include any value of the constant deviation *A* that may exist for the particular compass and location.

bearing of the celestial body is observed when the ship is steady on each heading, at the same moment the headings by the steering and other compasses are read and the time noted by a chronometer, or a watch compared with a chronometer whose error on G. M. T. is known. From the longitude and times the hour angles are found; then with the latitude, the body's declination, and the hour angle at the time a compass bearing was taken, the corresponding true bearing may be found in the azimuth tables. The difference between the true and the compass bearing of the body at the same instant is the compass error from which the deviation may be found by applying the variation from the chart. If the compass errors are obtained on equidistant headings, and the iron about the compass is symmetrical, the mean of the compass errors will give the variation. If the sun is the body observed, the true azimuth will be found in the azimuth tables; using latitude, declination, and local apparent time as arguments.

Ex. 6.—April 7, 1918, a U. S. torpedo-boat destroyer was swung for deviations of the standard compass, all correctors having been removed. Data as given in the form for a Compass Report on page 73. L 39° N., λ 76° 24′ W.

The watch time of observation and the compass bearing of the sun are recorded, respectively, in columns 2 and 5, opposite the proper heading per compass.

The error of the watch on L. A. T. is found from the data under caption "Local apparent time by chronometer" and is entered in column 3. This applied to the watch times in column 2 will give the corresponding local apparent times of observation recorded in column 4.

The work of finding the sun's true bearing from the azimuth tables may be facilitated, without loss of material accuracy, by computing a constant for the minutes of declination and latitude corresponding to the approximate middle instant of observation; the declination used being taken from the Nautical Almanac for the Greenwich time of the middle instant. The mean declination thus found in this example is 6° 39′+N=6°.65 N and the middle L. A. T. is $8^h\ 44^m\ 10^s$. The constant is thus found for minutes of latitude and declination, taking the L. A. T. equal to $8^h\ 40^m$.

$$\left.\begin{array}{r}\text{Lat. }39^\circ\text{ N}\\ \text{Dec. }6^\circ\text{ N}\\ \text{L. A. T. }8^h\ 40^m\end{array}\right\}\begin{array}{l}\text{on page 90 of azimuth tables.}\\ Z=\text{N }112^\circ\ 51'\text{ E}\end{array}$$

$$\text{For Dec. }7^\circ\text{ N}\qquad Z=\text{N }111\ \ 59\ \ \text{E}$$

Diff. for 1° of Dec. = (—)52′

Diff. for 0°.65 of Dec. = (—)34

For minutes of Dec. (—)34′

For minutes of Lat. = 0

Constant ....(—)34′

$$\left.\begin{array}{r}\text{Lat. }40^\circ\text{ N}\\ \text{Dec. }6^\circ\text{ N}\\ \text{L. A. T. }8^h\ 40^m\end{array}\right\}\begin{array}{l}\text{on page 92.}\\ Z=\text{N }113^\circ\ 29'\text{ E}\end{array}$$

$$\text{For Lat. }39^\circ\text{ N}\qquad Z=\text{N }112\ \ 51\ \ \text{E}$$

Diff. for 1° Lat. =(+)0 38′

Diff. for 0°.0 Lat. =0′

Then, using page 90 of the Azimuth Tables, with Lat. 39° N and Dec. 6° N, find the true azimuth interpolated for the local apparent time of column 4; apply the constant and enter result in column 6.

The difference between columns 5 and 6, marked in accordance with the rules of Article 53, will be the compass error on the heading opposite in column 1, and will be entered in column 7.

The difference between the mean of equidistant azimuths in columns 5 and 6 is the variation + constant *A*, as shown in column 8. The mean of the compass errors on equidistant headings should give the same result, variation + constant *A*, if no mistakes have been made. If accurately known, the value of *A* should be applied to the above-mentioned difference to obtain the variation by observation.

The algebraic difference between columns 7 and 8, that is the compass error and the variation, is the deviation to be

NOTE.—Before being entered in column 6, the true azimuth by tables should be expressed, like the compass azimuth, in the form of Z_λ which is the azimuth measured from North, around to the right, from 0° to 360°.

SOLUTION OF EX. 6, PAGE 71, AND FORM FOR COMPASS REPORT.

1	2	3	4	5	6	7	8	9
Ship's head by Standard Compass.	Times of observation by watch.	Watch correction on l. ap. t.	Local apparent times of observation.	Azimuth of sun by Standard Compass. No. 7203.	True azimuth of sun by Tables.	Errors of Compass. No. 7203.	Variation.	Deviations of Compass. No. 7203.
°	h m s		h m s	° ′	° ′	° ′		° ′
0	7 43 00	Error of watch on local apparent time, 3 m. 20 s. To be added to watch time of observation.	7 46 20	112 00	102 02	− 9 58	Mean of 24 equidistant azimuths, Col. 5, = 120° 32′ Mean of the corresponding azimuths of Col. 6, = 113 46 Variation by observation + Constant A, = −6 46 Constant A (apply when found accurately), = — 18 Variation by observation, = −6 28 (To be applied to Col. 7.)	− 3 30
15	7 47 25		7 50 45	113 10	102 49	−10 21		− 3 53
30	7 51 55		7 55 15	114 30	103 38	−10 52		− 4 24
45	7 57 00		8 00 20	116 30	104 33	−11 57		− 5 29
60	8 01 50		8 05 10	120 10	105 26	−14 44		− 8 16
75	8 06 42		8 10 02	124 00	106 20	−17 40		−11 12
90	8 11 46		8 15 06	128 20	107 19	−21 01		−14 33
105	8 17 10		8 20 30	131 20	108 21	−22 59		−16 31
120	8 23 10		8 26 30	133 20	109 32	−23 48		−17 20
135	8 28 42		8 32 02	133 10	110 38	−22 33		−16 04
150	8 33 41		8 37 01	128 40	111 39	−17 01		−10 33
165	8 39 10		8 42 30	123 30	112 48	−10 42		− 4 14
180	8 45 10		8 48 30	117 30	114 04	− 3 26		+ 3 02
195	8 50 42		8 54 02	113 15	115 17	+ 2 02		+ 8 30
210	8 56 41		9 00 01	109 00	116 37	+ 7 37		+14 05
225	9 00 46		9 04 06	103 30	117 33	+14 03		+20 31
240	9 04 47		9 08 07	106 30	118 28	+11 58		+18 26
255	9 08 46		9 12 06	109 40	119 25	+ 9 45		+16 13
270	9 13 47		9 17 07	113 10	120 38	+ 7 28		+13 56
285	9 19 50		9 23 10	118 40	122 09	+ 3 29		+ 9 57
300	9 24 40		9 28 00	124 50	123 23	− 1 27		+ 5 01
315	9 29 45		9 33 05	129 00	124 42	− 4 18		+ 2 10
330	9 34 15		9 37 35	133 10	125 54	− 7 16		− 0 48
345	9 38 40		9 42 00	135 50	127 05	− 8 45		− 2 17

Local apparent time by chron.	h	m	s	Location of compasses and particulars of compensation.
Chron. No. 10, Negus,	12	32	27	
C. C.,	+	5	03.7	
G. m. t. April 7,	0	37	30.7	
Eq. t.,	(−)	2	19.7	
G. ap. t.,	0	35	11	
Long. W.,	5	05	36	
Local ap. t.,	7	29	35	
Watch,	7	26	15	
Error of w. on l. a. t., slow,		3	20	

entered in column 9. Bearing in mind the fact that the + sign is given to easterly errors, variation, and deviation, and the — sign to westerly errors, variation, and deviation, and also that deviation equals compass error—variation, the sign of the deviation should be apparent. Easterly deviation may be marked either + or E., westerly deviation (—) or W.

The magnetic azimuth of the ship's head may be found by applying the deviations of the standard compass to the headings per standard.

At the instant of observing the sun's azimuth per standard, the ship's head by all other compasses on board should be noted. The readings of these compasses compared with the corresponding magnetic azimuths of the ship's head will give the deviations of the compasses on their particular headings, the deviations being marked as per rule Article 52.

59. (4) **By ranges.**—Ranges, whose magnetic bearings are known, may be found in various localities, having been specially laid out, or formed under natural conditions. The data concerning a number of such ranges have been published in a pamphlet by the U. S. C. and G. Survey.

When steaming across these ranges on various headings, the compass bearing of the range may be taken.

The deviation for any heading will be the difference between the compass bearing of the range on that heading and the known magnetic bearing, marked easterly when the magnetic bearing is to the right of the compass bearing, westerly when the magnetic bearing is to the left of the compass bearing.

60. Napier's diagram.—This is a graphic representation of deviations on either compass or magnetic headings, and it furnishes a ready method of finding the magnetic course corresponding to a given compass course, and vice versa.

It consists of a vertical line of convenient length divided into 24 equal parts representing the 24 15°-rhumbs of the

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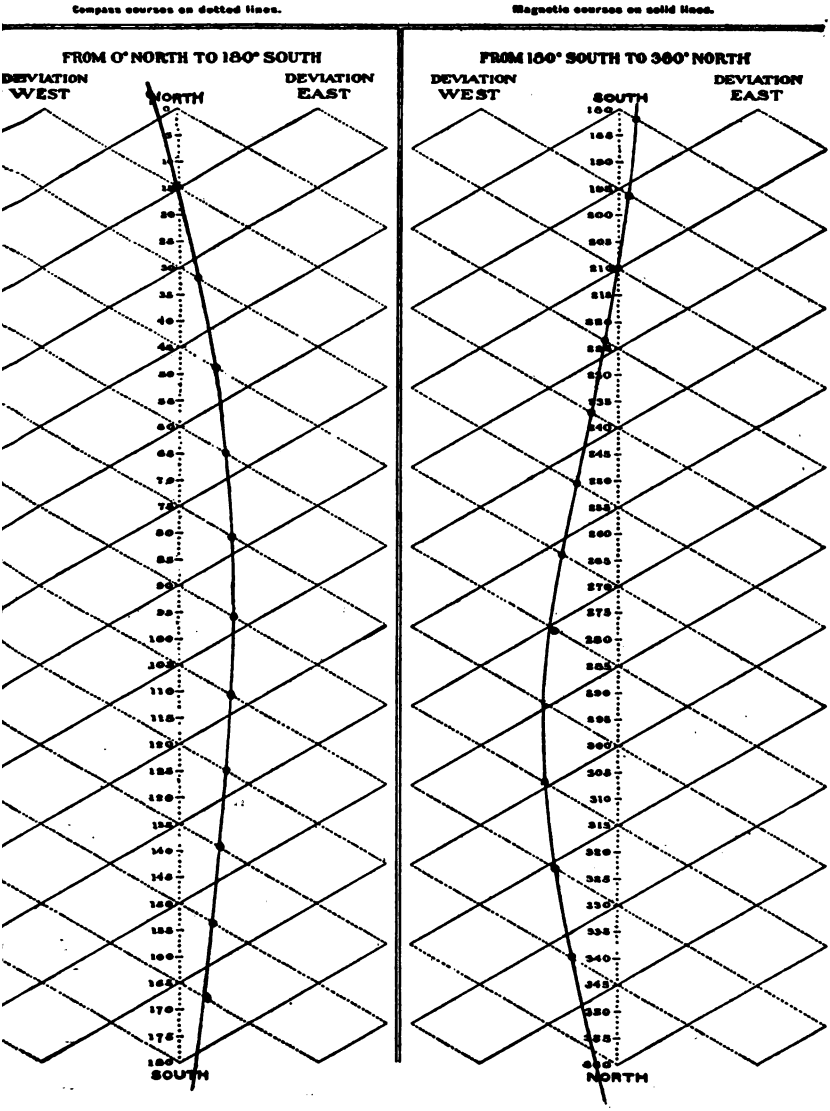


Curve of Deviations, Napier's Diagram.

compass; beginning at the top, these are numbered in order, 0°, 15°, 30°, 45° up to 360° from North around to the right. The line is also divided into 360 equal parts representing degrees, numbered at every fifth degree around to the right. Usually the curve is shown in two parts as on page 75.

The vertical line is intersected at 15° rhumbs by two lines, a plain line inclined upward and to the right, a dotted line inclined upward and to the left, each making an angle of 60° with the vertical.

To construct a curve.—Take on the vertical line the compass course for which the deviation has been obtained; lay off this deviation, to the scale of the vertical line, on the dotted line which passes through the course, or in a direction parallel to the dotted lines, to the right if the observed deviation is easterly, to the left if westerly, and mark the point so obtained with a dot. Having done this for each observed deviation, trace a fair curve through the points, and this will be the deviation curve. The deviations should be obtained on eight or more rhumbs equally distributed around the compass, but these need not be 15° rhumbs. If not possible to get the deviations on more than four rhumbs, these should be as near the quadrantal points as possible.

Rule I. From a given compass course to find the corresponding magnetic course.

Take the compass course on the vertical line; move thence on or parallel to the dotted lines till the curve is intersected, thence on or parallel to the plain lines till the vertical line is intersected. This point in the vertical line will be the required magnetic course.

Rule II. From a given magnetic course to find the corresponding compass course.

Find the magnetic course on the vertical line; from this point move on or parallel to the plain lines till the curve is intersected, thence on or parallel to the dotted lines meeting the vertical line in the required compass course.

SECTION IV.

THE THEORY OF COMPASS DEVIATIONS.

61. Soft iron and hard iron.—Preliminary to investigating the causes of deviations, it is essential to consider the character of the iron used in building a ship and the influence of the earth's magnetism on that iron.

Considering its physical characteristics, iron may be designated magnetically by the terms "soft" and "hard."

Soft iron is iron which, under the influence of a magnetic force, will instantly acquire magnetism by induction, but will as quickly lose it when that force is removed. In other words its magnetism is transient induced magnetism.

Hard iron is less susceptible to magnetic induction, but, when once magnetized, it retains a large part of its magnetism permanently. Furthermore, the greater the hardness and the less easily it can be magnetized, the greater the amount of magnetism it is capable of retaining and the longer it will retain it.

62. Effect of earth's magnetism on a soft iron rod.—If a rod of soft iron be held in the direction of the "line of force," it will instantly become magnetic. If in North latitude, the lower end will have induced in it North or red magnetism, and will repel the North end of the compass needle; and the upper end will have induced in it South or blue magnetism, and will attract the North end of the compass needle.

If inclined to the "line of force," its induced magnetism will be proportional to the cosine of the angle of inclination; therefore, at 90°, or at right angles with the "line of force," the rod will be in a neutral condition; beyond 90° the magnetism will be reversed, that end which was at first of North polarity will have South polarity, and the intensity of the magnetism induced will increase till at 180° it will again be a maximum.

If instead of being held in the direction of the "line of force," the rod is moved in a horizontal plane, it will be subject to induction only by the horizontal component of the total force, so that if placed East and West magnetic, the bar, being at right angles to the inducing force, will be in a neutral condition and have no effect on either end of the compass needle.

If held in any other position in the horizontal plane, its South end will attract, and its North end will repel the North end of the compass needle with a force proportional to the horizontal intensity multiplied by the cosine of the rod's magnetic azimuth.

If a soft iron rod be held in a vertical position, it will have magnetism induced in it by the vertical component of the earth's magnetism. In North latitude, the upper or South end will attract the North end of the compass needle; at the magnetic equator, being perpendicular to the line of force, it will be in a neutral condition; and in South magnetic latitude the polarity will be reversed, the upper end then having North polarity will repel the North end of the needle.

If a rod, held in a position favorable to induction, is hammered, twisted, bent, or otherwise subjected to mechanical violence, the amount of magnetism it will receive is increased. This magnetism diminishes more or less rapidly in the first few weeks but a portion of it is retained for months, perhaps for years, unless removed by similar mechanical violence applied in an opposite way. This condition is known as one of subpermanent magnetism.

A plate of iron has magnetism induced in it in a similar way, the magnetism being divided into regions of opposite polarity by a neutral plane at right angles to the direction of the earth's total force; and its permanency is dependent on the character of the iron and the treatment given it.

The law of induction, as explained for rods and plates, extends to bodies of a third dimension, whether of regular or irregular shape; the line connecting the induced poles, called the magnetic axis, lying in the direction of the line of force, with a neutral area whose plane is perpendicular to that axis.

63. Magnetic induction in an iron or steel ship.—Applying this law of induction to bodies of even the varied and complex form of an iron or steel ship, it is easy to understand how such a vessel should receive magnetism by induction and have it partially fixed in the course of construction by the processes of bending, twisting, hammering, or riveting to which the various parts are subjected.

Since the facility with which the induction takes place and the ability of the iron to retain the magnetism induced depend both on the character of the iron and the treatment it receives, it is convenient to consider separately the earth's effect on the soft and hard iron, and also their effects on the compass needle.

The iron in which only temporary magnetism is induced consists of the kind denominated as "soft iron," and in a ship this is either horizontal or vertical, or if not so, it may be resolved with components in those planes so that the earth's effect on soft iron will be:

(1) **Transient magnetism induced in horizontal soft iron,** or that developed in the horizontal soft iron of the ship by the inductive action of H, the horizontal component of the earth's total force.

It is transient in character and as it depends for its force upon H, which varies with the cosine of the dip, its force will be zero at the magnetic poles and a maximum at the magnetic equator.

(2) **Transient magnetism induced in vertical soft iron,** or that developed in the vertical soft iron of the ship by the inductive action of Z, the vertical component of the earth's total force. It is transient in character and as it depends for

its force upon Z, which varies with the sine of the dip, its force will be zero at the magnetic equator and a maximum at the magnetic poles.

Subpermanent magnetism induced in the ship while building.—The remainder of the ship's iron, consisting of that denominated "hard iron" and of that of a character intermediate between hard and soft iron, when acted upon by the earth's inducing forces in the process of building, assumes the character of a large magnet, more or less permanent, whose distribution of magnetism depends on the place of building and the azimuth of the ship's head at the time. Whilst building, the ship's polar axis and neutral plane respectively correspond, more or less, to the direction of the earth's total force and a plane at right angles to it. The magnetism thus developed is known as subpermanent magnetism, as it is not entirely permanent; suffering a diminution, after launching of the vessel and with change of direction from that in which the ship was built, for a lapse of several years till its magnetism settles down to practically a permanent condition. This state of affairs is in no wise due to induction in soft iron, and is modified if the vessel is launched before its hull is practically completed.

64. Forces acting on a compass needle in an iron or steel ship.—It is evident then that a compass needle, besides being acted upon by the earth's horizontal force which tends to keep the needle in the magnetic meridian, is subject to three distinct disturbing influences derived from the ship itself:

(1) Subpermanent magnetism;

(2) Transient magnetism due to vertical induction in vertical soft iron;

(3) Transient magnetism due to horizontal induction in horizontal soft iron;

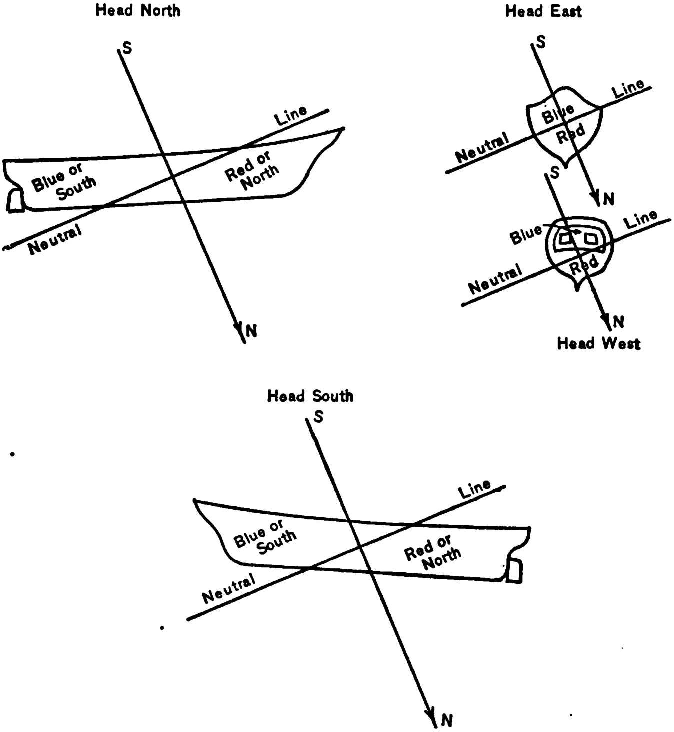
and that the resultant of these three forces, when not acting in the plane of the magnetic meridian, will deflect the needle, producing the total deviation for the particular heading of the ship.

65. Effect in producing deviation of each of the ship's disturbing forces when acting on the compass needle.

(1) **Effect of subpermanent magnetism.**—We have seen that the location of the poles of this subpermanent magnetism depends upon two things: 1st, the magnetic azimuth of the ship's head while building; 2d, the direction of the line of force at the place of building; hence the North (or red) pole will be in that part of the ship which was North in building and the South (or blue) pole will be in that part which was South in building. The repulsion of the North pole of the ship simply doubles the attraction of the South pole for the North end of the compass needle, therefore it may be laid down as a general rule that, under the influence of the subpermanent magnetism, the North end of the compass needle will be attracted to that part of an iron or steel ship which was South in building; hence in an iron ship built head North, the North end of the needle will be attracted toward the stern. Heading N. or S. there will be no deviation; in the former case the directive force is diminished, in the latter case, increased. As the ship swings in azimuth from these neutral points the needle is deflected; toward the East for westerly headings with a maximum of deviation about West, toward the West for easterly headings with a maximum about East.

In an iron ship built head South, the North end of the compass needle will be attracted toward the stem or head of the ship, and results just the reverse of the above will be obtained.

In an iron ship built head East, the North end of the compass needle will be attracted to the starboard side; heading East or West there will be no deviation, in the former case the directive force is diminished, in the latter case it is increased. As the ship swings in azimuth from these neutral points, the compass needle will be deflected toward the East for northerly headings with a maximum about North, toward the West for southerly headings with a maximum about South.



In an iron ship built head West, the North end of the needle will be attracted toward the port side, and results the reverse of those in a ship built head East will be obtained.

The accompanying diagrams will illustrate the distribution of magnetism in ships built head N., E., S., and W. (magnetic) at a place where the magnetic dip is 68° 30′, and the character of the deviations due to the subpermanent magnetism of these four ships is illustrated in the following curves. In reading these curves, the azimuths are taken on the vertical line and the deviations on the ordinates perpendicular thereto, the curve being to the right of the vertical line when the deviations are easterly, to the left when westerly.

Estimating the azimuth of the ship's head from the neutral points, these curves are "curves of sines"; they show the deviation to be the same in amount, but of opposite sign, on points differing 180° in azimuth, and the neutral points to correspond to those points of the compass on which the ship's head and stern were in building.

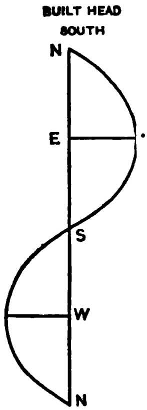


FIG. 21.

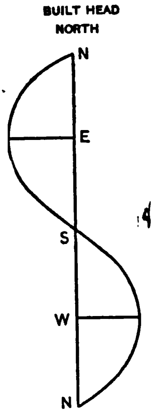


FIG. 22.

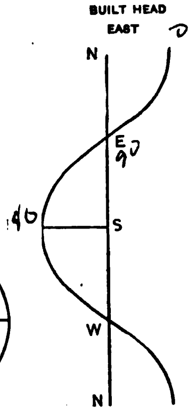


FIG. 23.

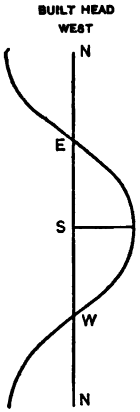


FIG. 24.

Semicircular deviation.—The above is called a semicircular deviation because it is easterly in one semicircle and westerly in the other, as the ship's head moves around a complete circle in azimuth.

If the compass is principally disturbed by the magnetic influences of the hull of the ship, the neutral points, or points of no deviation, will be opposite to each other and will correspond to those points of the compass towards which the ship's head and stern were directed in building; and the deviation, or more exactly the sine of the deviation, in each semicircle, is proportional to the sine of the azimuth of the ship's head measured from the neutral point, the azimuth being that shown by the disturbed compass.

Since the force due to subpermanent magnetism is constant for all latitudes, and its effect in producing deviation is inversely as H, the directive force of the earth, the semicircular deviation due to subpermanent magnetism varies with change of latitude.

66. (2) Effect of transient magnetism due to vertical induction in vertical soft iron.—The vertical component of the earth's force, Z, induces magnetism in the vertical soft iron and fittings of the ship, producing a resultant pole of South polarity towards which the North end of the compass needle is attracted. As the vertical inducing force remains the same at a given place, the magnetism induced by it does not vary as the ship turns in azimuth; therefore, it produces a semicircular deviation following the same law as that caused by subpermanent magnetism, with the exception that the effect produced in this case, being directly proportional to Z and inversely proportional to H, varies as the tangent of the magnetic dip, or as tan θ.

The semicircular deviation caused by induction in vertical soft iron is the kind formerly found in wooden ships; the neutral points being North and South, and the deviation easterly in the Eastern semicircle, westerly in the Western semicircle. It constitutes the smaller part of the semicircular deviation of iron or steel ships.

The ship's polar force and the starboard angle.—As the two forces we have just considered, those due to subpermanent magnetism and transient magnetism induced in vertical soft iron, produce the same kind of deviation, it is convenient to take them jointly and to consider the North point of the needle as acted upon by their resultant force, known as the ship's polar force; its horizontal component makes with the fore and aft line of the ship an angle, which measured from ahead around to the right (from 0° to 360°) is known as the starboard angle and designated by the letter *a*.

The semicircular component forces and the coefficients of semicircular deviation.—Now this resultant polar force of South polarity, attracting the North end of the compass needle toward a certain point in the ship, may itself be resolved into two component forces, one acting in the fore-and-aft line of the ship, the other in the athwartship line through the compass. Let 𝔅 represent the semicircular force acting in the fore-and-aft line and ℭ the semicircular force in the athwartship line; the former is marked + if acting toward the ship's head, (—) if toward the stern; the latter is marked + if acting to starboard, (—) if to the port side. The signs of these forces are dependent on the value of the starboard angle *a* as indicated in Fig. 25.

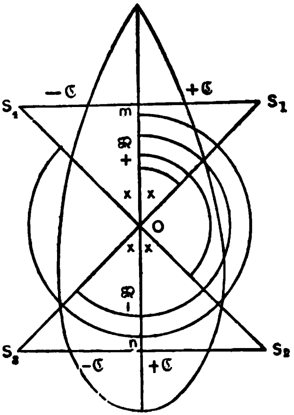


FIG. 25.

𝔅 and ℭ are known as the exact coefficients of the semicircular deviation; but + 𝔅 is the ship's polar force to head and + ℭ is the ship's polar force to starboard, both expressed in terms of the "mean value of the force of earth and ship to magnetic North" as a unit. This fact will be apparent from a study of section II, chapter IV.

Let *O* be the position of a compass; OS_1, OS_2, OS_3, or OS_4 be a ship's polar force represented in Fig. 25 as acting respectively from the starboard bow, starboard quarter, port quarter, or port bow, and in the horizontal plane.

For the force OS_1 or OS_4, $\mathfrak{B} = Om$, for the force OS_2 or OS_3, $\mathfrak{B} = On$; the sign of $\mathfrak{B}$ is $+$ when acting to head, $(-)$ when acting to stern as indicated in the figure.

For the forces OS_1, OS_2, OS_3, and OS_4, the values of $\mathfrak{C}$ are represented by mS_1, nS_2, nS_3, and mS_4 respectively; the sign of $\mathfrak{C}$ is $+$ when acting to starboard, $(-)$ when acting to port as indicated in the figure.

For the force

OS_1, $\mathfrak{B}$ is $+$, $\mathfrak{C}$ is $+$, and a is $< 90°$,
OS_2, $\mathfrak{B}$ is $(-)$, $\mathfrak{C}$ is $+$, and a is > 90 and $< 180°$,
OS_3, $\mathfrak{B}$ is $(-)$, $\mathfrak{C}$ is $(-)$, and a is > 180 and < 270,
OS_4, $\mathfrak{B}$ is $+$, $\mathfrak{C}$ is $(-)$, and a is > 270 and < 360,

all values of a being measured from ahead around to the right, as shown by circles in the figure.

It is convenient to find the angle $x = \tan^{-1} \frac{\mathfrak{C}}{\mathfrak{B}}$ and then to take $a = x$, $180° - x$, $180° + x$, or $360° - x$, according as the polar force acts from the starboard bow, starboard quarter, port quarter, or port bow respectively.

For illustration, consider the two components positive. The forces $\mathfrak{B}$ and $\mathfrak{C}$ exert each an attraction on the North end of the compass needle similar to that of a permanent magnet, producing semicircular deviation as the ship swings in azimuth. The force $+\mathfrak{B}$ causes no deviation on the heading North, but as the ship swings toward the East, the needle deviates toward the East, and the deviation increases with the azimuth by constant increments till the ship heads about East, when the force is at right angles to the direction of the needle and produces its maximum effect; then as the swinging continues the deviation diminishes by constant decrements till, when the ship heads South, the deviation is zero and the needle is again in the magnetic meridian. If the swinging is continued through the western semicircle, the effect is repeated except that the deviation is westerly and a maximum in amount at West.

Letting the maximum deviation produced be represented by *B* (which is approximately the $\sin^{-1} \mathfrak{B}$), the deviation on any other heading due to $\mathfrak{B}$ will be a fraction of *B*, the amount and sign depending on the azimuth of the ship's head per compass. *B* is known as the approximate coefficient of semicircular deviation due to the force $\mathfrak{B}$.

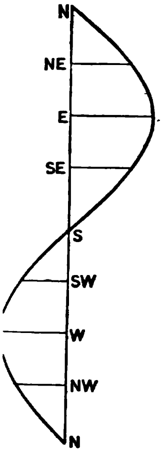


FIG. 26.

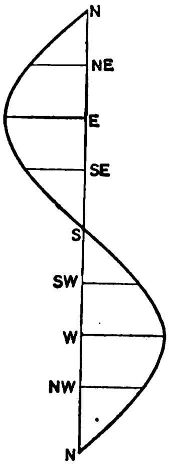


FIG. 27.

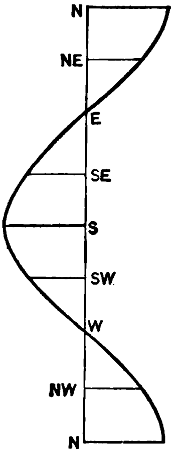


FIG. 28.

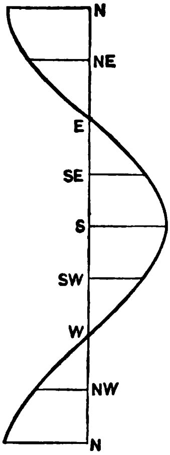


FIG. 29.

If as in Fig. 26, the azimuths are read off on the vertical line, and if, at the points representing the azimuths of the ship's head per compass, ordinates are erected perpendicular to the vertical line and representing according to a given scale of parts the corresponding deviations, the curve drawn through the extremities of the ordinates, showing the deviation to be zero at North and South and a maximum at East and West, will be a curve of sines; and the value of the deviation due to the fore-and-aft semicircular force $\mathfrak{B}$ for any heading of the ship z' per compass will be

$$B \sin z'.$$

Had the force $\mathfrak{B}$ attracted the North end of the needle to the stern, or if the sign of $\mathfrak{B}$ had been (—), a curve similar to Fig. 27 would have resulted, the deviations being westerly for those headings on which the force $+ \mathfrak{B}$ produced easterly deviations and easterly for those on which $+ \mathfrak{B}$ produced westerly deviations.

The force $+ \mathfrak{C}$ causes a maximum easterly deviation when heading North which diminishes as the ship swings in azimuth to the eastward till, when heading East, the force is in the magnetic meridian and produces no deviation. As the swinging is continued, the North end of the needle deviates to West and the amount of the deviation increases by constant increments till, when the ship heads South, the deviation is again a maximum in amount but westerly in sign.

If the swinging is continued, this effect is repeated, the sign of the deviation being opposite to that of the corresponding point 180° distant in azimuth.

Letting the maximum deviation be represented by C (which is approximately $\sin^{-1} \mathfrak{C}$), the deviation on any other heading due to $+ \mathfrak{C}$ will be a fraction of C, the amount and sign depending on the azimuth of the ship's head per compass. C is known as the approximate coefficient of semicircular deviation due to $+ \mathfrak{C}$, the athwartship semicircular force.

As seen in Fig. 28, the curve is one of cosines, and the value of the deviation on any heading (z') per compass due to the force $+ \mathfrak{C}$ will be

$$C \cos z',$$

and the deviation on that compass heading due to the resultant of the semicircular forces $+ \mathfrak{B}$ and $+ \mathfrak{C}$, or $\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$, will be

$$B \sin z' + C \cos z'.$$

67. (3) **Effect of transient magnetism due to horizontal induction in horizontal soft iron.**—Since the magnetism induced in horizontal soft iron by the earth's horizontal force varies as the cosine of the inclination of the iron to the direction of the earth's horizontal force, it is evident that we must now deal with an induced force which, unlike the forces producing semicircular deviation, is a variable one.

That the force induced may be a function of the azimuth of the ship's head, the horizontal soft iron of the ship should be considered as lying either in the fore-and-aft, or the athwartship direction through the compass. As a greater part of the horizontal soft iron is so situated, and as soft iron at intermediate angles may be represented by components parallel to those directions, the effect of all the soft iron in the ship, magnetized by the earth's horizontal component and acting in the horizontal plane through the compass, may be replaced by the effects of two systems of horizontal iron, one placed fore and aft and one athwartships. It will be shown later on that when the soft iron is symmetrically distributed on each side of the fore-and-aft line of the ship, and the ship is on an even keel, that the forces due to the induced magnetism of the two systems may be replaced by those of a single fore-and-aft and a single athwartship rod; the force due to the induced magnetism of the first rod will act in the fore-and-aft horizontal line, and the force due to the induced magnetism of the second rod will act in the athwartship horizontal line through the compass.

The character of each force, whether attracting or repelling the North end of the compass needle, will depend on the position of the rod; if entirely forward or abaft, to starboard or to port, of the compass, one end of the rod will act; if continuously extending above or below the compass needle, the opposite end will act. If a rod is entirely on one side of the compass, a similar rod, similarly situated but distant 180°, will simply double the effect of the first rod.

Depending on the location of the compass on board, the effect of the symmetrical horizontal soft iron may be that of one, or another, of the arrangements of the rods as shown in Figs. 30, 31, 32, and 33.

Taking the first rod (1) for consideration and regarding the magnetism induced in the rod on the various headings as represented in Fig. 34, we see that with the ship heading North no deviation is produced but the directive force is increased; as the ship swings to the eastward, the North point of the needle deviates to the East (or to the right); at NE., the maximum easterly deviation is caused, since at that angle with the meridian, the induced magnetic force, though equal only to H cos 45°, has a greater proportionate effect in causing deflection of the needle. As the ship continues to swing to the eastward, the rod gradually loses its magnetism, and the attraction on the needle diminishing it gradually returns toward the meridian. When the ship heads East, the rod is at right angles to the line of force and has no effect on the needle which is again in the meridian. For the NE. quadrant the curve of deviations is as represented in Fig. 35 from North to East.

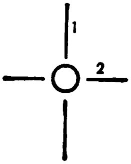


FIG. 30.

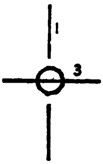


FIG. 31.

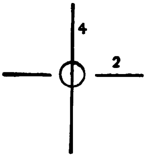


FIG. 32.

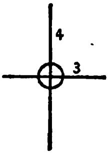


FIG. 33.

The ship continuing to swing, the end of rod (1) nearest the compass, having North magnetism induced in it, will repel the North end of the compass needle, and the curve of deviations traced will be exactly the same as in the NE. quadrant except that it will be on the opposite side of the vertical line. When the ship heads South there is no deviation, but the directive force is again increased. From South to West we will have the same curve (the deviations being the counterpart in amount and sign) as from North to East, at West the needle fails of effect. From West to North the deviations and the curve are the same as in the SE. quadrant.

Quadrantal deviation.—The deviation here illustrated is known as quadrantal deviation, and is so called because it is easterly and westerly alternately in the four quadrants as the ship's head moves around a complete circle in azimuth. Its zero points coinciding very nearly with the cardinal points,

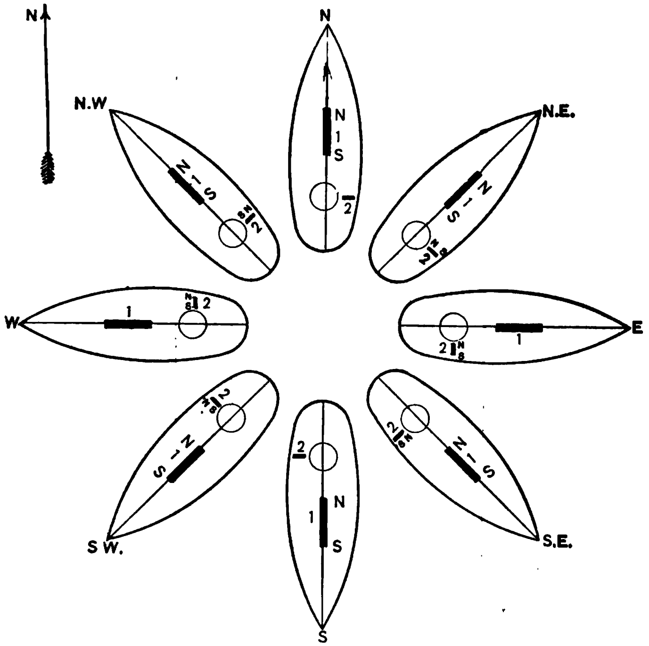


FIG. 34.

and the points of maximum deviation being at the quadrantal points, each characteristic occurring twice in a semicircle, the amount of the deviation (or more properly the sine of its amount) is proportional to the sine of twice the azimuth of the ship's head measured (as will be seen later) from a line half way between the magnetic North and the compass North. The quadrantal deviation usually found on board iron or steel ships is of the type represented by the curve of Fig. 35, easterly in the NE. and SW. quadrants, westerly in the SE. and NW. quadrants, or the type usually spoken of as a positive quadrantal deviation.

The horizontal component of the earth's total magnetic force is the directive force acting on the compass needle, the magnetism induced in the horizontal iron by the earth's horizontal force is the disturbing force acting on the needle, and the ratio between the two is constant; therefore the quad-

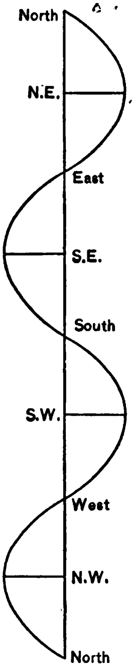


FIG. 35.

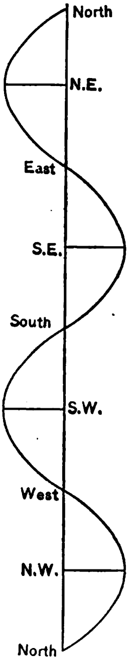


FIG. 36.

rantal deviation may be expected to remain unchanged in all magnetic latitudes, and even by lapse of time, so long as the distribution of the ship's horizontal soft iron remains unchanged.

Quadrantal force and coefficient of quadrantal deviation.— Let $\mathfrak{D}$ represent the quadrantal force, the sign of the force being + when producing a positive quadrantal curve as shown by Fig. 35; and letting *D* (which is approximately $\sin^{-1} \mathfrak{D}$) represent the maximum deviation, or that on the quadrantal points, the deviation on any other heading due to the force $+ \mathfrak{D}$ will be a fraction of *D* and a function of twice the azimuth; in other words *D* is the approximate coefficient of quadrantal deviation of the type represented by the curve (Fig. 35). (See Art. 79.)

If D_1 represents the coefficient of quadrantal deviation due to the induced magnetism of rod (1), then the quadrantal deviation on any heading z' per compass due to the effect of rod (1) will be

$$D_1 \sin 2z'.$$

Taking the rod (2) under consideration and regarding the magnetism induced in the rod on the various headings as represented in Fig. 34, it is seen that when the ship heads North no effect is produced, the rod being at right angles to the line of force; as the ship swings to eastward, polarities are developed as shown and the North end of the needle is repelled until, when the ship heads NE., there is a maximum westerly deviation. When the ship heads East, the rod is in the meridian, and its induced magnetism increases the directive force on the needle, but produces no deviation. It is thus seen that the effect of the magnetism induced in a rod lying wholly on one side of the compass, as the ship swings from North to East, is, in the case of an athwartship rod, just the reverse of that in a fore-and-aft rod (1). If the swinging is continued till the ship again heads North, the deviations on the various headings may be represented by a curve similar to Fig. 36, which shows for any azimuth of the ship's head a deviation of the same kind, but of the opposite sign, to that shown by the curve of Fig. 35 as in the case of rod (1). When the ship heads West the directive force is increased as was the case when the ship headed East.

The quadrantal deviation caused by rod (2), being westerly in the NE. and SW. quadrants and easterly in the SE. and NW. quadrants, is known as a negative quadrantal deviation.

In this case, the case of a negative quadrantal deviation, if (—) D_2 is the quadrantal coefficient, the quadrantal deviation on any heading z' per compass due to the induced magnetism of an athwartship rod (2) will be

$$-D_2 \sin 2z'.$$

The combined quadrantal deviation due to an arrangement of iron similar to that illustrated in Fig. 30 will be

$$\{+D_1 + (-D_2)\} \sin 2z' = \pm D \sin 2z'.$$

The deviation being easterly or westerly in the NE. and SW. quadrants (and hence westerly or easterly in the SE. and NW. quadrants), according as the effect of rod (1) is greater or less than the effect of rod (2). In cases where the rod (2) equals rod (1), an arrangement of iron similar to Fig. 30 will increase the directive force without causing deviation.

A rod (3), Fig. 31 and Fig. 33, continuously extending above or below the compass in a transverse plane, will have an effect directly the opposite of rod (2), diminishing the directive force at East and West and producing a positive quadrantal deviation as represented by Fig. 35.

If D_3 be the coefficient of quadrantal deviation due to rod (3), then the quadrantal deviation on any heading z' per compass will be

$$D_3 \sin 2z',$$

and the combined quadrantal deviation due to an arrangement of iron similar to Fig. 31 will be

$$(D_1 + D_3) \sin 2z' = D \sin 2z'.$$

A rod (4), Fig. 32 and Fig. 33, continuously extending above or below the compass, in the fore-and-aft plane, will exert on the North point of the compass needle an effect just the opposite to that exerted by rod (1) in the same plane but wholly on one side of the compass, diminishing the directive force at North and South and producing a negative quadrantal deviation as represented by Fig. 36. If $-D_4$ be the coefficient of the quadrantal deviation due to rod (4), then the quadrantal deviation on any heading z' per compass due to induced magnetism of rod (4) will be

$$(-)\ D_4 \sin 2z',$$

and the combined quadrantal deviation due to an arrangement of iron similar to Fig. 32 will be

$$(-D_2 + (-D_4)) \sin 2z' = -D \sin 2z',$$

and that due to an arrangement similar to Fig. 33 will be

$$(D_3 + (-D_4)) \sin 2z' = \pm D \sin 2z',$$

the sign of D depending on which is the greater, the magnetism due to rod (3) or that due to rod (4). In this arrangement when rod (3) equals rod (4), the directive force will be diminished, but no deviation will be produced.

Hence, we have for the general expression representing quadrantal deviation, on any heading z' per compass, due to horizontal induction in horizontal soft iron symmetrically situated, when D is the approximate coefficient, the term

$$D \sin 2z'.$$

The quadrantal deviation as represented by D is usually caused by the action of rods (1) and (3), Fig. 31, or excess of effect of (3) over (4), Fig. 33, since it is usually positive and the directive force on the needle is diminished.

68. In case the soft iron is unsymmetrically situated in the horizontal plane through the compass, an additional force due to horizontal induction in fore-and-aft iron may act on the needle, and it may be represented as that of a fore-and-aft rod to starboard or port of the compass, rod (5), Fig. 37; and an additional force due to horizontal induction in athwartship iron may also act on the needle, this force being represented as that of an athwartship rod forward or abaft the compass, rod (6), Fig. 37.

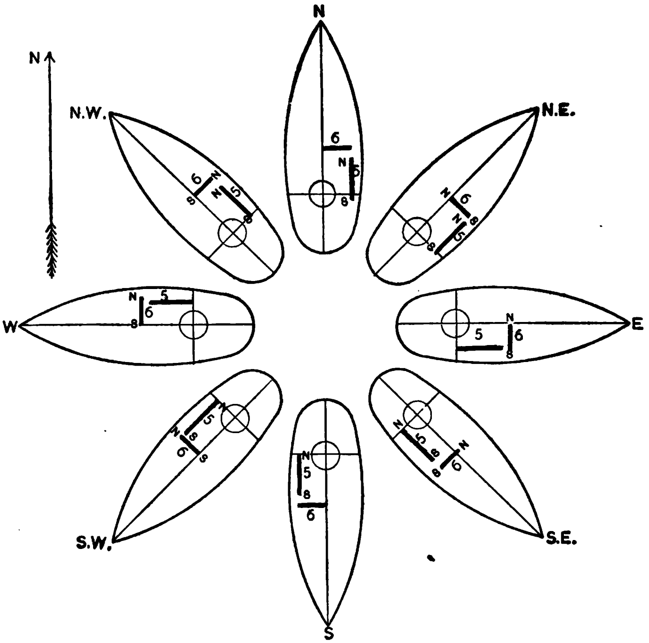


FIG. 37.

Regarding the magnetism induced in rod (5) as represented in Fig. 37 for the various headings per compass, it is seen that with the ship heading either North or South, the North point of the compass needle will have a maximum deflection to eastward, being most strongly attracted in the first case by the induced South pole, and equally repelled in the second case by the induced North pole of the rod (5).

When the ship heads either East or West, the rod (5) has no effect. At the quadrantal points the deviation is the same fraction of the maximum and also easterly. The curve representing the deviations is similar to the plain curve of Fig. 38.

From Fig. 37, it is seen that when the ship heads North or South rod (6) has no effect. When the ship heads East or West, the North point of the needle will have a maximum deflection to the westward, being most strongly repelled in the first case by the North pole of (6) and equally attracted in the second case by the South pole of (6). At the quadrantal points the deviation is the same fraction of the maximum and is also westerly. The curve of deviations in this case is similar to the dotted curve of Fig. 38.

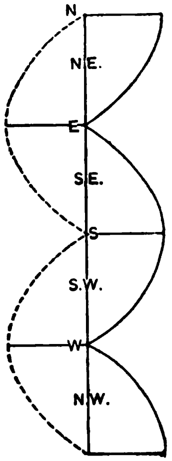


FIG. 38.

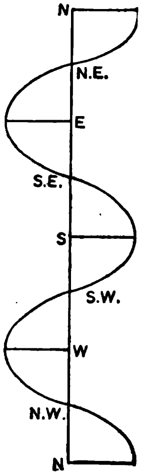


FIG. 39.

When the two rods (5) and (6), Fig. 37, act together, rod (5) causes a maximum easterly deviation when rod (6) has no effect, and rod (6) causes a maximum westerly deviation when (5) has no effect; at the quadrantal points the easterly deviation caused by (5) neutralizes the westerly deviation caused by (6); the resultant curve of deviation, as represented in Fig. 39, shows maxima when the ship heads on the cardinal points and minima when the ship heads on the quadrantal points.

This deviation, due to horizontal induction in soft iron unsymmetrically distributed about the compass, is of the quadrantal type and is in general very small.

Let $\mathfrak{E}$ represent the force producing this particular kind of deviation, + when it causes an easterly deviation between North and NE. as shown in Fig. 39; let E (which is approximately $\sin^{-1} \mathfrak{E}$) be the maximum deviation, or that on the cardinal points, then the deviation on any other heading z' per compass due to the force $+ \mathfrak{E}$ will be a fraction of E and will be found from the expression

$$E \cos 2z'.$$

E is known as the approximate coefficient of the quadrantal deviation due to horizontal induction in horizontal soft iron unsymmetrically distributed about the compass. (See Art. 80.)

If the rods (5) and (6) are in the port bow (Fig. 40.), the deviation due to their effect on any azimuth will be the same in amount but of the opposite sign to that produced in the case when they were in the starboard bow, the coefficient will be (—) E, and the deviation on any heading z' per compass will be

$$-E \cos 2z'.$$

If the rods are both in one quarter, the effect will be the same as if they were in the opposite bow; the effect of having one of the rods in the bow and one in the opposite quarter is the same as if both rods were in that bow, or both in that quarter.

So the general expression for the quadrantal deviation on any heading z' per compass due to horizontal induction in soft iron unsymmetrically distributed about the compass is

$$E \cos 2z'.$$

69. Constant deviation.—When the soft iron is not symmetrically distributed on each side of the fore-and-aft line through the compass, or the compass is not in the midship line, a constant term may be noted in the deviation.

It is usually very small and is called constant because it is the same in amount and direction on all headings; it is marked East or (+) when the easterly deviation is in excess, West or (—) when the westerly deviation is in excess. If due to the unsymmetrical soft iron of the ship, it is known as a real constant deviation which will not vary with change of latitude, as the ratio of the force induced in the iron and the directive force on the compass needle is a constant. The force producing constant deviation is represented by $\mathfrak{A}$, the approximate coefficient or deviation itself by A. (See Art. 80.)

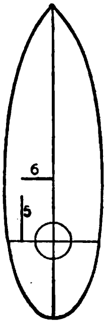


FIG. 40.

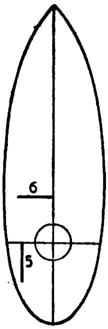


FIG. 41.

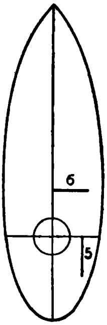


FIG. 42.

If rods (5) and (6) are situated as in Fig. 41, (5) will produce the plain curve of Fig. 38 when the ship is swung, and (6) will produce a curve similar to the dotted curve of Fig. 38 except of the opposite sign; that is to say (5) will cause a maximum of deviation on North and South, a minimum on East and West, all being easterly; (6) will cause a minimum of deviation on North and South, and a maximum on East and West, also easterly. In other words, as the effect of one rod increases, that of the other decreases, their resultant effect as the ship swings in azimuth being a constant easterly deviation.

If, however, rods (5) and (6) are situated as in Fig. 42, their resultant effect as the ship swings in azimuth will be a constant westerly deviation. A real value of A is rare on board ships with well-located compasses; more often there is an apparent value due to a badly placed lubber's line, index or other instrumental error, or an error in the assumed direction of the magnetic North. Other errors of a like nature may exist, but, whatever the causes, they may all be represented by the approximate coefficient of constant deviation A.

70. If δ represents the sum total of the deviations due to the various forces considered for the heading z' per compass we shall have

$$\delta = A + B \sin z' + C \cos z' + D \sin 2z' + E \cos 2z', \qquad (16)$$

$A, B, C, D,$ and E being known as the approximate coefficients of the deviation they respectively represent. (See Art. 77.)

Though the curve of total deviations, as shown on Napier's diagram (Art. 60), is irregular and unsymmetrical, the curves due to the separate forces considered are in themselves perfectly symmetrical, the irregularity arising from the superposition of all of them, the combined curve representing the resultant effect of all the forces.

71. Determination of coefficients by inspection.—The coefficients $A, B, C, D,$ and E may be found from the deviations observed with the ship's head on the 24 equidistant 15° compass rhumbs; also on any 12 or 8 of them if equidistant, as will be indicated in Art. 89, the process there followed being known as the "analysis of deviations"; but $A, B, C,$ and E

may be approximately determined by inspection from the deviations on the four cardinal points, and D may be also approximately obtained from the deviations on the four quadrantal points.

Using formula (16), Art. 70, and paying particular attention to the signs of functions of the azimuths, we have equations expressing the deviations on the eight principal points as follows, letting S_3 be the sine or cosine of 45°:

$$\begin{array}{lll} \text{On} & \text{North} & \delta_0 = A + C + E \\ & \text{NE.} & \delta_3 = A + BS_3 + CS_3 + D \\ & \text{East} & \delta_6 = A + B - E \\ & \text{SE.} & \delta_9 = A + BS_3 - CS_3 - D \\ & \text{South} & \delta_{12} = A - C + E \\ & \text{SW.} & \delta_{15} = A - BS_3 - CS_3 + D \\ & \text{West} & \delta_{18} = A - B - \mathrm{E} \\ & \text{NW.} & \delta_{21} = A - BS_3 + CS_3 - D \end{array}$$

and it is apparent from these equations that, using the word mean in its algebraic sense,

A is the mean of the deviations on the four cardinal points, or any four or more equidistant compass headings.

B is approximately the deviation at East, or the deviation at West with the sign changed; but more accurately the mean of these two values.

C is approximately the deviation at North, or the deviation at South with the sign changed; but more accurately the mean of these two values.

D is approximately the mean of the deviations at NE. and SW., or the mean of the deviations at SE. and NW. with the sign changed; but more accurately the mean of both these means.

E is the mean of the deviations on the four cardinal points of the compass after the signs of the deviations on East and West have been reversed.

Ex. 7.—Given the following deviation table, find by inspection the nearest approximation to the values of *A, B, C, D,* and *E.*

North	3° 12′ E			South	1° 36′ E	
NE	14 24 E			SW	7 00 W	
East	12 00 E			West	14 24 W	
SE	5 24 E			NW	12 12 W	

To find A.

At North 3° 12′ E
East 12 00 E
South 1 36 E
West 14 24 W

Algebraic Sum 2° 24′ E

$$A = \frac{2^\circ\ 24'\ E}{4} = 0^\circ\ 36'\ E$$

To find B.

At East 12° 00′ E
West (—) 14 24 W

2)26° 24′ E

B = 13° 12′ E

To find C.

At North 3° 12′ E
South (—) 1 36 E

2) 1° 36′ E

C = 0° 48′ E

To find D.

At NE 14° 24′ E
SW 7 00 W

2)7° 24′ E

1st value of D = 3° 42′ E
2d value of D(—) 3 24 W

2)7° 06′ E

D = 3° 33′ E

At SE 5° 24′ E
NW 12 12 W

2)6° 48′ W

2d value D 3° 24′ W

To find E.

At North 3° 12′ E
East (—) 12 00 E
South 1 36 E
West (—) 14 24 W

4)7° 12′ E

E = 1° 48′ E

72. Heeling error.—So far only those forces acting on the North point of the compass needle to produce deviation when the ship is upright have been considered, and it is necessary to consider other forces when an iron ship is inclined from the vertical.

There are certain forces which, acting only vertically and producing no deviation in the former case, will have in the latter case lateral components tending to draw the North point of the needle to one side or the other. Such are the vertical component of the ship's subpermanent magnetism and the vertical component of the magnetism induced in vertical soft iron. If they act vertically downward when the ship is on an even keel, the North end of the needle will go to windward when the ship heels, otherwise to leeward.

In addition, the horizontal deck beams and all other horizontal transverse iron become more or less magnetized by the earth's vertical inducing force, and the South polarity of their upper or weather ends will attract the North end of the needle to windward.

The resultant effect of these forces, when the ship is inclined, is known as the heeling error, the direction of which, depending on circumstances, may be to windward or to leeward.

The heeling error will vary with change of latitude because that part due to the vertical component of the subpermanent magnetism varies as $\frac{1}{H}$, and the parts due to magnetism induced by the earth's vertical force vary as tan θ.

This error is a maximum on northerly or southerly courses, a minimum on easterly or westerly courses, and, for intermediate headings, varies practically as the cosine of the azimuth of the ship's head.

The causes and effects of heeling error may be better understood after a careful study of the next chapter.

73. Mean directive force.—The ship's forces acting on the compass have been considered primarily as causing deviation, but they have an additional effect, increasing or diminishing the earth's directive force on the various headings as the ship swings in azimuth. This can be easily seen by resolving any force so that one component will be in the direction of the undisturbed needle and one at right angles to it in the horizontal plane.

Whilst the latter component acts to produce deviation, the former increases or diminishes the directive force according as it draws the North point of the needle to the northward or southward.

The force of earth and ship to magnetic North will vary with the azimuth of the ship's head, and its mean value for equidistant headings will be the mean directive force acting on the needle which, experience shows, is less than unity (H being unity) in nearly all iron or steel ships.

Other conditions being the same, the best location for a compass is that position where it will have the greatest mean directive force. (See Art. 76.)

CHAPTER IV.

MATHEMATICAL THEORY OF THE DEVIATIONS OF THE COMPASS.

SECTION I.

74. Mathematical theory of the deviations of the compass.—By considering all the iron of a ship as magnetically either hard or soft, it has been shown that on board an iron or steel ship, a freely suspended needle is acted upon by (1) the earth's total force; (2) the force due to the subpermanent magnetism induced in the ship in building; (3) those forces due to the transient magnetism induced in soft iron by the earth's force.

At a given place the force (1) is a constant force, though it does not draw the North point of the needle towards the same point in the ship on all azimuths of the ship's head. The force under (2) is a constant force and attracts the North point of the needle towards the same point in the ship for all azimuths. The forces under (3) are constant or variable, depending on whether the inducing force is the vertical or horizontal component of the earth's force.

In investigating mathematically the theory of compass deviations, it is necessary first to find the components of the various forces acting on the North point of the needle in certain definite directions through the compass, and the resultant of the components in each direction.

Let these directions be the fore-and-aft horizontal line, the transverse horizontal line, and the vertical line through the point of suspension of the needle, the length of which is regarded as infinitely small when compared with the distance of the nearest iron, or, what amounts to practically the same thing, the North point of the needle is considered the origin of coordinates.

Examining first the earth's force, let *O*, Fig. 43, represent the North point of a magnetic needle on board an absolutely

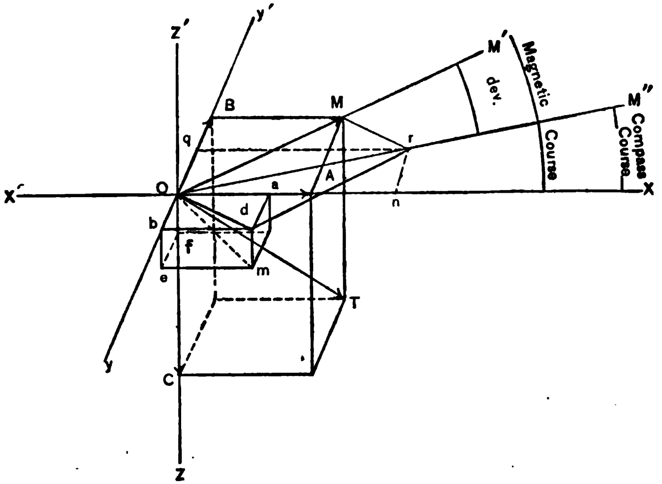


FIG. 43.

non-magnetic ship; let it be the origin of a system of rectangular coordinates of which *OX* the horizontal fore-and-aft line, *OY* the horizontal transverse line, and *OZ* the vertical line through the point *O* are the axes respectively denominated *X*, *Y*, and *Z*; the directions to head, to starboard, and vertically downward being regarded as positive. The South end of the needle is not considered, since an attraction or repulsion of the North end would be a repulsion or attraction of the south end, and the effect on the South end simply doubles that on the North end without changing the direction of the needle.

Therefore, we may confine the investigation to the action on the North point of the needle of the earth's total force which, for purposes of illustration, may be represented in intensity and direction by OT in the figure. The vertical plane of OT is that of the magnetic meridian OM', the angle $M'OT$ is the magnetic dip, OM represents in intensity and direction the earth's horizontal inducing force, and OC the intensity of the earth's vertical force through the origin of coordinates (see equation 15, Article 45).

If the needle is a freely suspended needle, it will point in the direction OT; if a compass needle, it will point in the direction OM; in other words, OM is the directive force acting on the compass needle, and is in the horizontal plane of XY.

Since a force represented by a vector or line of given length and direction, such as OM, can be resolved into two components at right angles to one another, all being in the same plane, then the force OM, or H, can be resolved into the forces OA and OB.

Therefore, letting $M'OX$ be the magnetic azimuth of the ship's head, *measured positively around to the right from the magnetic meridian,* then, if in Fig. 43, $\angle$'s OAM, BOA, and OMT are right angles, we shall have the components of the earth's total force in the direction of the three axes as follows:

$OA = H \cos z$, force of the earth's magnetism drawing the North point of the needle at O towards the ship's head;

$OB = H \cos (90° + z) = - H \sin z$, the force drawing it to starboard;

$OC = Z$, the force drawing it vertically downward.

Let the component OA in the axis of X be called X, the component OB in the axis of Y be called Y, the component OC in the axis of Z be called Z

Now, if the earth's total force be disregarded, and forces equivalent to X, Y, and Z be supposed to act in their proper axes on the North point of a freely suspended magnetic needle at O, it will take the direction of OT; a compass needle under the same forces would take the direction OM.

If instead of being mounted on board an absolutely non-magnetic ship, the magnetized needle is mounted on board an iron or steel ship in the same geographical locality, the forces due to the ship's magnetism will act on the North point of the needle in addition to the earth's force; a dipping needle will take the resultant direction of all the forces, and a compass needle the direction OM''. The angle $M'OM''$ is the deviation due to the ship's magnetic forces for the particular heading per compass z', and is marked East or + when OM'' is to the right of OM', that is, when the North point of the compass is drawn to the Eastward; otherwise, West or (—).

As the force due to the subpermanent magnetism of the hard iron of the ship alters neither its intensity nor its direction as the ship swings in azimuth, the components of this force in the three axes will be constant. They are represented by

P when drawing the North point of the needle towards the ship's head,

Q when drawing the North point of the needle to starboard,

R when drawing the North point of the needle vertically downward.

Expressed in terms of the earth's horizontal force, we have abstract quantities $\frac{P}{H}, \frac{Q}{H}, \frac{R}{H}$.

The greater portion of the soft iron on board ship lies in one, or another, of the three axes considered, and since the effect of soft iron lying in intermediate directions may be represented by other iron parallel to the three axes, the investigation of the effect of soft iron may be confined to the effects of iron parallel to those axes, and may properly begin with a consideration of the fore and aft system. It is plain that the northern portions of this iron will always have induced in them North polarity and the southern portions South polarity as the ship swings through a complete circle in azimuth, that the intensity of the force due to the induced magnetism will vary with the azimuth of the ship's head, and that the polarities will be reversed when the ship's head passes through an azimuth of 90° or 270°. If the ship heads North magnetic, the force induced in the fore and aft soft iron will be a maximum and will bear a certain specific ratio to the earth's horizontal force. If that ratio be l, then the force induced will be lH. With the ship on any magnetic azimuth z, the induced force will be $lH \cos z$. As $X = H \cos z$, the force on the heading z will be the same fraction of X that the possible maximum is of H, and as in this case it is desirable to express a force with a suggestion of the axis in which the iron may be, it is convenient to represent the force induced in the fore-and-aft iron as lX, or as a fraction of the fore-and-aft component of the inducing force; furthermore, the variations in the force induced and the reversal of polarities referred to will occur in the same order and at the proper time even though X is taken as the inducing force in the axis of X.

Therefore, it is mathematically correct to assume that the soft iron, lying in the direction of any one of the axes, has magnetic force induced in it by the earth's resolved component in the same axis and by that component only.

The fore-and-aft component X will induce magnetism in the fore-and-aft iron which may practically be considered as a fore-and-aft system of parallel magnets attracting the North point of the needle with a force lX towards a point or pole in the system, l being a constant dependent only on the soft iron in the ship.

This force will act in the fore-and-aft vertical plane through the compass only when the iron is symmetrically situated with reference to that plane, and in the fore-and-aft horizontal line only when the iron is symmetrically situated with reference to the horizontal plane through the compass. Such is not the usual case, and this force, instead of acting towards the ship's head, acts in some other direction.

The components of lX will be

aX to the ship's head,
dX to starboard,
gX vertically downward.

$\frac{a}{l}, \frac{d}{l}$, and $\frac{g}{l}$ are the direction cosines of lX; a, d, and g are not forces but are constant ratios, and do not change with azimuth or geographical position. These values depend only on the amount, arrangement, and capacity for induction of the soft iron of the ship.

Thus a is the ratio between the component X and the component in the same axis of the force induced in fore-and-aft soft iron; d is the ratio between the component X and the component to starboard of the force induced in fore-and-aft soft iron and is zero when that iron is symmetrically situated with reference to the fore-and-aft vertical section through the compass; g is the ratio between the component X and the component downward of the force induced in fore-and-aft soft iron.

In the same way it may be shown that the magnetism of the transverse iron is induced in it by Y and only by Y, and that the force so induced is a specific fraction of Y with components

bY to the ship's head,
eY to starboard,
hY vertically downward.

Also, that the earth's vertical force Z, and that only, will induce a force in the vertical soft iron which will be a specific fraction of Z, and the components of this force in the three axes will be

cZ to ship's head,
fZ to starboard,
kZ vertically downward.

b, e, h, c, f, and k, like a, d, and g, are abstract quantities or constant ratios that do not vary with change of azimuth or geographical position, and are similarly defined; b, f, and h become zero when the transverse and vertical soft iron is symmetrically situated with reference to the fore-and-aft vertical plane through the compass. If the transverse iron is so situated, and extends across the ship, there will be induced a pole of North polarity on one side of the compass and a pole of South polarity on the opposite side at an equal distance from the compass and the vertical fore-and-aft section through it. One pole will repel and the other will attract the North point of the needle with an equal force, the components of which in the fore-and-aft line will be equal and of opposite sign; in other words, b will be zero.

If the transverse iron does not extend across the ship, but is broken, the nearer pole on one side will be of one polarity and the nearer pole on the other side will be of the opposite polarity, and if the iron is symmetrical with reference to the fore-and-aft section through the compass, b will reduce to zero for the reasons above given.

In like manner it may be shown that under the same circumstances the components in the vertical direction will have equal values with contrary signs; in other words, h will reduce to zero.

It is evident that f will be zero when the vertical iron is symmetrically distributed on each side of the vertical fore-and-aft plane through the compass, since the pole of the system will lie in that plane.

Fundamental Equations.—It follows then that a compass needle on board an iron or steel ship is acted on by the following forces whose components are given for the axes of X, Y, and Z, in order:

(1) The earth's magnetic force whose components in the three axes are

$$X, Y, \text{ and } Z.$$

(2) The resultant pole of the ship's subpermanent magnetism whose components are

$$P, Q, \text{ and } R.$$

(3) The magnetic force due to transient induced magnetism in fore-and-aft soft iron whose components are

$$aX, dX, \text{ and } gX.$$

(4) The magnetic force due to transient induced magnetism in transverse soft iron, whose components are

$$bY, eY, \text{ and } hY.$$

(5) The magnetic force due to transient induced magnetism in vertical soft iron whose components are

$$cZ, fZ, \text{ and } kZ.$$

Therefore, if X', Y', and Z' represent respectively the combined forces due to the magnetism of earth and ship in the axes of X, Y, and Z acting on the North point of the needle, then

$$X' = X + aX + bY + cZ + P. \qquad (17)$$

$$Y' = Y + dX + eY + fZ + Q. \qquad (18)$$

$$Z' = Z + gX + hY + kZ + R. \qquad (19)$$

These equations, first used by M. Poisson and now known by his name, form the groundwork of all equations used in the mathematical theory of the deviations of the compass.

From the above equations it is plain that for the subpermanent magnetism of the ship one or more permanent magnets, P, Q, and R, may be substituted, and for the soft iron of the ship, whatever may be its amount or direction, nine soft iron rods, *a, b, c, d, e, f, g, h,* and *k,* illustrated in Plate III, may be substituted; the magnetism induced in the rods *a, b,* and *c* will produce a force in the axis of *X,* + if to head, (—) if toward the stern; the magnetism induced in *d, e,* and *f* will produce a force in the axis of *Y,* + if to starboard, (—) if to port; and the magnetism induced in *g, h,* and *k* will produce a force in the axis of *Z,* + if vertically downward, (—) if upward.

Parameters.—The quantities *a, b, c, d, e, f, g, h, k, P, Q,* and *R* are called parameters and are constant; the first nine depending on the amount, arrangement, and capacity for induction of the soft iron of the ship, and the last three on the amount and arrangement of the hard iron.

The parameters *a, b, c, d, e, f, g, h,* and *k* are ratios though physically represented by rods. They are not forces but become forces only when multiplied by the inducing forces acting on the rods they represent. Thus the force induced in the rod *a* by the inducing force *X* is aX. A distinction must be drawn between the case in which the coefficient of a rod is +, and that in which it is (—).

These parameters are ratios between two forces and their signs depend upon the signs of the two forces. The components of the earth's force in the axes are taken as the inducing forces in those axes, and since the cosines of angles between 90° and 270° are negative, the inducing force in any axis is negative if the azimuth of that axis is between 90° and 270°. Thus, if the ship heads NE., the inducing force from starboard is $H \cos (90° + z) = -.707H$, or a force equal to $.707H$ acts from the port beam. Now the force induced in a rod is positive if it draws the North point of the needle to head, to starboard, or vertically downward.

The following cases will serve to illustrate the points discussed. In Fig. 44, the ship heading North, the inducing

PLATE III.

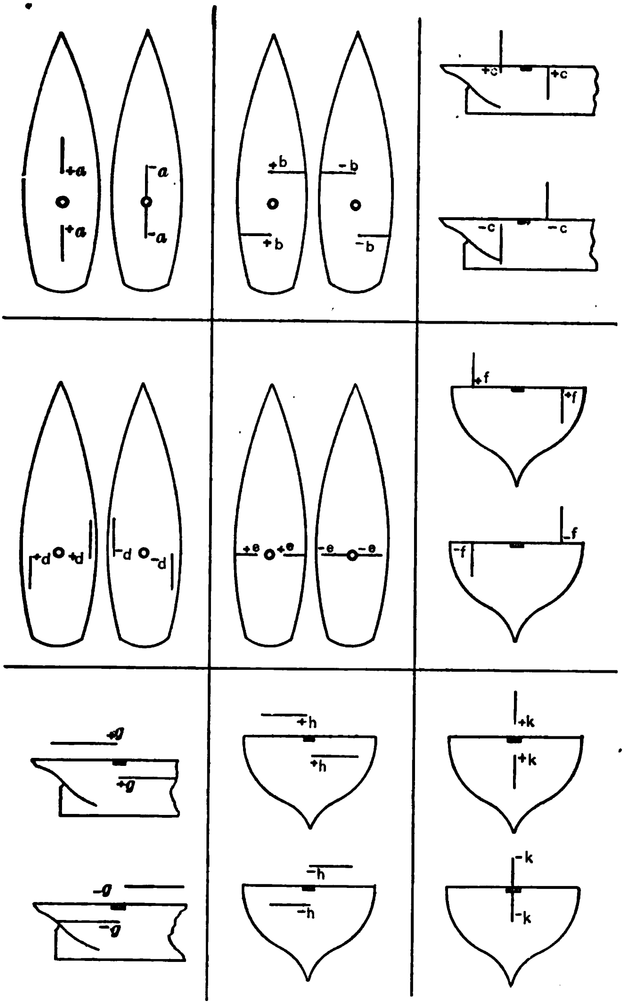


DIAGRAM SHOWING THE POSITIONS OF THE NINE SOFT IRON RODS WHICH REPRESENT THE WHOLE OF THE SOFT IRON OF A SHIP AS REGARDS ITS ACTION ON THE COMPASS.

force X from head is positive; the near end of the rod a has South polarity and draws the North end of the needle to head and the force induced aX is positive, therefore the ratio $= \frac{+aX}{+X} = +a$. In Fig. 45, the rod is acted upon by a negative force from ahead ($H \cos 180° = X$, or $H = (-)X$), or the inducing force acts from the stern; the near end of the rod has North polarity and repels the North end of the needle to the stern and the force induced aX is negative, therefore the ratio $= \frac{-aX}{-X} = +a$. The coefficient of this rod is thus seen to be $+a$.

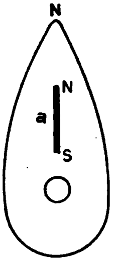


FIG. 44.

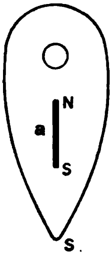


FIG. 45.

In the case of an a rod extending continuously through the compass, the signs of the induced forces would be the reverse of the above, whilst the signs of the inducing forces would be unaltered, and the ratio in both cases would be $-a$.

In Fig. 46, the ship heads NE., and the azimuth of the rod e is SE. The inducing force, ($H \cos (90° + z) = Y$), is $-Y$, and the starboard end has South magnetism induced in it. The force induced draws the needle to starboard and is $+eY$, therefore the ratio $= \frac{+eY}{-Y} = -e$. In Fig. 47, the ship heads NW. The inducing force is $+Y$. The starboard end of the rod has induced in it North polarity which repels the North point of the needle, the induced force is $-eY$, the ratio $= \frac{-eY}{+Y} = -e$. The coefficient of an athwartship rod extending on both sides of the compass is thus seen to be $-e$. In the case of an e rod entirely on one side of the compass the induced forces would have exactly the opposite effect, whilst the inducing forces would remain unchanged in sign and the ratio would be $+e$.

The signs of the coefficients of any of the remaining rods may, in a similar way, be proven to be as indicated in Plate III, remembering that it is convenient to consider the action on the North point of the needle of only the nearer end of a rod which is represented as entirely on one side of the compass, but that in the cases in which the rod continuously extends from one side of the compass to the other, as in the cases of $-a$, $-e$, and $-k$, either or both ends may be considered as acting.

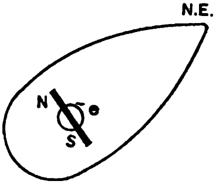


FIG. 46.

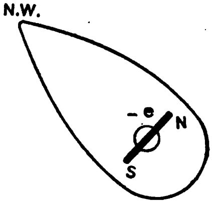


FIG. 47.

Five of the rods, a, c, e, g, and k, are symmetrically placed with reference to the fore-and-aft midship section; b, d, f, and h are not so placed.

If the soft iron of a ship may be supposed symmetrically distributed with reference to that plane, b, d, f, and h may each be considered equal to zero.

SECTION II.

75. Transformation of the fundamental equations.—To adapt Poisson's equations to the various uses to which a navigator may apply them, they must first be expressed in terms of quantities usually given or desired, that is, in terms of

H, the horizontal force of the earth;
H', the horizontal force of the earth and ship;
θ, the magnetic dip;
z, the magnetic course, or the azimuth of the ship's head measured eastward from the correct magnetic North;
z', the compass course, or the azimuth of the ship's head measured eastward from the direction of the compass North;
$\delta = z \sim z'$, the deviation of the compass due to the compass heading z' or the magnetic heading z.

Referring again to Fig. 43, if m is the resultant pole of the ship's magnetic force, attracting the North end of the needle, Od will be the force in the horizontal plane through the compass, and the components of Om in the three axes X, Y, and Z respectively will be Oa, Ob, and Of. The earth's directive force being OM, the ship's disturbing force Od, both in the horizontal plane, the needle will be acted upon by the resultant of these, $Or = H'$, and the components of Or in the axes of X and Y respectively will be $X' = On$ and $Y' = nr$. (For z' in Fig. 43, nr is to port, and hence is negative.)

Bearing in mind the definitions previously given, we have:

$$\begin{aligned} X &= H \cos z, \\ X' &= H' \cos z', \\ Y &= H \cos (90° + z) = -H \sin z, \\ Y' &= H' \cos (90° + z') = -H' \sin z', \\ Z &= H \tan \theta. \end{aligned}$$

Substituting these values in equations (17), (18), and

(19), dividing (17) and (18) by H and (19) by Z, we have:

$$\frac{H'}{H}\cos z' = (1+a)\cos z - b\sin z + c\tan\theta + \frac{P}{H}. \quad (20)$$

$$-\frac{H'}{H}\sin z' = d\cos z - (1+e)\sin z + f\tan\theta + \frac{Q}{H}. (21)$$

$$\frac{Z'}{Z} = \frac{g}{\tan\theta}\cos z - \frac{h}{\tan\theta}\sin z + 1 + k + \frac{R}{Z}. \quad (22)$$

Since H' is the force of the earth and ship in the direction of the disturbed needle, and since a force can be resolved into two components, in axes at right angles, all in the same plane, by multiplying it by the cosine of the angle its direction makes with each axis, and since the denominator in each equation denotes the unit of measure, equation (20) gives the force of earth and ship to head, equation (21) the force of earth and ship to starboard, each in terms of the earth's horizontal force as unit; equation (22) the force of earth and ship downward, in terms of the earth's vertical force as unit. As the azimuth of the ship's head may be anything from 0° to 360°, it is desirable to have these forces in two fixed directions,—in the magnetic meridian and at right angles to the meridian.

76. Force of earth and ship to magnetic North.—To find the components, in the direction of magnetic North, of the forces of earth and ship acting to head and to starboard, multiply (20) by $\cos z$; and (21) by $\cos(90° + z)$, or what amounts to the same thing, by $(-\sin z)$; and take the algebraic sum of the results. This will give the force of earth and ship to magnetic North.

Performing the operations indicated, we have:

$$\frac{H'}{H}\cos z'\cos z = (1+a)\cos^2 z - b\sin z\cos z$$
$$+ c\tan\theta\cos z + \frac{P}{H}\cos z.$$

Also,

$$\frac{H'}{H}\sin z'\sin z = -d\cos z\sin z$$
$$+ (1+e)\sin^2 z - f\tan\theta\sin z - \frac{Q}{H}\sin z.$$

And by algebraic addition,

$$\frac{H'}{H}(\cos z' \cos z + \sin z' \sin z) = -(d+b)\sin z \cos z + (1+a)\cos^2 z + (1+e)\sin^2 z + \left(c \tan \theta + \frac{P}{H}\right)\cos z - \left(f \tan \theta + \frac{Q}{H}\right)\sin z.$$

From plane trigonometry,

$$\cos z' \cos z + \sin z' \sin z = \cos(z \sim z') = \cos \delta,$$

$$\cos^2 z = \frac{1+\cos 2z}{2}, \sin^2 z = \frac{1-\cos 2z}{2},$$

$$\text{and } \sin z \cos z = \frac{\sin 2z}{2}.$$

Therefore, by substitution,

$$\frac{H'}{H}\cos \delta = -(d+b)\frac{\sin 2z}{2} + (1+a)\left(\frac{1+\cos 2z}{2}\right) + (1+e)\left(\frac{1-\cos 2z}{2}\right) + \left(c \tan \theta + \frac{P}{H}\right)\cos z - \left(f \tan \theta + \frac{Q}{H}\right)\sin z.$$

Collecting the terms involving the same function of the azimuth z, we have the force of earth and ship to magnetic North in terms of the earth's horizontal force as unit, expressed by the equation,

$$\frac{H'}{H}\cos \delta = 1 + \frac{a+e}{2} + \left(c \tan \theta + \frac{P}{H}\right)\cos z - \left(f \tan \theta + \frac{Q}{H}\right)\sin z + \frac{a-e}{2}\cos 2z - \frac{d+b}{2}\sin 2z. \qquad (23)$$

Force of earth and ship to magnetic East, and hence of ship alone, as the force of earth to East is always zero.—In a similar way, we may find the components in the direction of magnetic East of the forces of earth and ship acting to head and to starboard, multiplying (20) by cos (90° — z) = sin z, and (21) by cos z, and adding the results algebraically.

Performing these operations, making trigonometric substitutions and collecting the terms as before, we have the force of the ship alone to magnetic East in terms of the earth's horizontal force as unit, expressed by the equation,

$$\frac{H'}{H}\sin\delta = \frac{d-b}{2} + \left(c\tan\theta + \frac{P}{H}\right)\sin z + \left(f\tan\theta + \frac{Q}{H}\right)\cos z + \frac{a-e}{2}\sin 2z + \frac{d+b}{2}\cos 2z. \qquad (24)$$

If observations be made on four or more equidistant azimuths, the mean of results from (23) will be the mean value of the force of earth and ship to magnetic North; and the mean of results from (24) will be the mean value of the force of the ship to magnetic East.

The mean values from each equation will be represented by the constant terms, since the sum of two, four, or more equidistant values of sine or cosine is zero, and terms having sine or cosine as a factor will disappear.

Therefore, the mean value of the force of earth and ship to magnetic North in terms of H as unit is $1 + \frac{a+e}{2}$; this quantity, called λ, is generally less than unity in iron ships, a fact which indicates a mean directive force on the needle less than the earth's directive force in such ships.

The mean value of the force of ship to magnetic East in terms of H as unit is $\frac{d-b}{2}$. This quantity is + when the easterly deviations are in excess of the westerly deviations, (—) when the westerly deviations are in excess. This term reduces to zero when the soft iron of the ship is symmetrically distributed with reference to the fore-and-aft vertical section through the compass.

Letting $1 + \frac{a+e}{2}$ be represented by λ, and dividing (23) and (24) by λ, we have,

$$\frac{H'}{\lambda H}\cos\delta = 1 + \frac{1}{\lambda}\left(c\tan\theta + \frac{P}{H}\right)\cos z - \frac{1}{\lambda}\left(f\tan\theta + \frac{Q}{H}\right)\sin z + \frac{1}{\lambda}\,\frac{a-e}{2}\cos 2z - \frac{1}{\lambda}\,\frac{d+b}{2}\sin 2z, \tag{25}$$

and

$$\frac{H'}{\lambda H}\sin\delta = \frac{1}{\lambda}\,\frac{d-b}{2} + \frac{1}{\lambda}\left(c\tan\theta + \frac{P}{H}\right)\sin z + \frac{1}{\lambda}\left(f\tan\theta + \frac{Q}{H}\right)\cos z + \frac{1}{\lambda}\,\frac{a-e}{2}\sin 2z + \frac{1}{\lambda}\,\frac{d+b}{2}\cos 2z. \tag{26}$$

λH, which is the mean value of $H'\cos\delta$, or of the force of earth and ship to magnetic North, will be referred to hereafter as "the mean force to North."

To simplify these expressions, let their constant terms and the quantities connected with the various functions of the magnetic course be represented by the old English capital letters as follows:

$$\frac{1}{\lambda}\,\frac{d-b}{2} = \mathfrak{A},\quad \frac{1}{\lambda}\left(c\tan\theta + \frac{P}{H}\right) = \mathfrak{B},\quad \frac{1}{\lambda}\left(f\tan\theta + \frac{Q}{H}\right) = \mathfrak{C},$$

$$\frac{1}{\lambda}\,\frac{a-e}{2} = \mathfrak{D},\quad \frac{1}{\lambda}\,\frac{d+b}{2} = \mathfrak{E}.$$

Then by substituting these in (25) and (26), we have:

$$\frac{H'}{\lambda H}\cos\delta = 1 + \mathfrak{B}\cos z - \mathfrak{C}\sin z + \mathfrak{D}\cos 2z - \mathfrak{E}\sin 2z. \tag{27}$$

$$\frac{H'}{\lambda H}\sin\delta = \mathfrak{A} + \mathfrak{B}\sin z + \mathfrak{C}\cos z + \mathfrak{D}\sin 2z + \mathfrak{E}\cos 2z. \tag{28}$$

Equation (27) gives the combined force of earth and ship to magnetic North and equation (28) the force of the ship to magnetic East, both in terms of the "mean force to North" as unit of measure. The mean value of (28) is $\mathfrak{A}$, the magnitude and sign of which will indicate the excess of easterly over westerly deviations, or the reverse.

77. Formulæ for computing the deviations.—Dividing (28) by (27), we obtain

$$\tan \delta = \frac{\mathfrak{A} + \mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z + \mathfrak{E} \cos 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z - \mathfrak{E} \sin 2z} \qquad (29)$$

which will give the deviation (δ), by means of its tangent, on any magnetic course z when the five coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$ are known.

Substituting $\frac{\sin \delta}{\cos \delta}$ for $\tan \delta$ in (29) and clearing of fractions, we have:

$$\sin \delta + \mathfrak{B} \cos z \sin \delta - \mathfrak{C} \sin z \sin \delta + \mathfrak{D} \cos 2z \sin \delta - \mathfrak{E} \sin 2z \sin \delta = \mathfrak{A} \cos \delta + \mathfrak{B} \sin z \cos \delta + \mathfrak{C} \cos z \cos \delta + \mathfrak{D} \sin 2z \cos \delta + \mathfrak{E} \cos 2z \cos \delta.$$

Transposing and collecting terms,

$$\sin \delta = \mathfrak{A} \cos \delta + \mathfrak{B} (\sin z \cos \delta - \cos z \sin \delta) + \mathfrak{C} (\cos z \cos \delta + \sin z \sin \delta) + \mathfrak{D} (\sin 2z \cos \delta - \cos 2z \sin \delta) + \mathfrak{E} (\cos 2z \cos \delta + \sin 2z \sin \delta),$$

but from plane trigonometry,

since $z' = z - \delta$ and $2z' = 2 (z - \delta)$,

or $2 \left(z' + \frac{\delta}{2}\right) = (2z - \delta) = (2z' + \delta)$, we shall have

$$\sin \delta = \mathfrak{A} \cos \delta + \mathfrak{B} \sin z' + \mathfrak{C} \cos z' + \mathfrak{D} \sin (2z' + \delta) + \mathfrak{E} \cos (2z' + \delta). \qquad (30)$$

Equation (30) gives the deviations by means of its sine nearly, though not entirely, in terms of the compass course z'.

When the deviations are of moderate amount, say not more than 20°, equation (30) may be written with sufficient accuracy,

$$\delta = A + B \sin z' + C \cos z' + D \sin 2z' + E \cos 2z' \qquad (31)$$

in which the approximate coefficients, A, B, C, D, and E may be given in degrees and minutes and the deviation (δ) determined for the various compass courses in degrees and minutes, without introducing a greater error than 25′ in the computed deviation, even when near the agreed upon maximum limit of 20°.

In formula (30), the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$, called the exact coefficients of the various deviations produced, are in reality the forces producing those deviations expressed in terms of the "mean force to North," (λH), as unit; they are very nearly the natural sines of the angles represented by the approximate coefficients A, B, C, D, and E of equation (31), if δ is of moderate amount.[1]

When the soft iron is symmetrically arranged on each side of the fore-and-aft vertical plane through the compass, $b = 0$, $d = 0$, $f = 0$, $h = 0$, and therefore both $\mathfrak{A}$ and $\mathfrak{E}$ reduce to zero, and equations (29) and (31) become respectively,

$$\tan \delta = \frac{\mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z} \qquad (32)$$

and $$\delta = B \sin z' + C \cos z' + D \sin 2z'. \qquad (33)$$

Equation (30) will become, by developing the terms in the second member,

$$\sin \delta = \mathfrak{A} \cos \delta + \mathfrak{B} \sin z' + \mathfrak{C} \cos z' + \mathfrak{D} \sin 2z' \cos \delta$$
$$+ \mathfrak{D} \cos 2z' \sin \delta + \mathfrak{E} \cos 2z' \cos \delta - \mathfrak{E} \sin 2z' \sin \delta.$$

In this expression, δ being small, we can let $\cos \delta$ equal unity without material error, and the above becomes,

$$\sin \delta = \mathfrak{A} + \mathfrak{B} \sin z' + \mathfrak{C} \cos z' + \mathfrak{D} \sin 2z' +$$
$$\mathfrak{E} \cos 2z' + \mathfrak{D} \cos 2z' \sin \delta - \mathfrak{E} \sin 2z' \sin \delta.$$

$\mathfrak{E}$ is very small and $\sin \delta$ is very small, therefore the last term $\mathfrak{E} \sin 2z' \sin \delta$ is very small and may be neglected, hence, by transposing $\mathfrak{D} \cos 2z' \sin \delta$ and dividing through by the expression $(1 - \mathfrak{D} \cos 2z')$, we have:

$$\sin \delta = \frac{1}{1 - \mathfrak{D} \cos 2z'} \Big(\mathfrak{A} + \mathfrak{B} \sin z' + \mathfrak{C} \cos z' + \mathfrak{D} \sin 2z' + \mathfrak{E} \cos 2z' \Big), \qquad (34)$$

---

[1] The correct angular measure corresponding to any arc value is obtained by multiplying said arc by the radian 57°.3; thus $D = \mathfrak{D} \times 57°.3$.

which is very nearly exact and gives the deviations in terms of the exact coefficients and the compass courses.

When the soft iron is symmetrically distributed $\mathfrak{A}$ and $\mathfrak{E}$ will disappear in (34).

78. Subdivisions of the deviation.—In (31), the several parts of the deviation are:

A, the constant deviation due to transient magnetism induced in soft iron represented by parameters d and b, or to constant errors of observation, etc.;

$B \sin z' + C \cos z'$, the semicircular deviation due to subpermanent magnetism and the transient magnetism induced in vertical soft iron;

$D \sin 2z' + E \cos 2z'$, the quadrantal deviation. The first or larger part, $D \sin 2z'$, is due to transient magnetism in horizontal soft iron symmetrically situated with reference to the fore-and-aft vertical plane through the compass. The second, or smaller part, $E \cos 2z'$, is due to transient magnetism induced in horizontal unsymmetrically situated soft iron.

79. Relation between λ and $\mathfrak{D}$.—Both these coefficients depend for their value on the parameters a and e. The coefficient $\mathfrak{D}$, which equals $\frac{1}{\lambda}\frac{a-e}{2}$, represents the force in terms of λH which produces the larger part of the quadrantal deviation, the force itself being $\lambda H\mathfrak{D}$. This part is generally positive, being due to $+a$ and $-e$, or the excess of $-e$ over $-a$.

The horizontal force at the compass is that of the earth, the ship's subpermanent magnetism, and that induced in soft iron combined and acting in the direction of the compass needle, this resultant force being represented by H'. The component to magnetic North is $H' \cos \delta$ and the mean value of this component for an entire revolution of the ship's head is λH, which is generally less than unity, H itself being considered as unity. In the complete revolution of the ship's head, the increase of directive force due to the subpermanent magnetism in one semicircle exactly equals the decrease due to the same cause in the other semicircle, so the diminution in the value of λH is not due to subpermanent magnetism but arises from horizontal induction in soft iron represented as constituent parts of λ, therefore λ is generally less than unity and as it equals $1 + \frac{a+e}{2}$, the diminution is due to $-a$ and $-e$, or the excess of $-e$ over $+a$.

Since $\mathfrak{D}$ is usually positive, and λ is usually less than unity, it is to be inferred that the usual distribution of horizontal soft iron symmetrically situated may be represented by the arrangement of Fig. 48, or Fig. 49 in which the $+\mathfrak{D}$ would be due to the excess of effect of $-e$ over $-a$.

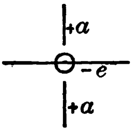


FIG. 48.

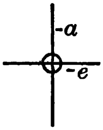


FIG. 49.

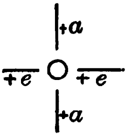


FIG. 50.

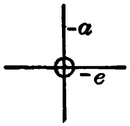


FIG. 51.

An arrangement of soft iron as shown by Fig. 50, in which $+a = +e$, would give λ a value greater than unity, or increase the "mean force to North"; that as shown by Fig. 51, in which $-a = -e$, would give λ a value less than unity, or diminish the "mean force to North"; but neither would cause any deviation.

When $\mathfrak{D}$ is caused by an arrangement of iron represented by parameters of one sign, it may be neutralized by compensators producing the effect of parameters of the opposite sign.

We may find a and e from λ and $\mathfrak{D}$ thus:

$$\left.\begin{aligned} \mathfrak{D} &= \frac{1}{\lambda}\ \frac{a-e}{2} \therefore 2\lambda\,\mathfrak{D} = a-e \\ \lambda &= 1+\frac{a+e}{2} \therefore 2\lambda-2 = a+e \end{aligned}\right\}\text{and}\ \begin{aligned} a &= \lambda(1+\mathfrak{D})-1. \qquad (35)\\ e &= \lambda(1-\mathfrak{D})-1. \qquad (36) \end{aligned}$$

80. Relation between $\mathfrak{A}$ and $\mathfrak{E}$.—These are the forces, in terms of λH, which cause respectively the constant deviation and the smaller part of the quadrantal deviation, and since $\mathfrak{A} = \frac{1}{\lambda}\frac{d-b}{2}$ and $\mathfrak{E} = \frac{1}{\lambda}\frac{d+b}{2}$, they are closely related; the forces causing these deviations arise from transient magnetism induced in horizontal soft iron unsymmetrically situated with reference to the fore-and-aft vertical plane through the compass.

They both reduce to zero when the soft iron is symmetrically distributed about the compass.

$\mathfrak{A}$ is marked + when the easterly deviation is in excess, otherwise (—).

The effect of $\mathfrak{E}$ is very small and is marked + when it causes easterly deviations between North and NE.

81. The coefficient $\mathfrak{B}$.—The fore-and-aft component of the magnetic force which causes semicircular deviation is $\lambda H \mathfrak{B}$, and $\mathfrak{B}$ represents that component if expressed in terms of λH as unit, in other words $\mathfrak{B}$ is the ship's polar force to head in terms of the "mean force to North" as unit. It is marked + if drawing the North point of the needle to head, otherwise (—).

Since $\mathfrak{B} = \frac{1}{\lambda}\left(c \tan \theta + \frac{P}{H}\right) = \frac{1}{\lambda H}(cZ + P)$, it is seen that the ship's polar force to head, expressed in terms of "mean force to North" as unit, is composed of (1) the fore-and-aft component of the ship's subpermanent magnetism P, + if acting to head, (—) if to the stern; (2) the fore-and-aft component of the force due to transient magnetism induced in vertical soft iron, or cZ, + if acting to head, (—) if to the stern.

That part due to subpermanent magnetism varies inversely as H, and that part due to vertical induction in vertical soft iron varies directly as the tangent of the magnetic dip.

The force $\mathfrak{B}$ may be counteracted by a fore-and-aft magnet with its center in a transverse vertical plane passing through the compass, so placed that the North end is forward if $\mathfrak{B}$ is $+$, aft if $\mathfrak{B}$ is $(-)$.

Regarding $\mathfrak{A}$ and $\mathfrak{E}$ as zero, equation (32) shows $\mathfrak{B}$ to be the only force causing deviation when the ship heads magnetic East or West, at which time it exerts almost its maximum effect; therefore to neutralize $\mathfrak{B}$, head the ship magnetic East or West, and move a fore-and-aft system of magnets, placed as in the preceding paragraph, towards or from the compass till the compass heading is East, or West, as the case may be.

82. The coefficient $\mathfrak{C}$.—The athwartship component of the magnetic force which causes semicircular deviation is $\lambda H\mathfrak{C}$, and $\mathfrak{C}$ represents that component if expressed in terms of λH as unit, in other words $\mathfrak{C}$ is the ship's polar force to starboard in terms of the "mean force to North" as unit. It is marked $+$ if drawing the North point of the needle to starboard, $(-)$ if to port.

Since $\mathfrak{C} = \frac{1}{\lambda}\left(f \tan\theta + \frac{Q}{H}\right) = \frac{1}{\lambda H}(fZ + Q)$, it is seen that the ship's polar force to starboard, expressed in terms of the "mean force to North" as unit, is composed of (1) the athwartship component of the ship's subpermanent magnetism Q, $+$ if acting to starboard, $(-)$ if acting to port; (2) the athwartship component of the force due to transient magnetism induced in vertical soft iron, or fZ, $+$ if acting to starboard, $(-)$ if to port.

That part due to subpermanent magnetism varies inversely as H, and that part due to vertical induction in vertical soft iron varies directly as the tangent of the magnetic dip.

The force $\mathfrak{C}$ may be counteracted by an athwartship magnet with its center in a fore-and-aft vertical plane passing through the compass, so placed that the North end is to starboard if $\mathfrak{C}$ is $+$, to port if $\mathfrak{C}$ is $(-)$.

Regarding $\mathfrak{A}$ and $\mathfrak{E}$ as zero, equation (32) shows $\mathfrak{C}$ to be the only force causing deviation when the ship heads magnetic North or South, at which time it exerts almost its maximum effect; therefore to neutralize $\mathfrak{C}$, head the ship magnetic North or South, and move an athwartship system of magnets, placed as in the preceding paragraph, towards or from the compass till the compass heading is North, or South, as the case may be.

83. The ship's polar force and the starboard angle α.— Both $\lambda H\mathfrak{B}$ and $\lambda H\mathfrak{C}$, in other words their resultant $\lambda H\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$, the ship's horizontal polar force, may be corrected by a single magnet, or a system of parallel magnets, whose center is immediately below the compass center and whose axis is horizontal and makes an angle α with the fore-and-aft line through the compass. This angle $= \tan^{-1}\dfrac{\mathfrak{C}}{\mathfrak{B}}$; it is called the starboard angle, and is measured from ahead around to the right. In other words, there is a magnetic force $\lambda H\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ drawing the North point of the needle toward a fixed point in the ship of South polarity, the direction of which with the fore-and-aft line, measured as above, is α. In compensation that field of South polarity must be neutralized by one of North polarity.

84. Variations in the parameters: The Gaussin error.— Poisson's equations are based on the hypothesis that the magnetism of a ship is partly permanent and partly transient, that in consequence all the parameters are constant and all the exact coefficients, except $\mathfrak{B}$ and $\mathfrak{C}$, are constant; the change in $\mathfrak{B}$ and $\mathfrak{C}$ taking place only on change of magnetic latitude, and then because they involve the dip and horizontal force. The hypothesis is not absolutely true, the magnetism due to hard iron is only subpermanent, and the transient magnetism is never that due to the inducing force at the time and place but that due to a force of some previous time and place; in other words there is a retardation in induction, and this occurs whether the ship is on a straight course or is turning in a circle.

These effects may be considered as unimportant in all soft iron except the parameters *a*, *e*, and *g*, which are not only comparatively large in amount, but cut the lines of force at all angles as the ship swings in azimuth, so that any slowness in receiving or parting with a full charge of magnetism has an important bearing on the deviation.

Take the rod (—) *e*, for example, in the case of a vessel swinging with starboard helm from a heading East; its port end having come from the North repels, and its starboard end having come from the South attracts the North end of the needle, even when the rod is at right angles to the line of force and when, by the hypothesis, it should fail of effect. The force due to this lagging of the magnetism causes a $+ \mathfrak{A}$ and a $+ \mathfrak{E}$ when the ship swings to the left, a (—) $\mathfrak{A}$ and a (—) $\mathfrak{E}$ when it swings to the right. This error is known as the "Gaussin error" after M. Gaussin who first called attention to it. For accuracy a ship should either be swung with both helms and a mean of the two deviations taken; or swung very slowly with one helm, the ship being kept steady on each rhumb at least four minutes. When only one helm has been used, it is proper to leave uncorrected at North and South a little easterly error if ship was swung to the left, a little westerly error if swung to the right, in all cases when the ship was not swung very slowly.

SECTION III.

85. Determination of the coefficients. Method of least squares.—It has been shown that the deviation (if of moderate amount, say not exceeding 20°), may be expressed for any heading z' per compass by the formula

$$\delta = A + B \sin z' + C \cos z' + D \sin 2z' + E \cos 2z',$$

which contains five unknown quantities.

If the deviation is observed on five different headings, these coefficients may be determined by elimination from the resulting equations; but, if observations are made on more than five headings, there will be more than five equations, and the values of the unknown coefficients found from any five of them will in general not satisfy the others by an amount called the error which may be + or (—).

The greater the number of observations to determine an unknown quantity, the more accurate will be its value, provided the observations are all carefully taken and are equally trustworthy; therefore if we can determine the five coefficients from observations on as many as 24 headings so much the better, and, if not on 24, then on as many as possible. The number of the observation equations being greater than the number of the unknown quantities, they must be so adjusted as to give the most probable values of the five quantities sought.

The "Theory of Probability of Errors" proves that the most probable values of unknown quantities are those that reduce the sum of the squares of the errors to a minimum.

Suppose the observations, greater in number than the number of the unknown quantities, give rise to equations of condition of the form

$$\left.\begin{array}{l} a_1 + b_1 x + c_1 y + d_1 z + \ldots\ldots = 0 \\ a_2 + b_2 x + c_2 y + d_2 z + \ldots\ldots = 0 \\ a_3 + b_3 x + c_3 y + d_3 z + \ldots\ldots = 0 \\ \qquad\qquad \text{etc., etc.,} \end{array}\right\} \qquad (37)$$

in which a_1, a_2, a_3, b_1, b_2, b_3, etc., are known quantities and x, y, z, etc., are unknown quantities.

When the most probable values of the unknown quantities are substituted in group (37), none of the equations will be fully satisfied; in other words each will fail to reduce to zero by a small error; E_1 in the first, E_2 in the second, and so on, and the equations will become

$$\left.\begin{array}{c} a_1 + b_1 x + c_1 y + d_1 z + \ldots\ldots = E_1 \\ a_2 + b_2 x + c_2 y + d_2 z + \ldots\ldots = E_2 \\ a_3 + b_3 x + c_3 y + d_3 z + \ldots\ldots = E_3 \\ \text{etc., etc.,} \end{array}\right\} \quad (38)$$

and the sum of the squares of the errors will be a minimum.

To find the most probable value of x from group (38), it is only necessary to consider x; so the terms independent of x may be represented in the left hand member of each equation in both groups (37) and (38) by M_1, M_2, M_3, etc., respectively, and we will have from (38),

$$\left.\begin{array}{c} b_1 x + M_1 = E_1 \\ b_2 x + M_2 = E_2 \\ b_3 x + M_3 = E_3 \\ \text{etc., etc.} \end{array}\right\} \quad (39)$$

Squaring both members of each equation and adding, we have

$$\begin{array}{c} (b_1 x + M_1)^2 + (b_2 x + M_2)^2 + (b_3 x + M_3)^2 + \ldots\ldots \\ = E^2{}_1 + E^2{}_2 + E^2{}_3 + \text{etc.} \end{array} \quad (40)$$

To find the most probable value of x, we must make the sum of the squares of the errors a minimum, or, what amounts to the same thing, make the left hand member of equation (40) a minimum.

Differentiating it with respect to x and placing the first differential equal to zero, after dividing through by the common factor $2dx$, we have

$$\begin{array}{c} (b_1 x + M_1)\, b_1 + (b_2 x + M_2)\, b_2 \\ + (b_3 x + M_3)\, b_3 + \ldots = 0 \end{array} \quad (41)$$

which gives the most probable value of x.

The equation (41) is called the normal equation in x and equals the sum of the equations in group (37) taken after each equation has been multiplied by the coefficient of x in that equation.

In the same way, letting N_1, N_2, N_3, etc., represent in the equations of group (38) all the terms independent of y, we will have the normal equation in y

$$\begin{aligned}(c_1 y + N_1)\, c_1 + (c_2 y + N_2)\, c_2 \\ + (c_3 y + N_3)\, c_3 + \ldots . = 0\end{aligned} \qquad (42)$$

which gives the most probable value of y. This normal in y equals the sum of the equations in group (37) taken after each equation has been multiplied by the coefficient of y in that equation.

Similarly, we may find the normal equation in z, and in all the other unknown quantities.

The normal equations will be the same in number as the unknown quantities, and the value of these quantities obtained therefrom will be the most probable value.

Comparing the normal equations with the equations of condition, the following rule for the formation of the normals is evident.

"Multiply each equation of condition by the coefficient of each unknown quantity in that equation taken with its sign. The sum of the resulting equations in which the coefficients of x are multipliers will be the normal equation in x, and similarly for the others."

86. Let δ_0, δ_1, δ_2, etc., be the deviations obtained from observations when the ship heads on the 15° rhumbs per compass represented by z'_0, z'_1, z'_2, etc., then (31) will become

$$\left.\begin{aligned}
\delta_0 &= A + B \sin z'_0 + C \cos z'_0 + D \sin 2z'_0 + E \cos 2z'_0 \\
\delta_1 &= A + B \sin z'_1 + C \cos z'_1 + D \sin 2z'_1 + E \cos 2z'_1 \\
\delta_2 &= A + B \sin z'_2 + C \cos z'_2 + D \sin 2z'_2 + E \cos 2z'_2 \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\delta_{23} &= A + B \sin z'_{23} + C \cos z'_{23} + D \sin 2z'_{23} + E \cos 2z'_{23}
\end{aligned}\right\} \quad (43)$$

In the above the sines and cosines may be replaced by the symbols S_0, S_1, S_2, etc., representing the sines of angles of 0°, 15°, 30°, etc., remembering, however, that the cosine of an angle is the sine of its complement, that is, $\cos z_1 = \sin z_5$, $\cos z_2 = \sin z_4$, etc., also that $\sin 2z_1 = \sin z_2$, or $\sin 2z_1 = S_2$, etc.

For uniformity's sake, the symbols S_0 and S_6 which are respectively zero and unity will appear as multipliers.

Strict attention must be paid to the signs of the functions on the azimuths used.

Making the proper substitutions we have

$$\left.\begin{array}{l} \delta_0 = A + BS_0 + CS_6 + DS_0 + ES_6 \\ \delta_1 = A + BS_1 + CS_5 + DS_2 + ES_4 \\ \delta_2 = A + BS_2 + CS_4 + DS_4 + ES_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \delta_{23} = A - BS_1 + CS_5 - DS_2 + ES_4 \end{array}\right\} \qquad (44)$$

which are the 24 observation equations, or equations of condition, from which the five normal equations must be found by the method of least squares as expressed in the rule of Article 85, before the coefficients A, B, C, D, and E can be found.

To form the normal equation in A.—Multiply each equation of group (44) by the coefficient of A in it; the coefficient in each case being unity, no change will be made in any term. Then adding all members on each side of the equality sign, we shall have.

$$\delta_0 + \delta_1 + \delta_2 + \delta_3 + \ldots . \delta_{23} = 24A. \qquad (45)$$

Since the sines and cosines of courses differing by 180° are numerically the same with opposite signs, their sum is zero, and the summation of sines, or cosines, on an even number of equidistant azimuths is zero; hence terms involving B, C, D, and E do not appear in (45).

Equation (45) may be put in the following form:

$$\left.\begin{aligned}&\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_0+\delta_{12})+\tfrac{1}{2}(\delta_6+\delta_{18})\right\}+\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_1+\delta_{13})+\tfrac{1}{2}(\delta_7+\delta_{19})\right\}\\+&\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_2+\delta_{14})+\tfrac{1}{2}(\delta_8+\delta_{20})\right\}+\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_3+\delta_{15})+\tfrac{1}{2}(\delta_9+\delta_{21})\right\}\\+&\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_4+\delta_{16})+\tfrac{1}{2}(\delta_{10}+\delta_{22})\right\}+\tfrac{1}{2}\left\{\tfrac{1}{2}(\delta_5+\delta_{17})+\tfrac{1}{2}(\delta_{11}+\delta_{23})\right\}\end{aligned}\right\}=6A, \qquad (46)$$

which explains the formation of columns (5) and (11) of the analysis sheet, Article 89, and the steps leading to the finding of A; for instance, $\frac{1}{2}(\delta_0+\delta_{12})$ will give the mean value of A and E on N. and S.; $\frac{1}{2}(\delta_6+\delta_{18})$ will give the mean value of A and E on E. and W.; the A will have the same sign in both cases, the E will have the contrary sign in each case; therefore, $\frac{1}{2}\left\{\frac{1}{2}(\delta_0+\delta_{12})+\frac{1}{2}(\delta_6+\delta_{18})\right\}$ will give the mean value of A on four rhumbs, and as each term in a bracket represents a mean value of A for four rhumbs, we shall have $6\,A$ on the opposite side of the equality sign in (46).

To form the normal in B.—Multiply each equation of group (44) by the coefficient of B in that particular equation, having due regard for the signs; in other words multiply the first equation by S_0, the second by S_1, etc., then add results on both sides of the equality sign. On the right hand side all the coefficients except B will disappear as the summation of the resulting multipliers for them will consist of two parts, each part containing identical terms with opposite signs; therefore,

$$\begin{aligned}&\delta_0S_0+\delta_1S_1+\delta_2S_2+\ldots\ldots-\delta_{23}S_1\\&\quad=B\left\{2\,\overline{(S_0^2+S_6^2)}+4\,(\overline{S_1^2+S_5^2}+\overline{S_2^2+S_4^2}+S_3^2)\right\}\\&\quad=B\left\{2+4\,(1+1+\tfrac{1}{2})\right\}=12B\end{aligned}$$

and the normal in B is, for observations on 24 15°-rhumbs,

$$\delta_0\,S_0+\delta_1\,S_1+\delta_2\,S_2\;\ldots\ldots-\delta_{23}\,S_1=12B. \qquad (47)$$

The grouping of the deviations in the analysis sheet, the finding of column (6) marked semicircular deviation, and the use of the divisor 6 in finding B from observations on 24 headings is thus explained; *i. e.,* since δ_0 is multiplied by the same quantity numerically as δ_{12}, δ_1 the same as δ_{13}, etc., but with a different sign, and since semicircular deviation is the algebraic difference of the deviations on opposite rhumbs divided by 2, if the results in (47) be grouped according to the multipliers and divided by 2, then (47) will become

$$\begin{aligned}\tfrac{1}{2}(\delta_0-\delta_{12})S_0+\tfrac{1}{2}(\delta_1-\delta_{13})S_1+\tfrac{1}{2}(\delta_2-\delta_{14})S_2&\\ +\;\ldots\;\tfrac{1}{2}(\delta_{11}-\delta_{23})S_1=6B,&\end{aligned}\tag{48}$$

which explains the steps pursued in the analysis sheet leading to the determination of B.

To form the normal equation in C.—Multiply each of the 24 equations of condition of group (44) by the coefficient of C in its own equation, and proceed as in finding the normal in B. All the coefficients, except C, will disappear as the summation of resulting multipliers for them will consist of two parts, each part containing identical terms with opposite signs, and we shall have the normal equation in C:

$$\delta_0 S_6+\delta_1 S_5+\delta_2 S_4+\;\ldots\;\delta_{23} S_5=12\,C. \tag{49}$$

This may be put under the following form:

$$\begin{aligned}\tfrac{1}{2}(\delta_0-\delta_{12})S_6+\tfrac{1}{2}(\delta_1-\delta_{13})S_5+\tfrac{1}{2}(\delta_2-\delta_{14})S_4&\\ +\;\ldots\;\tfrac{1}{2}(\delta_{11}-\delta_{23})(-S_5)=6\,C,&\end{aligned}\tag{50}$$

which explains the formation and use of column (6) of the analysis sheet and the steps leading to the determination of C from observations on 24 equidistant 15° rhumbs.

To find the normal equation in D.—By the same rule and similar methods, as in the cases of B and C, we will obtain the normal in D:

$$\delta_0 S_0+\delta_1 S_2+\delta_2 S_4+\delta_3 S_6+\ldots-\delta_{23} S_2=12\,D. \tag{51}$$

Since the functions of twice the azimuth are used, and since S_0 may be considered + or (—), the following will have negative multipliers, δ_6 up to δ_{11} inclusive, and δ_{18} to δ_{23} in clusive. If equation (51) is grouped by multipliers and divided twice by 2, we shall have

$$\left.\begin{array}{l} \frac{1}{2}\{\frac{1}{2}(\delta_0+\delta_{12})-\frac{1}{2}(\delta_6+\delta_{18})\}S_0+\frac{1}{2}\{\frac{1}{2}(\delta_1+\delta_{13})-\frac{1}{2}(\delta_7+\delta_{19})\}S_2 \\ +\frac{1}{2}\{\frac{1}{2}(\delta_2+\delta_{14})-\frac{1}{2}(\delta_8+\delta_{20})\}S_4+\frac{1}{2}\{\frac{1}{2}(\delta_3+\delta_{15})-\frac{1}{2}(\delta_9+\delta_{21})\}S_6 \\ +\frac{1}{2}\{\frac{1}{2}(\delta_4+\delta_{16})-\frac{1}{2}(\delta_{10}+\delta_{22})\}S_4+\frac{1}{2}\{\frac{1}{2}(\delta_5+\delta_{17})-\frac{1}{2}(\delta_{11}+\delta_{23})\}S_2 \end{array}\right\}=3D. \qquad (52)$$

which explains the formation of columns (5) and (12) and the steps leading to the determination of D in the analysis sheet.

The normal equation in E.—By the same rule and methods similar to those pursued in the case of D, we have the normal equation in E:

$$\delta_0 S_6+\delta_1 S_4+\delta_2 S_2 \ldots\ldots+\delta_{23} S_4=12E. \qquad (53)$$

If equation (53) is grouped by multipliers and divided twice by 2, we shall have

$$\left.\begin{array}{l} \frac{1}{2}\{\frac{1}{2}(\delta_0+\delta_{12})-\frac{1}{2}(\delta_6+\delta_{18})\}\quad S_6\quad+\frac{1}{2}\{\frac{1}{2}(\delta_1+\delta_{13})-\frac{1}{2}(\delta_7+\delta_{19})\}\quad S_4 \\ +\frac{1}{2}\{\frac{1}{2}(\delta_2+\delta_{14})-\frac{1}{2}(\delta_8+\delta_{20})\}\quad S_2\quad+\frac{1}{2}\{\frac{1}{2}(\delta_3+\delta_{15})-\frac{1}{2}(\delta_9+\delta_{21})\}\quad S_0 \\ +\frac{1}{2}\{\frac{1}{2}(\delta_4+\delta_{16})-\frac{1}{2}(\delta_{10}+\delta_{22})\}\{-S_2\}+\frac{1}{2}\{\frac{1}{2}(\delta_5+\delta_{17})-\frac{1}{2}(\delta_{11}+\delta_{23})\}\{-S_4\} \end{array}\right\}=3E \qquad (54)$$

which explains the steps leading up to the determination of E in the analysis sheet.

If observations are taken on only 12 equidistant headings, the divisors in the analysis sheet will be for A, B, and C, 3 instead of 6, and for D and E, $\frac{3}{2}$ instead of 3.

If taken on only the eight principal rhumbs, the divisors for $A, B,$ and C will be 2, and for D and E unity.

87. The expression

$$\delta=A+B\sin z'+C\cos z'+D\sin 2z'+E\cos 2z'$$

considers the deviation as composed of only the constant, semicircular, and quadrantal deviations, while in fact terms of a sextantal type, octantal type, etc., may exist.

This is shown by the following equation:

$$\begin{aligned} \delta = {} & A+B\sin z'+C\cos z'+D\sin 2z'+E\cos 2z' \\ & +F\sin 3z'+G\cos 3z'+H\sin 4z'+K\cos 4z' \qquad (55) \\ & +L\sin 5z'+M\cos 5z'+N\sin 6z'. \end{aligned}$$

which is obtained from (30) by a series of expansions, substitutions, and eliminations, and which is exact to terms of the third order inclusive.

88. Determination of exact coefficients.—For all practical purposes the deviation is expressed with sufficient accuracy by the five approximate coefficients; if, however, greater accuracy is required, as when the deviation much exceeds 20°, it should be expressed by the exact coefficients. These may be determined by the method of least squares, or from the following equations involving the approximate coefficients:

$$\mathfrak{A} = \sin A. \tag{56}$$

$$\mathfrak{B} = \sin B \left(1 + \tfrac{1}{2} \sin D + \tfrac{1}{12} \operatorname{versin} B - \tfrac{1}{4} \operatorname{versin} C\right) + \tfrac{1}{2} \sin C \times \sin E. \tag{57}$$

$$\mathfrak{C} = \sin C \left(1 - \tfrac{1}{2} \sin D - \tfrac{1}{4} \operatorname{versin} B + \tfrac{1}{12} \operatorname{versin} C\right) + \tfrac{1}{2} \sin B \times \sin E. \tag{58}$$

$$\mathfrak{D} = \sin D \left(1 + \tfrac{1}{3} \operatorname{versin} D\right). \tag{59}$$

$$\mathfrak{E} = \sin E - \sin A \times \sin D. \tag{60}$$

SECTION IV.

89. Analysis of deviations and the use of the form.—The deviations entered on this form should be for the 15° compass rhumbs, and, if the observations were not on these rhumbs, plot the deviations on a Napier diagram, and take off the total deviation on each 15° compass rhumb and transfer it to column 2 or 4, Table I, of the form. The following example will illustrate the process:

Ex. 8.—Compute the approximate and exact coefficients from the deviation table found in Ex. 4, Art. 56. Find also the starboard angle *a*.

(1) Write down the observed deviations in columns (2) and (4), opposite the proper rhumbs, prefixing the sign + to the easterly deviations, the sign (—) to the westerly deviations.

(2) Form column (5) by taking half the algebraic sum of columns (2) and (4). Since the constant and quadrantal deviations have the same sign and the semicircular has the opposite sign on azimuths differing 180°, this process eliminates the semicircular deviation, and column (5) records the constant and quadrantal deviation on the equidistant 15° rhumbs per compass from 0° to 165° inclusive, or from 180° to 345° inclusive.

(3) Form column (6) by taking half the algebraic difference of columns (2) and (4); or what is the same thing change mentally the signs of the quantities in column (4), then take half the sum of columns (2) and (4), entering results in column (6). This process eliminates the constant and quadrantal deviations, column (6) being the semicircular deviation on the equidistant 15° rhumbs per compass from 0° to 165° inclusive, or from 180° to 345° inclusive, if we consider the signs changed.

The correctness of columns (5) and (6) may be proved by adding them algebraically; the sum of the quantities opposite any heading should equal the quantity in column (2) opposite the same heading.

(4) Since the semicircular deviation on any compass azimuth of the ship's head z' is $B \sin z' + C \cos z'$, the quantities in column (6) are multiplied by the multipliers set opposite them in columns (7) and (8) to form respectively the products of columns (7) and (8), the first set of multipliers represented by S_0, S_1, S_2 . . . . S_6 being the natural sines of the rhumbs 0°, 15°, 30° . . . . 90° respectively, and the second set represented by S_6, S_5 . . . . S_0 being the natural cosines of the same rhumbs, S_0 being 0 and S_6 unity. The multiplication is facilitated by Table IV of this book, or by Table X of "Diehl's Compensation of the Compass."

When the angle is greater than the tabulated arc, as 42° 10′, it may be divided into two parts, each part to come within the limit of the table, as 30° and 12° 10′, and the sum of the results for the two parts taken.

In these multiplications careful attention must be paid to the rule of signs; that is, + multiplied by +, or (—) multiplied by (—) gives +, and + multiplied by (—) gives (—).

The algebraic sum of the products in each of the columns (7) and (8) divided by 6, 3, or 2, according as the observations were taken on 24, the 12 or 8 principal compass rhumbs, will give from column (7) the approximate coefficient B and from column (8) the approximate coefficient C.

An approximate check on B may be obtained by taking the mean of the deviations at East and at West, the sign of the latter being changed; an approximate check on C may be obtained by taking the mean of the deviations at North and South, the sign of the latter being changed.

(5) Proceed to find A, D, and E, following in Table II a process similar to that followed in finding B and C.

Write down the upper half of column (5) in column (9) and the lower half of column (5) in column (10). From columns (9) and (10), form columns (11) and (12) in the same way in which we formed columns (5) and (6) of Table I, and prove their correctness in the same manner.

It will be readily seen that by this process we have separated the constant and quadrantal deviations; column (11) is the constant part of the deviation, each of the eight values being derived from the deviations on four rhumbs of the compass 90° from each other, being the mean of the deviations as represented in the brackets of the left-hand member of equation (46).

(6) The sum of the quantities in column (11) divided by 6, 3, or 2 according as the observations were taken on the 24, 12, or 8 principal rhumbs will give the value of A. An approximate check on A may be obtained by taking the algebraic mean of the deviations on the 4 cardinal points.

Column (12) is the quadrantal deviation on 15° rhumbs from 0° to 75° and from 180° to 255°; or, with the signs changed, the quadrantal deviation from 90° to 165°, or from 270° to 345°. Each of the eight values in column (12) is derived from the deviations on four rhumbs of the compass 90° apart, as shown in equations (52) and (54).

(7) Since the quadrantal deviation on any compass heading of the ship z' is $D \sin 2z' + E \cos 2z'$, the quantities in column (12) are multiplied by the multipliers set opposite them in columns (13) and (14) to form respectively the products of columns (13) and (14), the first set of multipliers being the natural sines of twice the azimuth of the ship's head, the second set being the natural cosines of twice the same azimuths.

In these multiplications careful attention must be given to the signs.

The algebraic sum of the products in each of the columns (13) and (14) divided by 3, $\frac{3}{2}$, or unity, according as the observations were taken on 24, the 12 or 8 principal compass rhumbs, will give from column (13) the approximate coefficient D and from column (14) the approximate coefficient E.

An approximate check on D is the mean of the deviations on the quadrantal points, the signs of the deviations on SE. and NW. being changed before the mean is taken.

An approximate check on E. is the mean of the deviations on N., S., E., and W., the signs of the latter two being changed.

ANALYSIS OF DEVIATIONS OF THE STANDARD COMPASS OF A U. S. MONITOR.

Place of Observation, Annapolis, Md.; Latitude, 38° 58′ 53″ N.; Longitude, 76° 29′ W.

TABLE I.—COMPUTATION OF COEFFICIENTS B AND C.

(1)	(2)	(3)	(4)	(5)	(6)	(7) Computation of B.		(8) Computation of C.	
Ship's Head by Standard Compass.	Deviation. Easterly + Westerly—	Ship's Head by Standard Compass.	Deviation. Easterly + Westerly—	Half sum of Cols. (2) and (4).	Half sum of Cols. (2) and (4) [changing sign in Col. (4)]. Semicircular deviation.	Multi-pliers.	Products of Col. (6) by Multipliers.	Multi-pliers.	Products of Col. (6) by Multi-pliers.
°	° ′	°	° ′	° ′	° ′		° ′		° ′
0	− 6 15	180	+ 4 35	−0 50	− 5 25	0	0 00	1	−5 25
15	− 0 50	195	+ 2 30	+0 50	− 1 40	S_1	− 0 26	S_5	−1 37
30	+ 3 50	210	0 00	+1 55	+ 1 55	S_2	+ 0 58	S_4	+1 40
45	+ 8 25	225	− 3 25	+2 30	+ 5 55	S_3	+ 4 11	S_3	+4 11
60	+ 9 50	240	− 6 30	+1 40	+ 8 10	S_4	+ 7 04	S_2	+4 05
75	+11 20	255	− 9 20	+1 00	+10 20	S_5	+ 9 59	S_1	+2 40
90	+12 00	270	−12 20	−0 10	+12 10	1	+12 10	0	0 00
105	+11 20	285	−14 20	−1 30	+12 50	S_5	+12 24	$-S_1$	−3 19
120	+10 20	300	−16 00	−2 50	+13 10	S_4	+11 24	$-S_2$	−6 35
135	+ 8 55	315	−16 35	−3 50	+12 45	S_3	+ 9 01	$-S_3$	−9 01
150	+ 7 30	330	−13 30	−3 00	+10 30	S_2	+ 5 15	$-S_4$	−9 06
165	+ 6 15	345	−10 25	−2 05	+ 8 20	S_1	+ 2 09	$-S_5$	−8 03
					Sum of + terms =		+74° 35′		+12° 36′
					Sum of − terms =		− 0 26		−43 06
					Divisor 6)		+74° 09′	6)	−30° 30′
					B =		+12° 21′.5	C =	− 5° 05′

This form is adapted to a computation from observations on 24 equidistant compass headings. It may be used for 12 or 8 equidistant observations by omitting the intermediate headings, but the divisors for finding A, B, and C must be 3 or 2, respectively, and those for finding D and E must be $\frac{3}{2}$ or 1, respectively.

TABLE II.—COMPUTATION OF COEFFICIENTS A, D, E.

(9)	(10)	(11)	(12)	(13) Computation of D.		(14) Computation of E.		Multipliers for Computing B, C, D, E.
Upper half of Col. (5), Table I.	Lower half of Col. (5), Table I.	Half sum of Cols. (9) and (10). — Constant Deviation.	Half sum of Cols. (9) and (10). [changing sign in Col. (10)]. — Quadrantal Deviation.	Multipliers.	Products of Col. (12) by Multipliers.	Multipliers.	Products of Col. (12) by Multipliers.	
° ′	° ′	° ′	° ′		° ′		° ′	$S_1 = .259$
−0 50	−0 10	−0 30	−0 20	0	0 00	1	−0 20	$S_2 = .500$
+0 50	−1 30	−0 20	+1 10	S_2	+0 35	S_4	+1 01	$S_3 = .707$
+1 55	−2 50	−0 27	+2 22	S_4	+2 03	S_2	+1 11	$S_4 = .866$
+2 30	−3 50	−0 40	+3 10	1	+3 10	0	0 00	$S_5 = .966$
+1 40	−3 00	−0 40	+2 20	S_4	+2 01	$-S_2$	−1 10	$S_6 = 1$
+1 00	−2 05	−0 33	+1 33	S_2	+0 47	$-S_4$	−1 21	$S_0 = 0$
Sum of + terms = +0° 00′			Sum of + terms = +8° 36′				+2° 12′	
Sum of − terms = −3 10			Sum of − terms = −......				−2 51	
Divisor 6) −3° 10′			Divisor 3) +8° 36′			3) −0° 39′		
A = −0° 31′.7			D = +2° 52′			E = −0° 13′		

TABLE III.—COMPUTATION OF EXACT COEFFICIENTS.

$\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$.

	A	B	C	D	E
Angles	$-0^\circ 31'.7$	$+12^\circ 21'.5$	$-5^\circ 05'$	$+2^\circ 52'$	$-0^\circ 13'$
Sines	$-.0092$	$+.2140$	$-.0886$	$+.05001$	$-.00378$
Versines	* * *	$+.0232$	$.00393$	$.00125$	* * *

$$\mathfrak{A} = \sin A = -.0092 \qquad = (-).0092$$

$$\mathfrak{B} = \sin B\,[1 + \tfrac{1}{2}\sin D + \tfrac{1}{12}\text{versin } B - \tfrac{1}{4}\text{versin } C] + \tfrac{1}{2}\sin C \sin E$$

$$= +.214\,[1 + .0250 + .00193 - .00098] + .0443 \times .00378 \qquad = +.2197$$

$$\mathfrak{C} = \sin C\,[1 - \tfrac{1}{2}\sin D + \tfrac{1}{12}\text{versin } C - \tfrac{1}{4}\text{versin } B] + \tfrac{1}{2}\sin B \sin E$$

$$= -.0886\,[1 - .025 + .0003 - .0058] - .107 \times .00378 \qquad = (-).0863$$

$$\mathfrak{D} = \sin D\,[1 + \tfrac{1}{2}\text{ versin } D] = +.05001\,[1 + .00042] \qquad = +.0500$$

$$\mathfrak{E} = \sin E - \sin A \sin D = -.00378 + .0092 \times .05001 \qquad = (-).0033$$

$$\tan \chi = \frac{\mathfrak{C}}{\mathfrak{B}} = \frac{-.0863}{+.2197}\;\begin{array}{l} \text{log.} \quad 8.93601 \\ \text{log.} \quad 9.34183 \end{array}$$

$$\chi = 21^\circ 26'44'' \qquad \text{log. tan. } 9.59418$$

$$\alpha = 338^\circ 33'16''.$$

SECTION V.

90. Determination of λ.—This coefficient is the one which expresses the proportion of the mean horizontal force northward of the earth and ship to the horizontal force on shore and may be found from equation (27) written as follows,

$$\lambda = \frac{H'}{H}\,\frac{\cos \delta}{1 + \mathfrak{B}\cos z - \mathfrak{C}\sin z + \mathfrak{D}\cos 2z - \mathfrak{E}\sin 2z} \qquad (61)$$

when the horizontal force and the deviation for the magnetic azimuth z are known in addition to the exact coefficient $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$.

In the above equation

H' = the horizontal force of earth and ship combined;

H = the horizontal force of earth, considered as unity;

$\frac{H'}{H} = \frac{T^2}{T'^2}$, T being the time of n vibrations (say 10) of a small horizontal needle, 3 to 4 inches long, on shore in a place free from local magnetic disturbances; T' that of the same number of vibrations of the same needle, the center of which is in the same place exactly as that occupied by the center of the compass needle, when the compass is in place.

Great care should be exercised in taking the vibrations, and the mean of a number of determinations should be used, since the error of a single set might be comparatively large.

The equation $\frac{H'}{H}=\frac{T^2}{T'^2}$ is true only for infinitesimal arcs of vibration, but may be taken as sufficiently exact for all practical purposes if the arcs do not exceed 20°. However, the amplitude of arc should be as small as possible consistent with obtaining 10 well-defined vibrations.

The place on shore where the needle is vibrated should be free from local attraction, a fact that may be determined in the following way, namely: place a compass on its tripod and set up a staff about 50 yards distant, note the bearing of the staff per compass; interchange tripod and staff and again note the bearing of staff. Do the same thing on a line perpendicular to the first line. If the bearing and reverse bearing in each case differ by 180°, the locality may be assumed free from magnetic local influences.

The horizontal force instrument.—This instrument is used in finding the ratio of the horizontal force on board ship in the position of the compass to that on shore. It consists of a cylindrical brass case, with a removable glass cover, mounted upon a rectangular base which is provided with levels and leveling screws.

The case contains a horizontal circle graduated to degrees, and in the center a pivot which supports a small lozenge-shaped magnetic needle fitted with an adjustable sliding weight to counteract the dip and capable of vibrating freely in the horizontal plane.

Observations for horizontal force ashore.—Find a level spot free from local attraction, level the horizontal force instrument and orientate it. By means of a small magnet draw the needle aside about 20°, quickly removing the magnet to a proper distance. Then as the needle passes the zero line the first time "mark the time" or start the stop watch, as the needle passes the zero line the second time count "one," at the next passage "two," and so on till the count "ten," when the time is again noted or the stop watch stopped. The interval of time will be the time required by the needle to make 10 vibrations.

Observations on board.—Observations are similarly taken on board, the center of the horizontal force needle occupying the exact place usually occupied by the center of the compass, which with all correctors must be removed to a safe distance. The horizontal force instrument is leveled on a brass table in the compass chamber, the spindle of the table entering the central vertical tube of the binnacle.

The magnetic azimuth and deviation may be determined by any one of the usual methods.

The coefficients should have been determined as accurately as possible on equidistant compass courses; 24, 12, or 8 equidistant headings.

If the observations be taken on four equidistant magnetic azimuths we will have

$$\lambda = \tfrac{1}{4} \Sigma \frac{H'}{H} \cos \delta \qquad (62)$$

because, the summation of the sines and cosines being zero, the exact coefficients will disappear.

NOTE: In the service compass, the plane of the needles is three-fourths of an inch below the bottom of the wyes in which the compass rests when placed in the binnacle, and it may be located by placing in the wyes a straight edge at the center of which is pasted a piece of paper projecting vertically downward ¾ of an inch.

Ex. 9.—Given the exact coefficients of a "Monitor's Standard" $\mathfrak{A} = -.0092$, $\mathfrak{B} = +.2197$, $\mathfrak{C} = -.0868$, $\mathfrak{D} = +.0500$, $\mathfrak{E} = -.0033$, it is required to find λ from observations of horizontal force ashore and on board, the ship heading 58° (p. s. c.), deviation on that heading $+9° 30'$, T (the time of 10 vibrations of the horizontal needle ashore) $= 15^s.66$, T' (the time of 10 vibrations on board) $= 15^s.33$.

FOR COMPUTATION OF LAMBDA.

Ship's Head Mgtc.	By Compass.	T′	T	
67° 30′	58°	$15^s\,33$	$15^s\,66$	$\lambda = \frac{H'}{H} \times \frac{\cos\delta}{1 + \mathfrak{B}\cos z - \mathfrak{C}\sin z + \mathfrak{D}\cos 2z - \mathfrak{E}\sin 2z}$
$z = 67° 30'$			$\frac{H'}{H} = \frac{T^2}{T'^2}$	$= 1.0435 \times \frac{.986}{1 + .2197 \times .383 - (-.0868 \times .924) + .05(-.707) - (-.0033 \times .707)}$ $= \frac{1.0289}{1 + .0841 + .0797 - .0353 + .0023}$
$2z = 135°$				$= \frac{1.0289}{1.1308} = .9099 = \lambda.$
			$= 1.0435$	

Ex. 10.—It is required to find λ from the following observations for horizontal force made ashore and on board a monitor in the position of the standard compass, the magnetic courses and deviations being found by interpolation in the Standard's table of deviations, Art. 55.

Magnetic heading.	Deviations.	Horizontal vibrations.	
North	— 4° 35′	T' $14^{s}.60$	T 15.66
East	+ 12 00	16.28	
South	+ 4 54	18.17	
West	— 14 09	17.80	

For Head, North.

$15^{s}.66$.........log 1.19479
14 .60.........log 1.16435

0.03044
2

0.06088
δ_1 + 4° 35′....cos 9.99861

$\frac{H_1'}{H}$ cos δ_1 1.1468, log 0.05949

For Head, East.

$15^{s}.66$.........log 1.19479
16 .28.........log 1.21165

9.98314
2

9.96628
δ_2 + 12° 00′......cos 9.99040

$\frac{H_2'}{H}$ cos δ_2 .9051, log 9.95668

For Head, South.

$15^{s}.66$.........log 1.19479
18 .17.........log 1.25935

9.93544
2

9.87088
δ_3 + 4° 54′....cos 9.99841

$\frac{H_3'}{H}$ cos δ_3 .7401, log 9.86929

For Head, West.

$15^{s}.66$.........log 1.19479
17 .80.........log 1.25042

9.94437
2

9.88874
δ_4 — 14° 09′...cos 9.98662

$\frac{H_4'}{H}$ cos δ_4=.7505, log 9.87536

$$\lambda = \frac{1.1468 + .9051 + .7401 + .7505}{4} = .8856$$

91. Determination of a, e, b, and d, given $\mathfrak{A}$, $\mathfrak{E}$, $\mathfrak{D}$, and λ.

Ex. 11.—Given the following coefficients (Exs. 8 and 10), $\mathfrak{A} = -.0092$, $\mathfrak{E} = -.0033$, $\mathfrak{D} = .0500$, $\lambda = .8856$, it is required to find a, e, b, and d. See Art. 79 and Art. 80.

$$a = \lambda(1 + \mathfrak{D}) - 1 = .8856 \times 1.05 - 1. = .9299 - 1. = (-).0701$$

$$e = \lambda(1 - \mathfrak{D}) - 1 = .8856 \times .95 - 1 = .8413 - 1 = (-).1587$$

$$d - b = 2\lambda\mathfrak{A} = -2 \times .8856 \times .0092 = -.0163$$

$$d + b = 2\lambda\mathfrak{E} = -2 \times .8856 \times .0033 = -.0058$$

$$2d = -.0221 \therefore d = -.0110$$

$$2b = +.0105 \therefore b = +.0052$$

92. Determination of parameters g and h and the vertical force of the earth and ship.

—In equation (22),

$$\frac{Z'}{Z} = \frac{g}{\tan\theta}\cos z - \frac{h}{\tan\theta}\sin z + 1 + k + \frac{R}{Z},$$

the vertical force of the earth and ship is expressed in terms of the earth's vertical force as a unit of measurement.

The mean value of $\frac{Z'}{Z}$ on two or more equidistant azimuths will be the constant term $\left(1 + k + \frac{R}{Z}\right)$ of the second member. Letting μ be the mean value of $\frac{Z'}{Z}$, or the mean force downward of the earth and ship in terms of the earth's vertical force as unit, then

$\mu = 1 + k + \frac{R}{Z}$; therefore,

$$\frac{Z'}{Z} = \frac{g}{\tan\theta}\cos z - \frac{h}{\tan\theta}\sin z + \mu. \qquad (63)$$

From (63), the value of μ, $\frac{g}{\tan\theta}$, and $\frac{h}{\tan\theta}$ are derived from observations on 4, 8, 12, or 24 equidistant courses, using similar tabular forms to those used in finding the approximate coefficients.

As with the horizontal force instrument, the times of n vibrations of a dipping needle may be observed on board and on shore, the vibrations being made in a plane perpendicular to the compass meridian on board and perpendicular to the magnetic meridian ashore. The dipping needle is correctly placed for vibrations when, at rest, the magnetic axis of the needle is vertical. If T be the time of say 10 vibrations of this needle on shore, and T' that of the same number of vibrations on board, the center of the needle being in the exact position occupied by the center of the compass needle, then $\frac{Z'}{Z} = \frac{T^2}{T'^2}$. The times of vibrations are obtained in the same way as with a horizontal needle, the needle being deflected from the zero point about 10° in this case. The magnetic azimuth of the ship may be obtained at the same time in any one of the usual ways.

Owing to the fact that the sine and cosine of angles differing 180° have opposite signs, μ will be the mean of $\frac{Z'}{Z}$ observed on two opposite magnetic headings.

Regarding h as zero, if g is known, we may find μ from one observation of $\frac{Z'}{Z}$; if one observation is made on magnetic East or West, g will disappear and μ will equal $\frac{Z'}{Z}$.

If observations for $\frac{Z'}{Z}$ are made on four equidistant magnetic headings, then

$$\mu = \tfrac{1}{4}\,\Sigma\,\frac{Z'}{Z}\,. \qquad (64)$$

If the observations are made on N., E., S., and W., magnetic, then μ may be found from all four, g from the observations at N. and S., h from those at E. and W., a fact that is apparent from a consideration of equation (63).

***Ex.* 12.—It is required to find μ, g, and h from the following observations for vertical force made ashore and on board a "Monitor" in the position of the standard compass, tan θ being 2.86.**

Mag. heading.	**Vertical vibrations.**	
North	T' $18^s.43$	T $19^s.75$
East	18 .94	
South	19 .08	
West	19 .00	

For Head, North.

$$\begin{array}{lr}
19^s.75\ldots\ldots\ldots\log & 1.29557 \\
18\ .43\ldots\ldots\ldots\log & \underline{1.26553} \\
 & 0.03004 \\
 & \underline{2} \\
\frac{Z'_1}{Z} = 1.1484\ldots\log & 0.06008
\end{array}$$

For Head, East.

$$\begin{array}{lr}
19^s.75\ldots\ldots\ldots\log & 1.29557 \\
18\ .94\ldots\ldots\ldots\log & \underline{1.27738} \\
 & 0.01819 \\
 & \underline{2} \\
\frac{Z'_2}{Z} = 1.0874\ldots\log & 0.03638
\end{array}$$

For Head, South.

$$\begin{array}{lr}
19^s.75\ldots\ldots\ldots\log & 1.29557 \\
19\ .08\ldots\ldots\ldots\log & \underline{1.28058} \\
 & 0.01499 \\
 & \underline{2} \\
\frac{Z'_3}{Z} = 1.0715\ldots\log & 0.02998
\end{array}$$

For Head, West.

$$\begin{array}{lr}
19^s.75\ldots\ldots\ldots\log & 1.29557 \\
19\ .00\ldots\ldots\ldots\log & \underline{1.27875} \\
 & 0.01682 \\
 & \underline{2} \\
\frac{Z'_4}{Z} = 1.0805\ldots\log & 0.03364
\end{array}$$

$$\mu = \frac{1.1484 + 1.0874 + 1.0715 + 1.0805}{4} = 1.0969.$$

$$\begin{array}{lr}
\text{On N., } \frac{Z'_1}{Z} = & 1.1484 \\
\text{On S., } \frac{Z'_3}{Z} = & \underline{1.0715} \\
2\overline{)0.0769} & \\
\frac{g}{\tan\theta} = & 0.0384
\end{array}
\qquad
\begin{array}{lr}
\text{On West, } \frac{Z'_4}{Z} = & 1.0805 \\
\text{On East. } \frac{Z'_2}{Z} = & \underline{1.0874} \\
2) - & 0.0069 \\
\frac{h}{\tan\theta} = & -\ 0.0034
\end{array}$$

$$g = .0384 \times 2.86 = .1098 \qquad h = -\ .0034 \times 2.86$$

$$= (-)\ .0097$$

$\frac{2g}{\tan\theta}$ being the value of $\frac{Z'}{Z}$ at N. — the value of $\frac{Z'}{Z}$ at S.

$\frac{2h}{\tan\theta}$ being the value of $\frac{Z'}{Z}$ at W. — the value of $\frac{Z'}{Z}$ at E.

SECTION VI.

93. Other methods of finding the exact coefficients $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$.—In the case of a compass well located on board ship, $\mathfrak{A}$ and $\mathfrak{E}$ are either zero, or very small, and for all practical purposes may be neglected without appreciable error. The equations for deviations then become very simple on the two cardinal and the intercardinal points of any quadrant. From observations made on such points the compass may be fairly well compensated, and, in the case of one already compensated, a very good residual curve may be obtained by substituting the resulting values of $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ in equation (34), as illustrated in Art. 97. This is a good method whether at sea or at anchor in port swinging to tide. Even if at a dock, a vessel's head may be sprung around sufficiently to get the required observations.

When practicable, choose that quadrant in which the directive force on the needle is strong.

In NE. quadrant.—Letting $\mathfrak{A}$ and $\mathfrak{E}$ be zero, we shall have for compass courses North, NE., and East, from (30), the following:

(1) $\sin \delta_0 = \mathfrak{C} + \mathfrak{D} \sin \delta_0 \therefore \mathfrak{C} = \sin \delta_0 (1 - \mathfrak{D})$.

(2) $\sin \delta_3 = \mathfrak{B} S_3 + \mathfrak{C} S_3 + \mathfrak{D} \cos \delta_3$.

(3) $\sin \delta_6 = \mathfrak{B} - \mathfrak{D} \sin \delta_6 \therefore \mathfrak{B} = \sin \delta_6 (1 + \mathfrak{D})$.

Substituting value of $\mathfrak{B}$ and $\mathfrak{C}$ in (2),

$$\sin \delta_3 = S_3 [\sin \delta_0 (1 - \mathfrak{D}) + \sin \delta_6 (1 + \mathfrak{D})] + \mathfrak{D} \cos \delta_3.$$

$$\sin \delta_3 = S_3 (\sin \delta_0 + \sin \delta_6) + \mathfrak{D} \{\cos \delta_3 - S_3 (\sin \delta_0 - \sin \delta_6)\}$$

$$\mathfrak{D} = \frac{\sin \delta_3 - S_3 (\sin \delta_0 + \sin \delta_6)}{\cos \delta_3 - S_3 (\sin \delta_0 - \sin \delta_6)}$$

Multiplying through by S_3, and as $S_3^2 = \frac{1}{2}$, we have for the *NE. Quadrant,*

$$\left.\begin{aligned} \mathfrak{D} &= \frac{S_3 \sin \delta_3 - \frac{1}{2}(\sin \delta_0 + \sin \delta_6)}{S_3 \cos \delta_3 - \frac{1}{2}(\sin \delta_0 - \sin \delta_6)} \\ \mathfrak{B} &= (1 + \mathfrak{D}) \sin \delta_6 \\ \mathfrak{C} &= (1 - \mathfrak{D}) \sin \delta_0 \end{aligned}\right\} \tag{65}$$

Similarly for the *SE. Quadrant,*

$$\left.\begin{aligned} \mathfrak{D} &= \frac{- S_3 \sin \delta_9 + \frac{1}{2}(\sin \delta_6 + \sin \delta_{12})}{S_3 \cos \delta_9 - \frac{1}{2}(\sin \delta_6 - \sin \delta_{12})} \\ \mathfrak{B} &= \quad (1 + \mathfrak{D}) \sin \delta_6 \\ \mathfrak{C} &= -(1 - \mathfrak{D}) \sin \delta_{12} \end{aligned}\right\} \tag{66}$$

And for the *SW. Quadrant,*

$$\left.\begin{aligned} \mathfrak{D} &= \frac{S_3 \sin \delta_{15} - \frac{1}{2}(\sin \delta_{12} + \sin \delta_{18})}{S_3 \cos \delta_{15} - \frac{1}{2}(\sin \delta_{12} - \sin \delta_{18})} \\ \mathfrak{B} &= -(1 + \mathfrak{D}) \sin \delta_{18} \\ \mathfrak{C} &= -(1 - \mathfrak{D}) \sin \delta_{12} \end{aligned}\right\} \tag{67}$$

And for the *NW. Quadrant,*

$$\left.\begin{aligned} \mathfrak{D} &= \frac{- S_3 \sin \delta_{21} + \frac{1}{2}(\sin \delta_{18} + \sin \delta_0)}{S_3 \cos \delta_{21} - \frac{1}{2}(\sin \delta_{18} - \sin \delta_0)} \\ \mathfrak{B} &= -(1 + \mathfrak{D}) \sin \delta_{18} \\ \mathfrak{C} &= \quad (1 - \mathfrak{D}) \sin \delta_0 \end{aligned}\right\} \tag{68}$$

If observations have been made on the three cardinal points and two intercardinal points of one semicircle, consider each quadrant of that semicircle separately, find the values of $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$, from each, and take the mean of the two determinations of each coefficient; or, combine the formulæ in the proper quadrants before proceeding with the computation.

Ex. 13.—A distant object, the magnetic bearing of which was 326° 45′ bore (p. s. c.) respectively 355°, 335°, and 328°, as the ship headed (p. s. c.) successively South, SW., and West. Required $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$.

Deviation on South $= \delta_{12} = -28°\ 15'$,

$$\sin\delta_{12} = -.473,\ \cos\delta_{12} = +.881.$$

Deviation on SW. $= \delta_{15} = -8°\ 15'$,

$$\sin\delta_{15} = -.143,\ \cos\delta_{15} = +.990.$$

Deviation on West $= \delta_{18} = -1°\ 15'$, $\sin\delta_{18} = -.022$.

$$\mathfrak{D} = \frac{.707 \times (-.143) - \frac{1}{2}(-.473 - .022)}{.707 \times .990 - \frac{1}{2}\left\{-.473 - (-.022)\right\}}$$

$$= \frac{-.1011 + .2475}{.6999 + .2255} = \frac{.1464}{.9254} = +.1582$$

$$\mathfrak{B} = -(1 + .1582)(-.022) = 1.1582 \times .022 = +.0255$$

$$\mathfrak{C} = -(1 - .1582)(-.473) = +.8418 \times .473 = +.3982$$

94. Determination of $\mathfrak{B}$ and $\mathfrak{C}$ from observations of deviation and horizontal force on one heading.—Regarding $\mathfrak{A}$ and $\mathfrak{E}$ as zero, that is b and d as zero, we have from (20) and (21),

$$\frac{H'}{H}\cos z' = (1 + a)\cos z + c\tan\theta + \frac{P}{H}$$

$$\text{or, } \frac{H'}{H}\cos z' = (1 + a)\cos z + \lambda\mathfrak{B}. \tag{69}$$

$$-\frac{H'}{H}\sin z' = -(1 + e)\sin z + f\tan\theta + \frac{Q}{H}$$

$$\text{or } -\frac{H'}{H}\sin z' = -(1 + e)\sin z + \lambda\mathfrak{C}. \tag{70}$$

Substituting the values of $(1 + a)$ and $(1 + e)$ from (35) and (36), transposing and dividing through by λ, we have,

$$\mathfrak{B} = \frac{H'}{\lambda H}\cos z' - (1 + \mathfrak{D})\cos z. \tag{69a}$$

$$\mathfrak{C} = -\frac{H'}{\lambda H}\sin z' + (1 - \mathfrak{D})\sin z. \tag{70a}$$

z' is the azimuth of the ship's head per compass; z is the magnetic azimuth of the ship's head and may be determined by a time azimuth of the sun, from the bearing of a distant object of known magnetic bearing, or by reciprocal bearings; after the above data have been obtained, remove the compass to a sufficient distance and take vibrations of a horizontal needle in the exact place of the compass needle, calling T' the time of 10 vibrations. Take vibrations of the same needle ashore, calling T the time of ten vibrations there, then $\frac{H'}{H} = \frac{T^2}{T'^2}$. Therefore, if λ and $\mathfrak{D}$ can be obtained; $\mathfrak{B}$, $\mathfrak{C}$, and a may be found.

These coefficients, λ and $\mathfrak{D}$, are so nearly the same for compasses in similar positions in similar ships that, in the absence of any better values, they may be taken as the same as those in a sister ship, or assumed.

With the approximate values of $\mathfrak{B}$ and $\mathfrak{C}$ and the assumed value of $\mathfrak{D}$, the compass may be roughly corrected when in dry dock, moored to a wharf, or when it is impossible to get observations on more than one heading, provided, however, that there is no other iron vessel, nor other causes of disturbance, sufficiently near to exercise magnetic influence. If the compass should not be compensated, then a table of approximate deviations may be made by the formulæ of Art. 97.

Such observations may be valuable in determining the location of compasses for ships while still on the stocks.

95. Determination of $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, λ, a, and e from observations of deviation and horizontal force on two headings, $\mathfrak{A}$ and $\mathfrak{E}$ being neglected.

Let z'_1 and z'_2 be the two compass headings; z_1 and z_2 the two magnetic headings; $\frac{H'_1}{H} = \frac{T^2}{T_1'^2}$ the horizontal force of earth and ship on the first heading in terms of H; $\frac{H'_2}{H} = \frac{T^2}{T_2'^2}$ the same on the second heading. Then we have from (69) and (70) for the two headings:

$$\left.\begin{aligned}\lambda\mathfrak{B} &= \frac{H'_1}{H}\cos z'_1 - (1+a)\cos z_1 \qquad (71)\\ \lambda\mathfrak{B} &= \frac{H'_2}{H}\cos z'_2 - (1+a)\cos z_2 \qquad (72)\end{aligned}\right\}$$

$$\text{or } (1+a) = \frac{\frac{1}{2}\left\{\frac{H'_1}{H}\cos z'_1 - \frac{H'_2}{H}\cos z'_2\right\}}{\frac{1}{2}(\cos z_1 - \cos z_2)} \qquad (73)$$

$$\left.\begin{aligned}\lambda\mathfrak{C} &= -\frac{H'_1}{H}\sin z'_1 + (1+e)\sin z_1 \qquad (74)\\ \lambda\mathfrak{C} &= -\frac{H'_2}{H}\sin z'_2 + (1+e)\sin z_2 \qquad (75)\end{aligned}\right\}$$

$$\text{or } (1+e) = \frac{\frac{1}{2}\left\{\frac{H'_1}{H}\sin z'_1 - \frac{H'_2}{H}\sin z'_2\right\}}{\frac{1}{2}(\sin z_1 - \sin z_2)} \qquad (76)$$

From (73) and (76), (35) and (36),

$$\lambda = \tfrac{1}{2}\left\{(1+a) + (1+e)\right\} \qquad (77)$$

$$\text{and } \mathfrak{D} = \frac{1}{\lambda}\,\frac{a-e}{2} = \frac{1}{\lambda}\left\{\frac{1}{2}\Big[(1+a) - (1+e)\Big]\right\} \qquad (78)$$

Adding (71) and (72) and dividing by λ,

$$\mathfrak{B} = +\frac{1}{\lambda}\left\{\frac{1}{2}\left(\frac{H'_1}{H}\cos z'_1 + \frac{H'_2}{H}\cos z'_2\right) - (1+a)\left[\frac{1}{2}(\cos z_1 + \cos z_2)\right]\right\} \qquad (79)$$

Adding (74) and (75) and dividing by λ,

$$\mathfrak{C} = -\frac{1}{\lambda}\left\{\frac{1}{2}\left(\frac{H'_1}{H}\sin z'_1 + \frac{H'_2}{H}\sin z'_2\right) - (1+e)\left[\frac{1}{2}(\sin z_1 + \sin z_2)\right]\right\} \qquad (80)$$

$$\tan a = \frac{\mathfrak{C}}{\mathfrak{B}}. \qquad (81)$$

Since the sines and cosines of angles differing 180° have opposite signs, if the observations are taken on magnetic courses diametrically opposite, $(1 + a)$ in (79) and $(1 + e)$ in (80) will disappear, $\frac{1}{2}(\cos z_1 - \cos z_2)$ will become $\cos z_1$ and $\frac{1}{2}$ $(\sin z_1 - \sin z_2)$ will become $\sin z_1$; and equations (73), (76), (79), and (80), being much simplified, will become

$$1 + a = \frac{\frac{1}{2}\left\{\frac{H_1'}{H}\cos z_1' - \frac{H_2'}{H}\cos z_2'\right\}}{\cos z_1} \tag{82}$$

$$1 + e = \frac{\frac{1}{2}\left\{\frac{H_1'}{H}\sin z_1' - \frac{H_2'}{H}\sin z_2'\right\}}{\sin z_1} \tag{83}$$

$$\mathfrak{B} = \frac{1}{\lambda}\left\{\frac{1}{2}\left(\frac{H_1'}{H}\cos z_1 + \frac{H_2'}{H}\cos z_2'\right)\right\} \tag{84}$$

$$\mathfrak{C} = -\frac{1}{\lambda}\left\{\frac{1}{2}\left(\frac{H_1'}{H}\sin z_1 + \frac{H_2'}{H}\sin z_2'\right)\right\} \tag{85}$$

This method is strongly recommended, and most excellent results may be obtained when the vibrations are carefully taken.

In the steering compasses of some of the battleships, the ship's force exceeds that of the earth and it is impossible to obtain a curve of deviations for such compasses by swinging ship, and resort must be had to vibrations.

However, care must be exercised in selecting the rhumbs, as the formulæ will fail if the magnetic azimuths are equally distant from any one of the cardinal points; for if equally distant from North or South, $(1 + a)$ reduces to the form $\frac{0}{0}$, and if equally distant from East or West, $(1 + e)$ takes that form. As a general rule, select rhumbs on or near opposite quadrantal points, or on or near two adjacent cardinal points.

This method is valuable in locating compasses on board new ships and in determining beforehand the forces of the ship, and, if desired, a deviation table. Select the different places where, for other reasons, the compass might be located; obtain the compass and magnetic headings and the time of 10 vibrations of the horizontal needle in the exact position to be occupied by the compass needle when the ship is on the first heading; change the position of the ship and do the same on the second heading; note the time of 10 vibrations of the same horizontal needle ashore. Proceed with the computation, and all other considerations being equal, select that spot as the best location for the compass, where λ is greatest.

The following form not only facilitates the solution but indicates the data.

TWO OPPOSITE HEADINGS.

Ex. 14.—Data: 1st heading (p. s. c.), 199° 30′, dev. + 25° 30′, mean of 10 sets of horizontal vibrations $T'_1 = 38^s.6$; 2d heading (p. s. c.) 52° 30′, dev. —7° 30′, mean of 10 sets of horizontal vibrations $T'_2 = 34^s.1$. Mean of 10 sets of similar vibrations ashore $36^s.9$. Find λ, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$.

Heading.	z	z'	T'	$\frac{H'}{H} = \frac{T^2}{T'^2}$	$\cos z'$	$\sin z'$	$\frac{H'}{H}\cos z'$	$\frac{H'}{H}\sin z'$	$\cos z$	$\sin z$
1st 2d	225° 00′ 45 00	199° 30′ 52 30	38.6 34.1	0.91885 1.171	(—).9426 + .6088	(—).3338 + .7934	(—).8614 + .7129	(—).3051 + .9290	(—).7071 + .7071	(—).7071 + .7071
						½ Sum.	(1) (—).0742	(2) + .3120	(3) 0	(4) 0
						½ Diff.	(5) (—).7872	(6) (—).6171	(7) (—).7071	(8) (—).7071

$$(1 + a) = \frac{(5)}{(7)} = \frac{(-).7872}{(-).7071} = +\ 1.1133$$

$$(1 + e) = \frac{(6)}{(8)} = \frac{(-).6171}{(-).7071} = +\ 0.8727$$

$$\text{Sum} = 1.9860$$

$$\lambda = \frac{\text{Sum}}{2} = 0.993$$

$$\text{Diff.} = 0.2406$$

$$\tfrac{1}{2}\ \text{Diff.} = 0.1203$$

$$\mathfrak{D} = \frac{\text{Diff.}}{2\lambda} = \frac{.1203}{.993} = +\ .1211$$

$$\mathfrak{B} = \frac{1}{\lambda}\left\{(1) - (1 + a)\,(3)\right\} = \frac{(-).0742}{.993} = (-).0747$$

$$\mathfrak{C} = \frac{1}{\lambda}\left\{(-)\,(2) + (1 + e)\,(4)\right\} = \frac{(-).3120}{.993} = (-).3142$$

TWO HEADINGS NOT OPPOSITE.

Ex. 15.—From observations on board a U. S. Cruiser at Mare Island: 1st heading (p. s. c.) 205° 46′, dev. + 11° 20′, $T'_1 = 24^s.06$. 2d heading (p. s. c.) 307° 30′, dev. + 1°, $T'_2 = 27^s.32$. T (on shore) $= 22^s.43$. Find a, e, λ, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$.

Heading.	s	s'	T	T'	$\frac{H'}{H}=\frac{T^2}{T'^2}$	cos s'	sin s'	$\frac{H'}{H}$ cos s'	$\frac{H'}{H}$ sin s'	cos s	sin s
			s	s							
1st	217° 06′	205° 46′	22.43	24.06	0.869	(−).901	(−).435	(−).783	(−).378	(−).798	(−).603
2nd	308 30	307 30	22.43	27.32	0.674	+ .609	(−).793	+ .410	(−).534	+ .623	(−).783

½ Sum	(1) (−).187	(2) (−).456	(3) (−).088	(4) (−).693
½ Diff.	(5) (−).597	(6) +.078	(7) (−).711	(8) +.090

$$(1+a)=\frac{(5)}{(7)}=\frac{(-).597}{(-).711}=.840\ \therefore\ a=-.160$$

$$(1+e)=\frac{(6)}{(8)}=\frac{.078}{.090}=\ \underline{.867}\ \therefore\ e=-.133$$

$$\text{Sum}=1.707$$

$$\lambda=\frac{\text{Sum}}{2}=0.854$$

$$\text{Diff.}=-.027$$

$$\tfrac{1}{2}\ \text{Diff.}=-.0135$$

$$\mathfrak{D}=\frac{\text{Diff.}}{2\lambda}=\frac{-.0135}{.854}=(-).016$$

$$\mathfrak{B}=\frac{1}{\lambda}\left\{(1)-(1+a)\ (3)\right\}=\frac{1}{.854}\left\{(-).187-(.840\times(-).088)\right\}=\frac{(-).187+.074}{.854}=-.133$$

$$\mathfrak{C}=\frac{1}{\lambda}\left\{(-)\ (2)+(1+e)\ (4)\right\}=\frac{1}{.854}\left\{+.456+(.867\times(-).693)\right\}=\frac{+.456-.601}{.854}=-.170$$

96. Determination of the forces of hard and soft iron causing semicircular deviation.—We have seen that semicircular deviation is caused by two separate forces: (1) the force due to subpermanent magnetism, the components of which to head and to starboard in terms of the "mean force to North" as unit are respectively $\frac{P}{\lambda H}$ and $\frac{Q}{\lambda H}$; (2) the force due to transient magnetism induced in vertical soft iron, the components of which in the same axes in terms of "mean force to North" as unit are respectively $\frac{c}{\lambda} \tan \theta$ and $\frac{f}{\lambda} \tan \theta$.

In the above expressions for the forces, the values $\frac{P}{\lambda}$, $\frac{Q}{\lambda}$, $\frac{c}{\lambda}$ and $\frac{f}{\lambda}$ are the parts that do not change, and it is desirable to determine them at the first opportunity, and then the forces due to hard and soft iron separately at the place selected as that of compensation.

These quantities may be determined from two different values of 𝔅 and ℭ observed at places, in widely different latitudes, where the dip and horizontal force are known. When they are once determined, it will be possible to correct separately the deviation due to the two kinds of iron, that due to hard iron being corrected by magnets and that due to soft iron by what is known as a Flinders bar. This bar is of soft iron and is about 36 inches long. It is placed in the starboard angle (a_v) of the forces due to induction in c and f with the lower end level with the compass needles; or in the angle $180° + a_v$, the upper end in this case level with the compass needles. When f is zero, a_v is 0° or 180°.

The Flinders corrector consists preferably of a bundle of rods of about ¼ inch diameter and about 36 inches in length, so that, at a fixed distance, the intensity of its induced force may be varied as desired by increasing or decreasing the number of the rods, instead of varying the distance of a single rod.

If whilst a vessel is on the magnetic equator, the compass be carefully compensated, the force causing semicircular deviation, being there due to subpermanent magnetism alone, should be entirely neutralized by magnets. Then, if on change of magnetic latitude, a Flinders bar be so placed as to correct any semicircular deviation that appears, the compensation should be general for existing conditions.

If the compass is not compensated, with the values of $\frac{P}{\lambda}$, $\frac{c}{\lambda}$, $\frac{Q}{\lambda}$, and $\frac{f}{\lambda}$ known, it will be possible to predict changes in the deviations and to make out a table of deviations for the locality of other cruising grounds, H and θ being known.

Letting the values of the quantities proper to the problem be, at the first place $\mathfrak{B}_1$, $\mathfrak{C}_1$, H_1, $\tan\theta_1$, and at the second place $\mathfrak{B}_2$, $\mathfrak{C}_2$, H_2, $\tan\theta_2$, we shall have

$$\left.\begin{aligned} H_1\mathfrak{B}_1 &= \frac{P}{\lambda} + \frac{c}{\lambda}\tan\theta_1 H_1 \\ H_2\mathfrak{B}_2 &= \frac{P}{\lambda} + \frac{c}{\lambda}\tan\theta_2 H_2 \\ H_1\mathfrak{C}_1 &= \frac{Q}{\lambda} + \frac{f}{\lambda}\tan\theta_1 H_1 \\ H_2\mathfrak{C}_2 &= \frac{Q}{\lambda} + \frac{f}{\lambda}\tan\theta_2 H_2 \end{aligned}\right\} \qquad (86)$$

Ex. 16.—At Cape Henry $\mathfrak{B}_1 = +.697$, $\mathfrak{C}_1 = -.119$, $H_1 = .219$, $\tan\theta_1 = 2.605$; at Key West $\mathfrak{B}_2 = +.393$, $\mathfrak{C}_2 = -.08$, $H_2 = .311$, $\tan\theta_2 = 1.393$. It is required to find

$$\frac{P}{\lambda},\ \frac{c}{\lambda},\ \frac{Q}{\lambda},\ \text{and}\ \frac{f}{\lambda}.$$

(2) What were the values of $\mathfrak{B}$ and $\mathfrak{C}$ at New York, $H = .185$, $\tan\theta = 3.14$?

(3) What was the total force at New York due to subpermanent magnetism, and what was the total force due to transient magnetism induced in vertical soft iron?

(4) A Flinders bar having been placed while the ship was at New York, it is required to find the direction in which it was placed, and the compass heading after correction, if before, the vessel headed 48° 30′ (p. c.), dev. + 41° 30′, the azimuth of the ship's head remaining the same.

By substitution in (86) we have

$$.219 \times .697 = \frac{P}{\lambda} + \frac{c}{\lambda} \times 2.605 \times .219$$
$$.153 = \frac{P}{\lambda} + .570\frac{c}{\lambda}$$
$$.122 = \frac{P}{\lambda} + .433\frac{c}{\lambda}$$
$$.031 = .137\frac{c}{\lambda}$$
$$\therefore \frac{c}{\lambda} = .226$$

$$.311 \times .393 = \frac{P}{\lambda} + \frac{c}{\lambda} \times 1.393 \times .311$$
$$.122 = \frac{P}{\lambda} + .433\frac{c}{\lambda}$$
$$.122 = \frac{P}{\lambda} + .098$$
$$\frac{P}{\lambda} = .024$$

$$.219 \times (-.119) = \frac{Q}{\lambda} + \frac{f}{\lambda} \times 2.605 \times .219$$
$$(-)\,.026 = \frac{Q}{\lambda} + .570\frac{f}{\lambda}$$
$$(-)\,.025 = \frac{Q}{\lambda} + .433\frac{f}{\lambda}$$
$$(-)\,.001 = .137\frac{f}{\lambda}$$
$$\frac{f}{\lambda} = (-)\,.007$$

$$.311 \times (-.08) = \frac{Q}{\lambda} + \frac{f}{\lambda} \times 1.393 \times .311$$
$$(-)\,.025 = \frac{Q}{\lambda} + .433\frac{f}{\lambda}$$
$$(-)\,.025 = \frac{Q}{\lambda} - .003$$
$$\frac{Q}{\lambda} = (-)\,.022$$

(2) To find the value of 𝔅 and ℭ at New York,

$$\mathfrak{B} = \frac{P}{\lambda H} + \frac{c}{\lambda}\tan\theta \qquad \mathfrak{C} = \frac{Q}{\lambda H} + \frac{f}{\lambda}\tan\theta$$

$$\mathfrak{B} = \frac{.024}{.185} + .226 \times 3.14 \qquad \mathfrak{C} = \frac{(-)\,.022}{.185} + \left((-)\,.007 \times 3.14\right)$$

$$\mathfrak{B} = .130 + .710 = +.840 \qquad \mathfrak{C} = (-)\,.119 - .022 = (-)\,.141$$

(3) + .130 = force in keel line due to subpermanent magnetism at New York.

(—).119 = force transverse to keel line due to subpermanent magnetism at New York.

+ .710 = force in keel line due to vertical soft iron at New York.

(—).022 = transverse force due to vertical soft iron at New York.

$$\tan a_{s.p.} = \frac{(-)\,.119}{+\,.130} = (-)\,.9154$$

$$\text{therefore } a_{s.p.} = 317^\circ\ 31'\ 45''$$

$$\tan a_v = \frac{(-)\,.022}{.710} = (-)\,.03099$$

$$\text{therefore } a_v = 358^\circ\ 14' + .$$

Total force due to subpermanent magnetism

$$= \sqrt{(.130)^2 + (-.119)^2} = .176.$$

Total force due to vertical soft iron

$$= \sqrt{(.710)^2 + (-.022)^2} = .710.$$

(4) The Flinders bar is placed in the angle $a_v = 358^\circ\ 14'+$ with its lower end on a level with the compass needles, or in the angle $a_v + 180^\circ = 178^\circ\ 14' +$ with its upper end on a level with the compass needles, and at such a distance as to neutralize the deviation due to the vertical soft iron. In case the corrector is to be at a fixed distance from the compass, then increase or decrease the number of rods till the desired effect is produced.

The deviation resulting from the equation

$$\tan \delta = \frac{\frac{c}{\lambda}\tan\theta\sin z + \frac{f}{\lambda}\tan\theta\cos z}{1 + \frac{c}{\lambda}\tan\theta\cos z - \frac{f}{\lambda}\tan\theta\sin z} \qquad \text{(a)}$$

is the deviation due to vertical soft iron on the magnetic heading z.

In the example $z' = 48^\circ\ 30'$, dev. $= +\,41^\circ\ 30'$, and therefore z is East.

Substituting the values of $\frac{c}{\lambda}\tan\theta$ and $\frac{f}{\lambda}\tan\theta$ found in part (2) of this example, we have

$$\tan \delta = \frac{+\,.710 \times 1 + (-\,.022) \times 0}{1 + .710 \times 0 - (-\,.022 \times 1)} = \frac{.710}{1.022}$$

$$= .6947 \therefore \delta = +\,34^\circ\ 47'.$$

The deviation due to vertical soft iron being + 34° 47′ on the given heading, that amount should be removed by the Flinders corrector; therefore, after the correction has been made, the compass heading should be 83° 17′.

97. Computation of deviations from the coefficients.—Various methods have been explained for obtaining new values of the coefficients, especially the changing ones; having obtained these it may be desirable to compute a deviation table; or knowing the values of $\frac{P}{\lambda}$, $\frac{c}{\lambda}$, $\frac{Q}{\lambda}$, and $\frac{f}{\lambda}$, it may be necessary to determine the deviations for certain localities to be visited where there may be no opportunities for swinging ship.

From the approximate coefficients the deviations may be obtained from the equation

$$\delta = A + B \sin z' + C \cos z' + D \sin 2z' + E \cos 2z';$$

and from the exact coefficients, they may be found for magnetic azimuths from equation

$$\tan \delta = \frac{\mathfrak{A} + \mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z + \mathfrak{E} \cos 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z - \mathfrak{E} \sin 2z},$$

then by use of Napier's diagram the deviations may be found for the compass headings.

However, deviations are desired for compass headings and may be found for such from equation (34),

$$\sin \delta = \frac{1}{1 - \mathfrak{D} \cos 2z'} (\mathfrak{A} + \mathfrak{B} \sin z' + \mathfrak{C} \cos z' + \mathfrak{D} \sin 2z' + \mathfrak{E} \cos 2z').$$

The following form facilitates the computation.

Ex. 17.—Given $\mathfrak{B} = (-).1352$, $\mathfrak{C} = (-).1088$, $\mathfrak{D} = +.1084$, $\mathfrak{A}$ and $\mathfrak{E}$ zero. Find the deviations on the 8 principal rhumbs per compass.

Head (p. c.)	$\mathfrak{B} = (-).1352$		$\mathfrak{C} = (-).1088$		$\mathfrak{D} = +.1084$		Sum.	$\mathfrak{D} = +.1081$		$1 - \mathfrak{D} \cos 2z'$	$\sin \delta =$ sum divided by $1 - \mathfrak{D} \cos 2z'$	Deviations.
	$\sin z'$	$\mathfrak{B} \sin z'$	$\cos z'$	$\mathfrak{C} \cos z'$	$\sin 2z'$	$\mathfrak{D} \sin 2z'$		$\cos 2z'$	$\mathfrak{D} \cos 2z'$			
°												° ′
0	0	.0000	1	−.1088	0	.0000	−.1088	1	+.1084	.8916	−.1220	− 7 00
45	S_8	−.0956	S_8	−.0772	1	+.1084	−.0644	0	.0000	1.0000	−.0644	− 8 42
90	1	−.1352	0	.0000	0	.0000	−.1352	−1	−.1084	1.1084	−.1220	− 7 00
135	S_8	−.0956	$-S_8$	+.0772	−1	−.1084	−.1268	0	.0000	1.0000	−.1268	− 7 17
180	0	.0000	−1	+.1088	0	.0000	+.1088	1	+.1084	.8916	+.1220	+ 7 00
225		+.0956		+.0772		+.1084	+.2812		.0000	1.0000	+.2812	+16 20
270		+.1352		.0000		.0000	+.1352		−.1084	1.1084	+.1220	+ 7 00
315		+.0956		−.0772		−.1084	−.0900		.0000	1.0000	−.0900	− 5.10

It may be noted that the second halves of columns $\mathfrak{B} \sin z'$ and $\mathfrak{C} \cos z'$ are the same as the upper halves with a change of signs. Column $\mathfrak{D} \sin 2z'$ is the same in the lower half as in the upper half, no signs changed; the same is true for the column $\mathfrak{D} \cos 2z'$.

SECTION VII.

Heeling Error.

98. In Art. 74 it was shown that, with the ship upright, the magnetic forces acting on the compass needle to head, to starboard, and vertically downward were expressed by Poisson's equations, in which X, Y, and Z were the components of the earth's force in the three directions called respectively the axes of X, Y, and Z, the first two being in the horizontal plane; that magnetism was induced in the parameters $a, b, c \ldots . k$ by the earth's component parallel to the direction in which the parameters lay, the induced force in each case being a linear function of the inducing force.

When the ship heels, the transverse and vertical iron alter their directions and make with the axes of Y and Z, respectively, an angle equal to the angle of heel; for Poisson's equations to express, under the new conditions, the forces to head, in the inclined transverse direction of the deck, and in the inclined direction of the keel, these directions must be taken as new axes, and the earth's force resolved parallel to them.

Let the resolved components of the earth's force be Y_i in the inclined transverse direction of the deck, Z_i in the inclined direction of the keel, the force X to head being unchanged by heeling.

The force induced in the fore-and-aft iron will be the same as before the ship was heeled, the force induced in the transverse iron will be the same linear function of Y_i as it formerly was of Y, and that induced in the iron formerly vertical will be the same linear function of Z_i as it formerly was of Z, because, the axes and the iron being parallel, the ratio of the earth's component in the axis and the force induced in the iron of that axis will not be changed by heeling, and hence the values of the parameters in the equations for the new axes remain unchanged.

In other words, the rods a, d, and g will be magnetized by force X; b, e, and h by force Y_i; c, f, and k by force Z_i; whilst the components of the subpermanent magnetism remain unchanged.

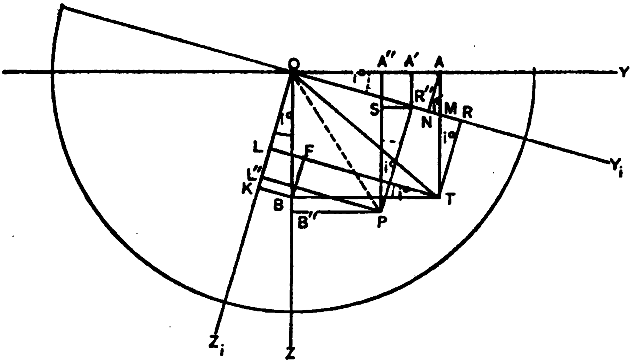


FIG. 52.

Therefore, letting X', Y_i', and Z_i' represent respectively the forces of earth and ship in the new axes to head, to starboard, and to keel, Poisson's equations become:

$$X' = X + aX + bY_i + cZ_i + P. \tag{87}$$
$$Y_i' = Y_i + dX + eY_i + fZ_i + Q. \tag{88}$$
$$Z_i' = Z_i + gX + hY_i + kZ_i + R. \tag{89}$$

The next step is to express the forces represented by equations (87), (88), and (89), in terms of the components X, Y, and Z; to do which it will be necessary to substitute the values of Y_i and Z_i in terms of those quantities.

In Fig. 52, let OY and OZ be the transverse and vertical axes, ship upright; OY_i and OZ_i the corresponding axes, ship heeled $i°$.

$OA = Y$, the horizontal component of earth's force to starboard;

$OB = Z$, the vertical component of earth's force; then

OT is that component of the earth's total force that acts in an athwartship plane through the compass, on the North point of the needle at O, which changes neither in direction nor intensity when the ship heels; and hence

$OR = Y_i$ is the component of the earth's force in the new axis to starboard, and

$OL = Z_i$ is the component of the earth's force in the new axis to keel.

But from the figure

$$OR = ON + NM + MR = OA \cos i$$
$$+ (AM + MT) \sin i = OA \cos i + OB \sin i$$
$$\text{or } Y_i = Y \cos i + Z \sin i, \tag{90}$$
$$\text{and } OL = OK - LK = OK - FB = OB \cos i - BT \sin i$$
$$\text{or } Z_i = Z \cos i - Y \sin i. \tag{91}$$

Substituting (90) and (91) in (87), (88), and (89), and collecting the terms with common factors, we have

$$X' = X + aX + (b \cos i - c \sin i) Y$$
$$+ (b \sin i + c \cos i) Z + P, \tag{92}$$
$$Y'_i = (\cos i + e \cos i - f \sin i) Y + dX$$
$$+ (\sin i + e \sin i + f \cos i) Z + Q, \tag{93}$$
$$Z'_i = (\cos i + h \sin i + k \cos i) Z + gX$$
$$- (\sin i - h \cos i + k \sin i) Y + R, \tag{94}$$

which are the forces acting still in the new axes, though in terms of X, Y, and Z.

As the compass needle is constrained to move in the horizontal plane, to obtain an expression for deviation due to the above forces, we must obtain their components in the horizontal plane.

Since the ship is heeled about the axis of X, the force X' in equation (92) is already acting in the horizontal plane, and it is only necessary to obtain the horizontal component of earth and ship to starboard represented by Y'.

Referring again to figure 52,

Let OP represent, in intensity and direction, that component of the total force of earth and ship which acts in an athwartship plane through the compass after the ship has heeled i°, then

OR'' is the component of that force in the new axis to starboard, and

$Y' = OA''$ is the component of that force in the horizontal plane to starboard.

But from the figure:

$$OA'' = OA' - A''A' = OA' - R''S$$
$$= OR'' \cos i - R''P \sin i = OR'' \cos i - OL'' \sin i;$$
$$\text{or } Y' = OA'' = Y'_i \cos i - Z'_i \sin i. \qquad (95)$$

Substituting in (95) the values of Y'_i and Z'_i from (93) and (94), we have

$$\left.\begin{aligned} Y' = {} & (\cos^2 i + e \cos^2 i - f \sin i \cos i)\, Y + d \cos i\, X \\ & + (\sin i \cos i + e \sin i \cos i + f \cos^2 i)\, Z + Q \cos i \\ & - (\sin i \cos i + h \sin^2 i + k \sin i \cos i)\, Z - g \sin i\, X \\ & + (\sin^2 i - h \sin i \cos i + k \sin^2 i)\, Y - R \sin i. \end{aligned}\right\}$$

$$\left.\begin{aligned} Y' = {} & \{ \sin^2 i + \cos^2 i \}\, Y + \{ d \cos i - g \sin i \}\, X \\ & + \{ e \cos^2 i - (f + h) \sin i \cos i + k \sin^2 i \}\, Y \\ & + \{ f \cos^2 i + (e - k) \sin i \cos i - h \sin^2 i \}\, Z \\ & + Q \cos i - R \sin i. \end{aligned}\right\}$$

Since $\sin^2 i + \cos^2 i = 1$, and by substituting $1 - \sin^2 i$ for $\cos^2 i$, we have

$$\left.\begin{aligned} Y' = Y + {} & \{ d \cos i - g \sin i \}\, X + \{ e - (f + h) \sin i \cos i \\ & - (e - k) \sin^2 i \}\, Y + \{ f + (e - k) \sin i \cos i \\ & - (f + h) \sin^2 i \}\, Z + Q \cos i - R \sin i. \end{aligned}\right\} \quad (96)$$

Equations (92) and (96) are of the form

$$X' = X + a_iX + b_iY + c_iZ + P_i\,, \tag{97}$$

$$Y' = Y + d_iX + e_iY + f_iZ + Q_i\,, \tag{98}$$

if

$$\left.\begin{aligned}
a_i &= a.\\
b_i &= b\cos i - c\sin i.\\
c_i &= b\sin i + c\cos i.\\
d_i &= d\cos i - g\sin i.\\
e_i &= e - (f+h)\sin i\cos i - (e-k)\sin^2 i.\\
f_i &= f + (e-k)\sin i\cos i - (f+h)\sin^2 i.\\
P_i &= P.\\
Q_i &= Q\cos i - R\sin i.
\end{aligned}\right\} \tag{99}$$

99. The coefficients when ship is heeled.—As the values of the parameters and magnets have changed in consequence of the ship's heeling, so have the magnetic coefficients which depend on them.

Therefore, if λ_i, $\mathfrak{A}_i$, $\mathfrak{B}_i$, $\mathfrak{C}_i$, $\mathfrak{D}_i$, $\mathfrak{E}_i$, respectively, represent the altered values of the coefficients λ, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, due to the heeling of the ship through $i°$, by substitution in the equations for the exact coefficients, we have

$$\lambda_i = \lambda - \frac{f+h}{2}\sin i\cos i - \frac{e-k}{2}\sin^2 i. \tag{100}$$

$$\lambda_i\mathfrak{A}_i = \lambda\mathfrak{A}\cos i + \frac{c-g}{2}\sin i. \tag{101}$$

$$\lambda_i\mathfrak{B}_i = \lambda\mathfrak{B} + \{\, b\sin i - c\,\mathrm{versin}\, i \,\}\tan\theta. \tag{102}$$

$$\left.\begin{aligned}
\lambda_i\mathfrak{C}_i = \lambda\mathfrak{C} + \{\, &(e-k)\sin i\cos i - \frac{R}{Z}\sin i\\
&- (f+h)\sin^2 i \,\}\tan\theta + \frac{Q}{H}\mathrm{versin}\, i.
\end{aligned}\right\} \tag{103}$$

$$\lambda_i\mathfrak{D}_i = \lambda\mathfrak{D} + \frac{f+h}{2}\sin i\cos i + \frac{e-k}{2}\sin^2 i. \tag{104}$$

$$\lambda_i\mathfrak{E}_i = \lambda\mathfrak{E}\cos i - \frac{c+g}{2}\sin i. \tag{105}$$

Approximate values of the coefficients when the ship is heeled.—If the soft iron is symmetrically arranged on each side of the fore-and-aft plane through the compass, b, d, f, h, $\mathfrak{A}$ and $\mathfrak{E}$ will be zero; and as a steady angle of heel would be small, we may without much sacrifice of accuracy replace $\sin i$ by i, letting $\cos i = 1$, $\operatorname{versin} i = 0$, and $\sin^2 i = 0$. Equations (100) to (105) will then give

$$\left.\begin{aligned}
\lambda_i &= \lambda. \\
\mathfrak{A}_i &= +\frac{c-g}{2\lambda}i. \\
\mathfrak{B}_i &= \mathfrak{B}. \\
\mathfrak{C}_i &= \mathfrak{C} + \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right)\tan\theta i. \\
\mathfrak{C}_i &= \mathfrak{C} + Ji, \text{ if } J = \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right)\tan\theta. \\
\mathfrak{D}_i &= \mathfrak{D}. \\
\mathfrak{E}_i &= -\frac{c+g}{2\lambda}i.
\end{aligned}\right\} \quad (106)$$

100. Deduction of the equation expressing heeling deviation.*—If δ represents the deviation for a given compass course z' when the ship is on an even keel, δ_i the deviation for the same compass course when the ship heels $i°$ to starboard, then equation (34) becomes in each case, the approximate heeling coefficients being substituted,

$$\delta\ (1 - \mathfrak{D}\cos 2z') = \mathfrak{A} + \mathfrak{B}\sin z' + \mathfrak{C}\cos z' + \mathfrak{D}\sin 2z' + \mathfrak{E}\cos 2z'$$

$$\delta_i\ (1 - \mathfrak{D}\cos 2z') = +\frac{c-g}{2\lambda}i + \mathfrak{B}\sin z' + (\mathfrak{C} + Ji)\cos z' + \mathfrak{D}\sin 2z' - \frac{c+g}{2\lambda}i\cos 2z'.$$

Therefore, since by the hypothesis $\mathfrak{A}$ and $\mathfrak{E}$ are zero,

$$(\delta_i - \delta)\ (1 - \mathfrak{D}\cos 2z') = \frac{c-g}{2\lambda}i + Ji\cos z' - \frac{c+g}{2\lambda}i\cos 2z'.$$

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* In an analogous manner, the deviation of the compass, due to a change of trim in a fore and aft direction, has been deduced and expressed in the following form by Lt. Comdr. Lloyd H. Chandler, U. S. N., in which t is the angle of inclination:

$$\delta_t = \frac{2\lambda}{2\lambda-(c+g)t}\delta + \left(\frac{2}{2\lambda-(c+g)t}\right)\left\{\left(a-k-\frac{R}{Z}\right)\tan\theta\right\}t\sin z' - \left\{\frac{c+g}{2\lambda-(c+g)t}\right\}t\sin 2z'.$$

It has its field of application when necessity arises to navigate under abnormal trim, as when compartments at one end of the ship have become flooded. (Proceedings of the U. S. Naval Institute, Vol. 34, 1908.)

Substituting $\cos^2 z' - \sin^2 z'$ for $\cos 2z'$, multiplying $\frac{c-g}{2\lambda}i$ by $(\sin^2 z' + \cos^2 z')$, rearranging the terms in the right member of the equation, also regarding $(\delta_i - \delta)\ \mathfrak{D}\cos 2z'$ as zero, and transposing δ we have

$$\delta_i = \delta + Ji\cos z' + \frac{c}{\lambda}i\sin^2 z' - \frac{g}{\lambda}i\cos^2 z', \qquad (107)$$

which will give the heeling deviation on the compass heading z' when δ, J, c, and g are known. It has already been shown how to find δ (Art. 55), also c and g (Arts. 96 and 92). The method of finding J will be explained in Art. 103.

101. General effect of ship's heeling on the deviation of the compass.—Equation (107) shows that the effect of heeling, besides altering the term $\mathfrak{C}$ by the expression Ji, is to introduce a constant term and a quadrantal term of the $\mathfrak{E}$ type; iron which is symmetrical with the ship upright becoming unsymmetrical when the ship heels. It is readily seen that c introduces a $(-b)=ci$, and g, a $(+d)=-gi$, when the ship heels, and that these cause the $\mathfrak{A}_i$ and $\mathfrak{E}_i$. c represents vertical soft iron in the midship line, as the smokestack; the effect of c depends on the proximity of the compass to the pole of c, and is a minimum when the ship heads North or South, a maximum when the ship heads East or West. $\frac{c}{\lambda}$ is generally $+$ and seldom exceeds .100 for the usual positions of compasses. The parameter g represents soft iron parallel to the axis of X, as the keel or propeller shaft; the effect of g depends on the proximity of the compass to the pole of g, being greatest when the compass is well forward or well aft and the ship's head North or South. The effect of g is a minimum so far as location of the compass is concerned when the compass is at equal distances from each end of the ship, and disappears in all cases when the ship heads East or West. $\frac{g}{\lambda}$ is generally $+$ and seldom exceeds .100.

The effects of both c and g may be neglected in the ordinary cases in navigation.

The term $Ji \cos z'$, however, cannot be neglected, as it is often large in amount.

When J is (—), as it usually is for compasses on the upper deck, it represents a deviation of the North point of the needle to windward; when +, as it may be for a compass on the main deck, it represents a deviation to leeward. (—) J is called the heeling coefficient to windward, (—) $Ji \cos z'$ being the heeling error to windward, a maximum on North and South courses and a minimum on East and West courses. J is generally a fractional number and indicates the heeling deviation, for each degree of heel, arising from a change in the value of $\mathfrak{C}$ due to heeling, on N. and S. courses (p. c.).

102. General effect of the ship's heeling on the coefficients determined when the ship is upright.—An inspection of equations in group (106) shows that the coefficients depending on fore-and-aft action, $\mathfrak{B}$ and $\mathfrak{D}$, are unaltered; that $\mathfrak{A}$ and $\mathfrak{E}$ undergo a slight change; and that $\mathfrak{C}$ is considerably altered. As $\mathfrak{C}$ has its maximum effect when the ship heads North or South and its minimum effect when the ship heads East or West, it is apparent that the heeling error is a maximum or a minimum under the same circumstances.

103. Different ways of expressing the heeling coefficient and the use of each.—The expression $J = \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right) \tan \theta$ may be written (—) $J = \frac{1}{\lambda}\left(-e + k + \frac{R}{Z}\right) \tan \theta$, and then, if the right hand member of the equation is positive, it indicates (as is usual in North magnetic latitude) a deviation of the North end of the needle to windward.

Since $\tan \theta = \frac{Z}{H}$, $e = \lambda\,(1 - \mathfrak{D}) - 1$, and $\mu - 1 = k + \frac{R}{Z}$, the expression $-J = \frac{1}{\lambda}\left(-e + k + \frac{R}{Z}\right) \tan \theta$ may be put in the forms below, each of which will be shown to serve a special purpose:

$$(a)\ -J = \frac{1}{\lambda}\left\{(-e+k)\tan\theta + \frac{R}{H}\right\}.$$

$$(b)\ -J = -\frac{eZ}{\lambda H} + \frac{kZ+R}{\lambda H}.$$

$$(c)\ -J = \left(\mathfrak{D} + \frac{1}{\lambda} - 1\right)\tan\theta + \frac{\mu-1}{\lambda}\tan\theta.$$

$$(d)\ -J = \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1\right)\tan\theta.$$

Form (a) shows the changes which may be expected in $(-)J$ on change of magnetic latitude; λH is always $+$; $\tan\theta$ is $+$ in the northern, $(-)$ in the southern hemisphere; at the usual position of a standard compass on a ship built in North magnetic latitude, $(-)\ e$, $+\ k$, and $+\ R$ are positive. $(-)\ e$ and $+\ k$ will change sign in South latitude, $+\ R$ will not, therefore the heeling error will be to windward unless the ship is so far South in the southern hemisphere that $(-e+k)\tan\theta$ is greater than $\frac{R}{H}$.

Form (b) shows how the heeling deviation is caused; $(-)\ \frac{eZ}{\lambda H}\ i$ expresses the effect due to vertical induction in horizontal soft iron represented by the rod $(-e)$, inclined at an angle i to the horizontal plane, and acting against a directive force λH; the effect being a heeling error to windward in North magnetic latitudes, to leeward in South magnetic latitudes. $\frac{kZ+R}{\lambda H}\ i$ expresses the combined effect of vertical induction in vertical soft iron represented by the rod $+\ k$ and the component to keel of the subpermanent magnetism represented by $+\ R$, both acting at an angle i from the vertical and against the directive force λH. The effect of this part is a heeling error to windward or leeward according as the resultant force of $kZ + R$ is $+$ or $(-)$.

Form (*c*) is useful for computing separately the heeling deviation due to (1) vertical induction in horizontal transverse soft iron, (2) vertical induction in vertical soft iron and vertical subpermanent magnetism; $\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta = \frac{-eZ}{\lambda H}$ expressing the first part, and $\frac{\mu-1}{\lambda}\tan\theta=\frac{kZ+R}{\lambda H}$ expressing the second part.

Form (*d*) is the most convenient form for computing the heeling coefficient; the value of $\mathfrak{D}$ having been determined by analysis of a table of deviations or from observations on two headings, μ and λ by vertical and horizontal vibrations on board and ashore, and θ taken from a chart of magnetic dip.

In order that there may be no semicircular heeling error (—) J must be zero; therefore,

$$\mathfrak{D}+\frac{\mu}{\lambda}-1=0,$$
$$\mu=\lambda(1-\mathfrak{D})=1+e,$$
$$\mu-1=e.$$

That is to say

$$\mu=\frac{\text{Mean vertical force of earth and ship}}{\text{Vertical force of earth}}=\lambda(1-\mathfrak{D})=1+e,$$

and

$$\mu-1=\frac{\text{Mean vertical force of ship}}{\text{Vertical force of earth}}=e.$$

Ex. 18.—With the following data from examples 11 and 12, viz., $\lambda = .8856$, $\mathfrak{D} = +.05$, $\tan\theta = 2.86$, $\mu = 1.0969$, $g = .1098$, c being neglected, it is required to find $(-)J$ and the total deviation on courses South and NW. (p. c.) when the ship is heeled 1st 10° to starboard, 2d 10° to port. Deviation, when ship is upright, on South $+4°\ 30'$, on NW. $-16°$.

$$-J = \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1\right)\tan\theta = \left(.05 + \frac{1.0969}{.8856} - 1\right)2.86 = 0.8254$$

$$\text{and } +J = -\ 0.8254$$

$$\frac{g}{\lambda} = \frac{.1098}{.8856} = +\ 0.124.$$

The ship heels 10°, therefore $i = 10\begin{cases} + \text{ if heeled to starboard} \\ - \text{ if heeled to port..}\end{cases}$

At South, $Ji\cos z' - \frac{g}{\lambda}i\cos^2 z' = (-).8254 \times 10 \times (-1) - .124 \times 10 \times 1$

$$= +\ 8°.254 - 1°.24 = +\ 7°.014 = +\ 7°\ 01'.$$

At NW., $Ji\cos z' - \frac{g}{\lambda}i\cos^2 z' = (-).8254 \times 10 \times .707 - .124 \times 10 \times .5$

$$= (-)\,5°.836 - 0°.620 = (-)\,6°.456 = (-)\,6°\ 27'.$$

Ship heeled to starboard, on South,

$$\delta = +\ 4°\ 30' + (+)\ 7°\ 01' = +\ 11°\ 31'.$$

Ship heeled to port, on South,

$$\delta = +\ 4°\ 30' - (+)\ 7°\ 01' = (-)\ 2°\ 31'.$$

Ship heeled to starboard, on NW.,

$$\delta = (-)\ 16° + (-)\ 6°\ 27' = (-)\ 22°\ 27'.$$

Ship heeled to port, on NW.,

$$\delta = (-)\ 16° + (+)\ 6°\ 27' = (-)\ 9°\ 33'.$$

104. Determination of heeling error by listing and then swinging the ship.—The deviations may be determined by swinging the ship, first, upright, then heeled $i°$. The difference on any compass azimuth of the two results will be the heeling error for that angle of heel on that course.

The results of this practical method no doubt would be more satisfactory after the work was done; it is a tedious process, however, and the heeling error is usually determined theoretically from observations already shown to be of a simple character, when not corrected by the tentative method.

If swinging takes place both before and after listing, c may be found from the observations at East or West, as then $\delta_i = \delta + \frac{c}{\lambda} i$; and g may be determined from the observations at North or South, as then $\delta_i = \delta + Ji - \frac{g}{\lambda} i$, λ and J having been computed.

105. Correction of heeling error by vibrations.—As e is minus and less than unity when the quadrantal deviation has not been corrected, it is thus evident, in order that there may be no heeling deviation under such circumstances, that the mean vertical force at the position of the compass must be less than that on shore, and that the time of n vibrations of a vertical needle at the position of the compass represented by T' must be greater than the time of the same number of vibrations of the same needle ashore represented by T.

Regarding h as zero, (63) becomes

$$\frac{Z'}{Z} = \frac{T^2}{T'^2} = \mu + g \cot\theta \cos z$$

$$\text{and } T' = \frac{T}{\sqrt{\mu + g \cot\theta \cos z}} \tag{108}$$

but when the heeling error is corrected $\mu = \lambda(1 - \mathfrak{D}) = 1 + e$.

$$\text{Then } T' = \frac{T}{\sqrt{\lambda(1 - \mathfrak{D}) + g \cot\theta \cos z}} = \frac{T}{\sqrt{1 + e \pm g \cot\theta \cos z}}. \tag{109}$$

When g is unknown, the ship's head may be placed on East or West magnetic,

$$\text{then } T' = \frac{T}{\sqrt{\lambda(1 - \mathfrak{D})}} = \frac{T}{\sqrt{1 + e}}. \tag{110}$$

If the spheres are in position there will be a new value of λ and perhaps a residual value of $\mathfrak{D}$.

Let the altered values be λ_2 and $\mathfrak{D}_2$.

Then with the spheres in place equations (109) and (110) become

$$T' = \frac{T}{\sqrt{\lambda_2 (1 - \mathfrak{D}_2) + g \cot \theta \cos z}}. \qquad (111)$$

$$T'' = \frac{T}{\sqrt{\lambda_2 (1 - \mathfrak{D}_2)}}. \qquad (112)$$

It is thus seen that the heeling error may be corrected by so altering the vertical force that the vertical vibrations of a dipping needle shall take place in the proper time found, according to the circumstances, from equations (109), (110), (111), or (112).

The vertical force is altered as desired by the vertical movement of a vertical magnet in the binnacle tube below the center of the needle.

It is customary now to correct the heeling error by what is known as the heeling adjuster, or by the tentative method (Art. 108 (5)).

106. Correction of heeling error by using the heeling adjuster.—The heeling adjuster is a small brass box provided with levels and leveling screws, mounting on a horizontal axis a needle which is free to vibrate in the vertical plane, its tendency to dip being counteracted by a small sliding platinum weight whose distance from the axis of suspension may be measured by a scale on the glass cover. There is a small glass window in each end provided with an index line to mark the horizontal plane. Without the small weight, the needle before being magnetized was exactly balanced, so the weight is intended to balance the vertical magnetic force ashore or on board. Letting b and a, respectively, denote the distance between the weight and the center of the needle when the needle is exactly balanced on board and ashore, the heeling adjuster being properly placed in the magnetic meridian, then $\frac{b}{a} = \frac{Z'}{Z}$; and, when the ship's head is East or West magnetic, $\frac{b}{a} = \mu$.

In order that there may be no heeling error we must have $\mu = 1 + e = \lambda(1 - \mathfrak{D})$.

$$\text{Therefore, } b = a\lambda(1 - \mathfrak{D}) \qquad (113)$$

before the quadrantal spheres are placed.

If λ_2, $\mathfrak{D}_2$, μ_2 be the altered values of λ, $\mathfrak{D}$, and μ after the spheres are in place,

$$\text{then } \mu_2 = \lambda_2(1 - \mathfrak{D}_2) \text{ or } b = a\lambda_2(1 - \mathfrak{D}_2). \qquad (114)$$

To correct the error.—Place the weight at reading b from (113) or (114), according as the spheres are off or on the brackets, head ship East or West magnetic, put the adjuster on the brass table provided for this purpose, in the exact position of the compass needle, the adjuster properly placed in the meridian. If the needle remains horizontal there is no heeling error. If one end dips, place the heeling corrector magnet in the tube, proper pole up; raise or lower it till the adjuster needle is horizontal. The heeling error is then corrected.

SECTION VIII.

COMPENSATION OF THE COMPASS.

107. Principles and object of compensation.—It has been shown that each kind of deviation is due to certain forces, either of attraction or repulsion, acting on the North point of the compass needle, and it is evident from the known laws of magnetic action that these forces can be neutralized and hence deviation reduced to zero, by introducing other forces of the same magnitudes, but such as to act in the opposite directions. By compensation, the deviations are not only reduced to zero, or to convenient amounts, so that a change in azimuth of the ship's head is represented by a similar apparent movement of the compass, but the directive force of the needle is equalized on the different headings. *After compensation, all correctors should be secured in place and their positions noted in the Compass Journal.*

108. Order of compensation.—Since the correctors, when in place, exert a mutual action on each other and thereby create forces additional to those of the ship, it is essential that the semicircular correction, which is the largest and most important one, should be made when the magnetic conditions approximate as nearly as possible to those when the compensation is complete; therefore, the quadrantal and heeling correction should precede the semicircular correction. The quadrantal spheres, when in place, correct a portion of the heeling error and for this reason it is desirable that the spheres should be in place before the heeling correction is made. However, if the values of λ and $\mathfrak{D}$, before the spheres are in place, are known by computation, the heeling correction may properly be made by the method of Art. 106, as the first correction, the distance used for the position of the weight on board being $b = a\lambda\ (1 - \mathfrak{D})$.

If the correction should be made by this method after the spheres have been placed, we must find the distance b from the equation $b = a\lambda_2\ (1 - \mathfrak{D}_2)$, λ_2 and $\mathfrak{D}_2$ being altered values of λ and $\mathfrak{D}$.

The heeling error may, however, be corrected by the tentative method and, in that case, the following will be the order of compensation:

(1) Correction for quadrantal deviation (approximate).
(2) Correction for heeling error (approximate).
(3) Correction for semicircular deviation.
(4) Correction for quadrantal deviation.
(5) Correction for heeling error.
(6) Swing for residuals.

It being assumed that the ship is on an even keel; all movable local masses of iron or steel in the vicinity of the compass secured in their normal positions for sea; and the binnacle of Type VI stripped of all correctors, which are placed at a safe distance; we will proceed to compensate the standard compass.

From data by computation.—Having obtained a curve of deviations for the standard compass by any of the methods previously referred to, take from the Napier's diagram the compass headings corresponding to North, NE., and East magnetic, then head the ship successively on those rhumbs, steadying on each at least four minutes. Note carefully the reading of the steering compass when the ship is steadied on those rhumbs.

Then proceed with

(1) **The approximate correction of the quadrantal deviation.**—If the value of $\mathfrak{D}$ is known or can be estimated, place the spheres on the brackets according to Table III of "Diehl's Compensation of the Compass," or Table V of this book.

If $\mathfrak{D}$ is unknown, place them at a mean position of 13.5 inches for the 7-inch spheres. If spheres of this size overcorrect at the outer limit, use smaller ones, remembering that one sphere will correct half as much as two of the same size.

(2) **The approximate correction of the heeling error.**—Having no means to determine λ_2 and $\mathfrak{D}_2$, place the ship's head East or West magnetic by means of the steering compass. The needle of the heeling adjuster having been made horizontal on shore, with the weight in a given position, must be made nearly horizontal on board, position of weight unchanged, by means of the vertical correcting magnet in the central tube, the non-weighted end inclined perceptibly upwards.

In case no observations were made ashore, place the heeling magnet in its tube, North end up in North magnetic latitude unless there is reason to know that the ship's vertical force acts upward, and lower it to the bottom.

(3) **To correct the semicircular deviation.**—Neglecting the values of $\mathfrak{A}$ and $\mathfrak{E}$, it is apparent from the equation

$$\tan \delta = \frac{\mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D}_2 \sin 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D}_2 \cos 2z},$$

in which $\mathfrak{D}_2$ is

the coefficient of quadrantal deviation left uncorrected, that when the ship heads magnetic North or South, $\tan \delta = \frac{\pm \mathfrak{C}}{1 \pm \mathfrak{B} + \mathfrak{D}_2}$; that the forces $\mathfrak{B}$ and $\mathfrak{D}_2$ act in the meridian; that the transverse force $\mathfrak{C}$ is the only one acting to produce deviation; and that in order to reduce the deviation on those headings to zero, it is only necessary to neutralize $\mathfrak{C}$, which may be done by introducing an equal but opposing force in the transverse line.

When the ship heads East or West magnetic, the above equation becomes $\tan \delta = \frac{\pm \mathfrak{B}}{1 \mp \mathfrak{C} - \mathfrak{D}_2}$, the forces $\mathfrak{C}$ and $\mathfrak{D}_2$ act in the meridian, and the fore-and-aft force $\mathfrak{B}$ is the only one acting to produce deviation; to reduce the deviation on those headings to zero, it is only necessary to neutralize $\mathfrak{B}$, which may be done by introducing an equal but opposing force in the fore-and-aft line. Therefore,

To neutralize the force $\mathfrak{C}$.—Head the vessel North magnetic and keep it steady by the steering compass. Run the athwartship carrier down.

If the compass shows easterly deviation, the force $\mathfrak{C}$ draws the North point of the needle to starboard; enter one or more magnets on each side of the athwartship carrier, North or red ends to starboard; move the carrier up or down until the compass points North magnetic.

If the compass shows westerly deviation, the force $\mathfrak{C}$ draws the North point of the compass needle to port; enter the athwartship magnets with North or red ends to port; raise or lower the carrier till the compass points North magnetic.

Or head the vessel South magnetic, enter the athwartship magnets with North or red ends to port if the deviation is easterly, or to starboard if the deviation is westerly; raise or lower the carrier till the compass points South magnetic.

To neutralize the force $\mathfrak{B}$.—Head the vessel East magnetic and keep it steady by the steering compass. Run the fore-and-aft carrier down.

If the compass shows easterly deviation, the force $\mathfrak{B}$ draws the North point of the needle to head; enter one or more magnets on each side of the fore-and-aft carrier, North or red ends forward; move the carrier up or down till the compass heading of the ship is East. If the compass shows westerly deviation, the force $\mathfrak{B}$ draws the North point of the needle to stern; enter the fore-and-aft magnets with North or red ends aft and raise or lower the carrier till the compass heading is East.

Or head the vessel West magnetic; enter the fore-and-aft magnets with North or red ends aft, if the deviation is easterly, or forward if the deviation is westerly; raise or lower the carrier till the compass heading is West.

(4) **To correct the quadrantal deviation.**—With the semicircular forces neutralized there remains only $\mathfrak{D}_2$ to cause deviation, and when the ship heads NE., SE., SW., or NW., magnetic, $2z$ being 90° or 270°, the equation for deviation becomes $\tan \delta = \pm \mathfrak{D}_2$, and to reduce the deviation on those headings to zero, it is only necessary to neutralize the force $\mathfrak{D}_2$, which may be done by introducing an equal but opposing force.

The quadrantal deviation is usually positive, and hence is corrected by placing the quadrantal spheres to starboard and port of the compass in which position they produce a negative quadrantal deviation, the soft iron of the sphere having the effect of the $-a$ and $+e$ rods; therefore,

To neutralize the remaining quadrantal force.—Having corrected the semicircular deviation, head the vessel NE. (or SE., SW., NW.) magnetic and keep it steady by the steering compass. If any deviation is shown, move the spheres on the side brackets in or out until the compass heading is NE. (SE., SW., NW.).

If the spheres over-correct at the outer limits of the brackets, use smaller ones; if they undercorrect when placed at the inner limits, use larger ones.

(5) **To correct the heeling error.**—In case the heeling corrector has not been placed by shore observations, and is in the bottom of the central tube of the binnacle, if there is sufficient sea on to give a moderate roll on a North or South course, steer North or South per compass and observe the vibrations of the card as the ship rolls from side to side. These will be greater than those due to the ship's real motion in azimuth when the heeling error is material, therefore raise the heeling corrector slowly till the vibrations almost disappear, leaving an amplitude of 1° or 2° to avoid over-correction. It must not be forgotten, however, that the correction once made is for that particular latitude only and the position of the heeling corrector must be changed for any considerable change of magnetic latitude.

In the case of a vessel heeling steadily on a North or South course, the deviations observed when heeled may be compared with those when the ship is upright on the same course, and the difference removed by raising or lowering the corrector.

If the conditions are not favorable for the final placing of the heeling corrector, reserve it for a future time.

(6) Swing ship on the sixteen principal rhumbs and obtain a table of residual deviations; either readjust the correctors, proceeding as in the original correction, or use the residual deviations to run on. In case of re-compensation, the vessel must be again swung for a final table of deviations.

The ship may now be placed with its head on any two adjacent cardinal points magnetic by the standard compass, and the other compasses corrected for semicircular deviation; then on the intercardinal point magnetic by the standard compass, and the others corrected for quadrantal deviation.

109. Determination of the magnetic courses when compensating compasses whose deviations are unknown.—Select ahead of time the locality, the date, and the limits of local apparent time between which the observations must be made.

Take from the Nautical Almanac the sun's declination for the instant midway between the time limits, and, by the method of Art. 58, find from the azimuth tables the sun's true bearing at intervals, say, of ten minutes of time, for the known latitude and declination. Apply the variation for the locality and obtain the magnetic bearings. Make a table with a column of magnetic bearings opposite a column of local apparent times, or construct a curve; the ordinates representing local apparent times at intervals of ten minutes; the abscissæ, the corresponding magnetic azimuths for the given latitude and declination.

On the date selected proceed to the locality, set the watch to local apparent time, and shortly before the first selected time, set that rhumb of the pelorus corresponding with the desired magnetic heading to the ship's head and clamp the plate. Set the sight vanes to correspond with the sun's magnetic bearing at the selected time and clamp the vanes.

Working the engines slowly and using the helm, bring the sight vanes to bear on the sun and keep them there till the watch shows the selected local apparent time when the ship heads on the desired magnetic point, and the ship's head per standard should be noted, also the ship's head per steering compass. Let it be assumed that we have obtained the headings by the steering compass corresponding to any two adjacent cardinal points and the intervening quadrantal point, all magnetic, and that a careful record has been made of the same. Then to compensate, it is only necessary to proceed as explained in Art. 108.

Ex. 19.—Having decided to compensate the compass on April 18, 1918, off Cape May, in latitude 39° N., longitude 74° 30′ W., between the hours of 8 a. m. and 10 a. m., local apparent time, it is required to make a table of magnetic bearings of the sun at intervals of ten minutes between the limits named, and to determine the compass readings corresponding to magnetic North, NE., and East. Variation −8°.

	h m s
L. A. T. of middle instant	9 00 00
Longitude of locality	4 58 00W.
G. A. T. of middle instant	1 58 00
Equation of time	− 00 33
G. M. T. of middle instant	1 57 27
or April 18.	$1^h.96$

		h m m s
Equation of Time at G. (+ to Mean Time)		1 58=0 33.0
☉'s declination		H. D.
At G. M. noon	N. 10° 38′.2	0′.9
Corr.	1.8	1.96
	Dec.=N. 10° 40′	1′.8
	=N. 10°.7	

$$\left.\begin{array}{l}\text{Lat. } 39^\circ \text{ N.}\\ \text{Dec. } 10^\circ \text{ N.}\\ \text{L. A. T. } 9^h\end{array}\right\}\begin{array}{l}\text{On page 90, azimuth tables.}\\ \\ Z = \text{N. } 113^\circ\ 32' \text{ E.}\end{array}$$

For Dec. 11° N. $Z = \text{N. } 112\ \ 34$ E.

Diff. for 1° of Dec. (—) 58′
Diff. for 0°.7 of Dec. (—) 41′

Hence we have for Lat. 39° N. and Dec. 10°.7 N. as follows:

L. A. T.	Sun's true azimuth.	Sun's magnetic azimuth.
8^h 00^m a. m.	100° 55′	108° 55′
8 10 a. m.	102 44	110 44
8 20 a. m.	104 36	112 36
8 30 a. m.	106 33	114 33
8 40 a. m.	108 34	116 34
8 50 a. m.	110 40	118 40
. .	. .	. .
. .	. .	. .
. .	. .	. .
. .	. .	. .
10 00 a. m.	128 32	136 32

To head magnetic North.—The ship being on the station ahead of time—before 8 a. m., local apparent time, set the North point of the pelorus to correspond with the ship's head, clamp the plate; set the sight vanes to the magnetic bearing of the sun 108° 55′ (by table or curve) and clamp the vanes. So manœuvre the ship by the engines and helm that the sight vanes will point directly toward the sun. By helm and engines, keep the sight vanes on the sun till the watch set to local apparent time indicates 8 a. m.

At that instant the ship heads North magnetic.

The standard compass reads 6° (for example).

The steering compass reads 8° (for example).

Note carefully the heading by the steering compass at this time.

To head NE. magnetic.—Say it is desired to be on this heading at $8^h\ 20^m$ a. m. Set the NE. point of the pelorus to the ship's head, clamp the plate; set the sight vanes to the magnetic bearing of the sun 112° 36′ (by table or curve) and clamp the vanes. Proceed as before, keeping the vanes on the sun till $8^h\ 20^m$ a. m., when the ship heads NE. magnetic and

The standard compass reads 38° 30′ (for example).

The steering compass reads 22° 30′ (for example).

Note carefully the heading by steering compass.

To head magnetic East.—Let $8^h\ 40^m$ a. m. be the selected time. A short time before this clamp the pelorus plate with the East point on the forward keel line or indicator and clamp the vanes to indicate magnetic bearing of the sun 116° 34′ (by table or curve). Proceed as before, keeping the vanes on the sun till the watch set to local apparent time shows $8^h\ 40^m$ a. m. At that instant the ship will be heading East magnetic and

The standard compass reads 78° 00′ (for example).

The steering compass reads 56° 00′ (for example).

Again note carefully the heading by steering compass.

To head a magnetic course by azimuth circle.—Knowing the magnetic bearing of the sun for a given instant at the place of observation, or of a distant object, set the direct sight vanes of the azimuth circle to the right or left of the ship's head by compass by an angle equal to that which the sun or object is to the right or left of the magnetic heading desired at the selected instant. By using helm and engines bring the sight vanes on the sun, keeping them on it till the watch shows the selected time, when the ship will be on the desired magnetic heading. In the case of a distant object the time is not considered.

110. To compensate on one heading, as when riding to a tide, in a dry dock, or when alongside a wharf, etc.*—Having obtained the exact coefficients $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ by any of the methods already referred to, also the magnetic heading, and knowing the compass heading and deviation, compute the deviation due to each coefficient by substituting that coefficient alone in the equation,

$$\tan \delta = \frac{\mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z},$$

then find what should be the compass heading as each amount is successively neutralized.

If the value of λ is known, make observations with the heeling adjuster ashore and on board, finding $b = a\lambda\ (1 - \mathfrak{D})$; and, neglecting g, place the heeling corrector magnet in place (Art. 106). However, if the values of λ_2 and $\mathfrak{D}_2$ may be determined after the quadrantal spheres are in place, first put the spheres on the brackets and correct the quadrantal deviation; then, finding $b = a\lambda_2\ (1 - \mathfrak{D}_2)$, place the heeling corrector.

Move the quadrantal correctors in or out, keeping them equally distant from the compass needles, till the amount of deviation due to $\mathfrak{D}$ is corrected.

Then correct the amount of deviation due to that force, $\mathfrak{B}$ or $\mathfrak{C}$, which, for the ship's heading, is more nearly at right angles to the direction of the compass needle. Thus, if the ship's head is more nearly North or South, eliminate the deviation due to $\mathfrak{C}$ by means of the athwartship magnets first, and then eliminate that due to $\mathfrak{B}$. If the ship's heading is more nearly East or West, reverse this procedure, eliminating first the deviation due to $\mathfrak{B}$ and then that due to $\mathfrak{C}$.[1]

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* For procedure in special cases, as when heading North (E., S. or W.), magnetic, etc., see Appendix B.

When the forces $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ have been neutralized, compass and magnetic courses should be the same ($\mathfrak{A}$, $\mathfrak{E}$, zero).

Ex. 20.—In example 14, Art. 95, the following coefficients were found for a standard compass by observations of deviation and horizontal force on two opposite headings, viz.: $\mathfrak{B} = (-)\,.0747$, $\mathfrak{C} = (-)\,.3142$, $\mathfrak{D} = +\,.1211$.

It is required to find the deviation due to each force when the ship heads 199° 30′ (p. s. c.), dev. + 25° 30′, and the compass heading per standard as each is successively neutralized, the ship being kept steady on the corresponding magnetic heading by the steering compass.

To find the deviation due to $\mathfrak{D}$,

$$\tan \delta_q = \frac{\mathfrak{D} \sin 2z}{1+\mathfrak{D} \cos 2z} = \frac{+.1211\times 1}{1+0} = +.1211 \text{ and } \qquad \delta_q = +6^\circ\, 54'\, 16''.$$

To find deviation due to $\mathfrak{B}$,

$$\tan \delta_B = \frac{\mathfrak{B} \sin z}{1+\mathfrak{B} \cos z} = \frac{(-).0747\times(-).7071}{1+((-).0747\times(-).7071)} = \frac{+.0528}{1.0528} = +.05015 \text{ and } \delta_B = +2^\circ\, 52'\, 28''.$$

To find deviation due to $\mathfrak{C}$,

$$\tan \delta_C = \frac{\mathfrak{C} \cos z}{1-\mathfrak{C} \sin z} = \frac{(-).3142\times(-).7071}{1-((-).3142\times(-).7071)} = \frac{+.2222}{.7778} = +.28568 \text{ and } \delta_C = +15^\circ\, 56'\, 35''.$$

Therefore, note the corresponding heading by the steering compass and keep the ship steadied on that heading, in case the vessel is not secured in dock or to a wharf. Then,

(1) By means of the spheres neutralize the force $+\mathfrak{D}$, making the ship's head per standard compass 206° 24′ 16″.

(2) By means of the athwarthship magnets, North or red ends to port, neutralize the force $(-)\,\mathfrak{C}$, making the compass headings 222° 20′ 51″.

(3) By means of the fore-and-aft magnets, North or red ends aft, neutralize the force $(-)\,\mathfrak{B}$, making the compass heading 225°, which is the magnetic heading.

As $\delta_B = +\,2^\circ\, 52'\, 28''$, the amount of error is only about 13′.

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[1] If found, by neutralizing $\mathfrak{B}$ and $\mathfrak{C}$ in a certain order, that the elimination of one force leaves the other at a small angle with the needle, a condition unfavorable for its elimination, consider the effect of a reversal of that order with a view to improving conditions.

111. Values of *A* and *E* to be left uncorrected.—In all cases of the compensation of the compass, when *A* or *E* or their algebraic sum is as much as 1°, the amount should be left uncorrected. *A* has a constant value and sign on all headings, the quadrantal deviation represented by *E* varies as cos $2z'$ and changes sign at East and West.

Therefore, if compensating semicircular deviation on North or South, the amount to be left uncorrected for *A* and *E* would be the algebraic sum of the amounts due to their signs by analysis; if on heading East or West, it would be the algebraic sum of the amounts due to *A* with sign unchanged and to *E* with sign changed from that by analysis.

SECTION IX.

112.[1] The Dygogram; Its Construction, Description, and Use.—The dygogram is one of the graphic methods of representing the deviations of the compass for magnetic headings; it also shows the horizontal components of the magnetic force acting on the compass needle, the directions in which they act, and the deviations produced by each component as well as the total deviation for any magnetic heading.

The word "dygogram" is a contraction for "dynamo-goniogram," meaning a "force and angle diagram." It is a geometrical construction fulfilling the conditions of the general expression

$$\tan\delta = \frac{\mathfrak{A} + \mathfrak{B}\sin z + \mathfrak{C}\cos z + \mathfrak{D}\sin 2z + \mathfrak{E}\cos 2z}{1 + \mathfrak{B}\cos z - \mathfrak{C}\sin z + \mathfrak{D}\cos 2z - \mathfrak{E}\sin 2z},$$

as will be shown further on.

To construct the Dygogram when $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$ are known.—Navigators of the U. S. naval service have blank

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Art. 112[1] taken from an article by Comdr. John Gibson, U. S. N., in Proceedings of U. S. Naval Institute, Vol. XX, No. 3.

forms supplied upon which there is a vertical scale, *OP*, representing unity, which is divided into 100 equal parts, and by estimate into 1000 parts; and an arc, with *O* as a center and radius equal to *OP*, divided into degrees, upon which deviations may be read off. When no blank is at hand, a similar scale may readily be constructed. In all cases the line *OP* is equal to unity and is vertical, and at *P* there is a horizontal line.

Example.—Let $\mathfrak{A} = +.053$, $\mathfrak{B} = +.222$, $\mathfrak{C} = +.220$, $\mathfrak{D} = +.226$, $\mathfrak{E} = +.063$.

For reference, see Fig. 53.

From *P* **lay off** *PA* **=** $\mathfrak{A}$ **to the right if** $\mathfrak{A}$ **is +, to the left if −.**
" *A* **"** *AE* **=** $\mathfrak{E}$ E **" " "** $\mathfrak{E}$ **" +, " " −.**
" *E* **"** *ED′* **=** $\mathfrak{D}$ **upwards "** $\mathfrak{D}$ **" +, (as it usually is).**
" *D′* **"** *D′B′* **=** $\mathfrak{B}$ **" "** $\mathfrak{B}$ **" +, downwards if −.**
" *B′* **"** *B′N* **=** $\mathfrak{C}$ **to the right. "** $\mathfrak{C}$ **" +, to the left if −.**

With *A* as a center, and a radius equal to $AD' = \sqrt{\mathfrak{D}^2 + \mathfrak{E}^2}$, describe a circle, called the "generating circle." From *N* draw a straight line through *D′* and produce it until it intersects the generating circle a second time, which point mark *Q*. The point *Q* is called the "pole" of the dygogram and is one of the necessary points to have. From *D′* produce *ND′* for the distance *D′S* equal to *D′N*. Take a straight-edge of paper of sufficient length and lay it down on *NS;* mark, on the edge of the paper, dots opposite the points *N, D′*, and *S;* move the paper around so that the center dot moves on the circumference of the generating circle and with the edge always passing through the pole *Q;* by means of pencil dots opposite the end marks on the paper-edge, a sufficient number of points may be obtained for drawing in free hand the curve of the dygogram.

To mark the dygogram for magnetic headings, lay a protractor on the line *NS*, its center at *Q*, and dot off the headings required (usually every 15° rhumb); through each of

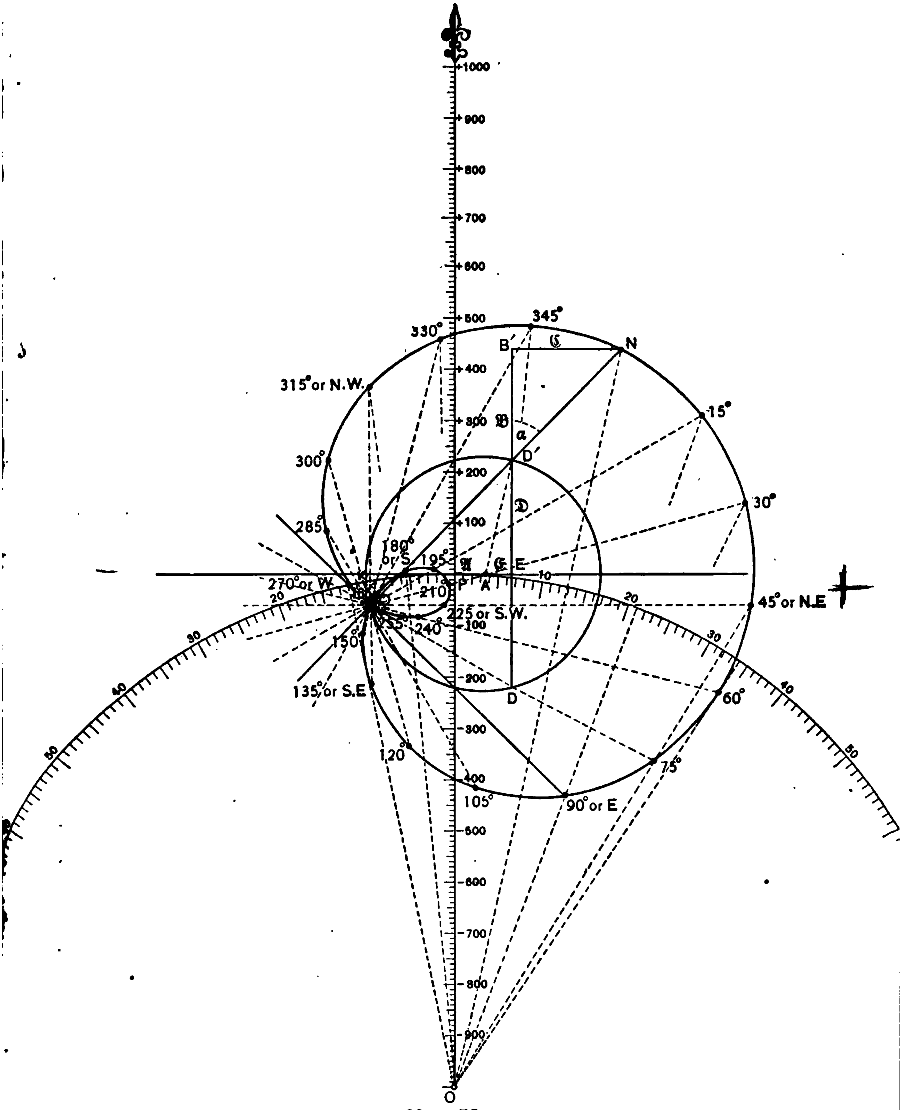
+1000
+900
+800
+700
+600
+500
+400
+300
+200
+100
-100
-200
-300
-400
-500
-600
-700
-800
-900
O
345°
330°
315° or N.W.
300°
285°
270° or W.
180° or S
195°
210
225 or S.W.
240
255
150°
135° or S.E
120°
105°
90° or E
75°
60°
45° or N.E
30°
15°
B
ℭ
N
𝔅
a
D′
𝔇
𝔄
𝔈
E
P
A
D
10
20
30
40
50

FIG. 53.

these points and through Q draw a line and extend it across the dygogram. Where the lines cut the dygogram are the points required; the first cut to the right of N. (looking from S. to N.) is, say, 15°, the 2d, 30°, the 3d, 45°, the 4th, 60°, the 5th, 75°, etc. Draw small circles around the points and mark each one correctly.

To construct a table of deviations for magnetic headings when only the exact coefficients are known.—Having proceeded so far as to find the required points of the curve and marked each correctly, as explained above, draw a line from each point to O (or until it intersects the graduated arc); the deviation then for each magnetic heading is shown by the angle which the line drawn from that point makes at O with the vertical graduated line, and is read off from the graduated arc; if to the right of the vertical line the deviation is East or +, and if to the left, West or —. It is usual to record the deviations in a tabular form as follows (as example, case of Fig. 53 is taken):

Magnetic Heading.	Deviation.	Magnetic Heading.	Deviation.
0°	+13° 00′	180°	— 5° 30′
15	+20 10	195	— 2 15
30	+26 30	210	— 0 45
45	+31 55	225	— 1 00
60	+34 00	240	— 4 30
75	+31 30	255	— 8 45
90	+21 00	270	—12 00
105	+ 4 00	285	—13 15
120	— 7 30	300	—11 15
135	—11 50	315	— 7 00
150	—11 30	330	— 1 00
165	— 9 00	345	+ 5 50

To construct a table of deviations for compass headings when only the exact coefficients are known.—In practice it is necessary to have a table of deviations for "compass headings," and to get it when only the exact coefficients are known, proceed as explained for constructing the dygogram and the table of deviations for magnetic headings. Then construct the Napier's curve by laying off on the Napier's diagram along the "full lines" the deviations for the magnetic headings; draw in the curve and then take off the deviations along the "dotted lines" for the "compass headings," and record them in tabular form, one column being "Compass Headings" and the other one "Deviations."

To show that the dygogram satisfies the conditions of the general expression.—

$$\tan\delta = \frac{\mathfrak{A} + \mathfrak{B}\sin z + \mathfrak{C}\cos z + \mathfrak{D}\sin 2z + \mathfrak{E}\cos 2z}{1 + \mathfrak{B}\cos z - \mathfrak{C}\sin z + \mathfrak{D}\cos 2z - \mathfrak{E}\sin 2z}.$$

For reference, see Fig. 54.

Construct the dygogram, as previously explained, and take any point *R* of the dygogram corresponding to the magnetic heading *z*. The position of the different coefficients, or the lines representing the forces, as laid down in constructing the dygogram, are for a magnetic heading North; for any other heading *z* the lines and different triangles remain of the original size, but assume new positions in regard to the center *A*. As the ship swings through the magnetic azimuth *z*, the keel line *DD′B′* swings around *D* as a center, through the angle *z*, in the new position *DD″K*, cutting the generating circle at the point *D″*. According to the construction of the dygogram a line, *QD″R*, making an angle *z* at *Q* with the line *NS*, will cut the dygogram at the point for the azimuth *z;* this line will pass through *D″* because the angles *D′QD″* and *D′DD″* are each equal to *z*, and, as both *Q* and *D* are on the circumference of the circle, the angles are each measured by half the same arc, *D′D″*. According to the construction, $D''R = D'N = \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$, and by geometry, the angle $B'D'N = KD''R = a$; therefore, a perpendicular let fall from *R* upon *DD″* produced will cut at *K* such that $D''K = \mathfrak{B}$ and $KR = \mathfrak{C}$. Thus it is seen that, in swinging through an azimuth *z*, the triangle of polar forces, *D′B′N*, has assumed the position *D″KR*.

The triangle *AED′* will revolve around *A* as a center in such a manner that while the ship turns through an angle z, the triangle AED' will turn through an angle $2z$. Above it was seen that half the arc $D'D''$ measured the angle $D'QD''=z$; therefore, the angle at the center, measured by the same arc, would be equal to $2z$; that is, $D'AD'' = 2z$, and therefore the other sides of the triangle, AE and ED', will turn through the angle $2z$; or $EAE' = 2z$ and $E'D''D''' = 2z$ (D''' being vertically below D'' on the line PBC).

The forces, as represented by the coefficients, have kept their original values or strength, but now act in new directions to produce deviation and to affect the directive force of the needle. $PA = \mathfrak{A}$ remains constant; $AE' = \mathfrak{E}$, but acts at the angle $2z$ with its former position; $E'D'' = \mathfrak{D}$ acts at an angle $2z$; $D''K = \mathfrak{B}$ and $KR = \mathfrak{C}$, but each acts at the angle z with its former position.

From each of the points E', D'', K, and R let fall perpendiculars upon the two axes having P as an origin.

As R is the point of the dygogram for azimuth z, the deviation $\delta = POR$; then, $\tan \delta = \dfrac{LR}{OL}$ in which LR = force of earth and ship to magnetic east in terms of mean force to N. as unit; OL = force of earth and ship to magnetic north in terms of mean force to N. as unit.

By referring to the figure it is seen that the angles

$$NQR,\ D'DD'',\ D'QD'',\ DD''D''',\ D''KB,\ KRL$$

are all equal to each other and to the azimuth z. It may also be seen that

$$D'AD'' = EAD' - EAD''; E'AS = E'AD'' - EAD''; \text{ or}$$
$$EAD'' = E'AD'' - E'AS; \text{ hence } D'AD'' = EAD' - E'AD'' + E'AS.$$

But

$$EAD' = E'AD''; \therefore D'AD'' = E'AS = 2z = MD''D''' = ME'S.$$

Now, $LR = PC = PA + AS + SD''' + D'''B + BC$;

but $PA = \mathfrak{A}$; $AS = \mathfrak{E} \cos 2z$.

∅ 2

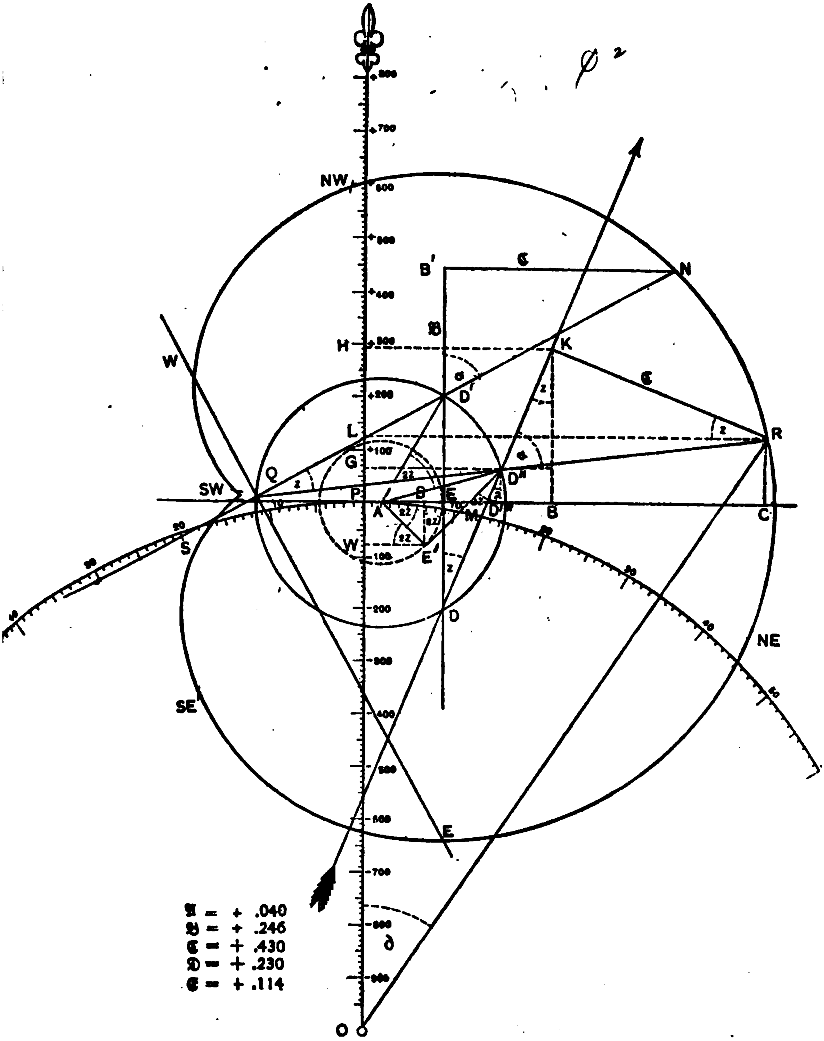

FIG. 54.

GEOMETRICAL DEMONSTRATION OF THE DYGOGRAM.

Let $E'M = r$ and $MD'' = s$; then $r + s = \mathfrak{D}$;

$$SD''' = SM + MD''' = r \sin 2z + s \sin 2z;$$

$$\therefore\ SD''' = (r + s) \sin 2z = \mathfrak{D} \sin 2z.$$

$$D'''B = \mathfrak{B} \sin z;\ BC = \mathfrak{C} \cos z;$$

$$\therefore\ LR = \mathfrak{A} + \mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z + \mathfrak{E} \cos 2z.$$

Again, $OL = OP + (WG - WP) + (GH - LH)$, G being the point where the horizontal line from D'' cuts the vertical axis.

But $OP = 1$; $WG = E'S + D'''D'' = r \cos 2z + s \cos 2z$

$$= (r + s) \cos 2z = \mathfrak{D} \cos 2z.$$

$$WP = \mathfrak{E} \sin 2z;\ GH = \mathfrak{B} \cos z;\ LH = \mathfrak{C} \sin z;$$

$$\therefore\ OL = 1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z - \mathfrak{E} \sin 2z.$$

Hence $\tan \delta = \dfrac{\mathfrak{A} + \mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z + \mathfrak{E} \cos 2z}{1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z - \mathfrak{E} \sin 2z}$.

The line NS, although taken as the zero line for laying off the magnetic headings, does not represent the direction of the keel line of the ship for magnetic North or South. The vertical line represents the keel line for magnetic North or South (magnetic meridian) and the direction for any other heading z is represented by drawing from D (vertically below E), a line $DD''K$ making an angle $z = D'DD''$ with the vertical. As $\mathfrak{B}$ is laid off to head and $\mathfrak{C}$ to starboard (opposite, if negative), the angle $B'D'N = KD''R = \alpha$ (the starboard angle).

It must be seen that the points marked on the curve of the dygogram are not really directions of the ship's head, but are the points on the curve which show, for the headings designated, the deviations of the compass and the position of the forces in regard to the meridian, for those headings.

To represent the direction of the ship's head on the dygogram for any designated point of the curve, join the point on the curve with Q and note the intersection of this line with the circumference of the generating circle; a line drawn from D through this point of intersection will give the keel line of the ship. Confusion may be avoided by drawing around the point of intersection with the generating circle the outlines of a ship with its head in the proper direction.

$OP =$ unity $=$ mean force to North $=$ mean directive force in the compass needle $= \lambda H$. Where λ is unity, the mean force to North becomes H, the horizontal force of the earth at that place.

$OL = \frac{1}{\lambda} \frac{H'}{H} \cos \delta =$ force of earth and ship to magnetic North, in terms of mean force to North as unit (for any particular azimuth z of ship's head) $=$ directive force of needle.

$LR = \frac{1}{\lambda} \frac{H'}{H} \sin \delta =$ force of earth and ship to magnetic East (in terms of λH, the mean force to North as unit, for any particular azimuth z) $=$ force tending to draw the needle from the magnetic meridian, thus causing deviation.

$OR = \frac{1}{\lambda} \frac{H'}{H} =$ force in the direction of the disturbed needle; the needle being drawn by the force to east (LR), out of the meridian, through the angle $POR = \delta$ for that particular azimuth z.

The angle $POA =$ deviation due to constant force $\mathfrak{A}$ (same for all headings).

$AOE' =$ deviation due to induced force in unsymmetrical soft iron, represented by coefficient $\mathfrak{E}$.

$E'OD'' =$ deviation due to induced force in symmetrical soft iron, represented by coefficient $\mathfrak{D}$.

$D''OK =$ deviation due to polar force to head, represented by coefficient $\mathfrak{B}$.

$KOR =$ deviation due to polar force to starboard, represented by coefficient $\mathfrak{C}$.

Of course, the sum of any two or more of these angles is equal to combined deviation caused by the combined forces designated. Thus the deviation for magnetic azimuth z caused by the forces represented by $\mathfrak{A}$, $\mathfrak{E}$, and $\mathfrak{D}$ is

$$POD'' = POA + AOE' + E'OD''.$$

A correct idea of what the dygogram is may be obtained from the following, viz.: Suppose a compass needle pivoted at O (see Fig. 54), its half length when equal to OP being considered as unity, that is, equal to the mean force to North, λH. Suppose the needle capable of assuming a length proportional to the force in the direction of its length, for each heading. From an inspection of the dygogram, it is seen that the force in the direction of the needle $\left(OR = \frac{1}{\lambda}\frac{H'}{H}\right)$ varies in amount or length as the ship swings in azimuth. Now, as the ship swings in azimuth, through a complete circle, the end of the needle will trace out the curve of the dygogram, its end, at any azimuth z being at the point R of the dygogram, showing a deviation $\delta = POR$.

Various cases might be given where a knowledge of the dygogram would be of great assistance; such as where the values of $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$ are all known, and it is desired to compensate on any heading while at the dock; in which case the deviation due to $\mathfrak{A}$, $\mathfrak{E}$, $\mathfrak{B}$, and $\mathfrak{C}$ has to be left uncorrected in compensating the quadrantal deviation, and that due to $\mathfrak{A}$ and $\mathfrak{E}$ left uncorrected in compensating that due to $\mathfrak{B}$ and $\mathfrak{C}$.

Fig. 55 shows the manner in which the various forces revolve, by which the final curve of the dygogram is traced out, when all the forces have appreciable values as represented by the exact coefficients.

OP is equal to unity $= \lambda H$.

PA is the dygogram due to the constant force represented by $\mathfrak{A}$.

The inner circle is the dygogram due to the induced force represented by $\mathfrak{E}$, standing to one side of the meridian line on account of the constant force $\mathfrak{A}$; the circle is properly marked.

The next circle, having a radius equal to $\sqrt{\mathfrak{D}^2 + \mathfrak{E}^2}$, is the dygogram due to both induced forces, represented by $\mathfrak{E}$ and $\mathfrak{D}$, standing to one side of the meridian line on account of the constant force $\mathfrak{A}$, and if $\mathfrak{A}$ is zero its center is at P.

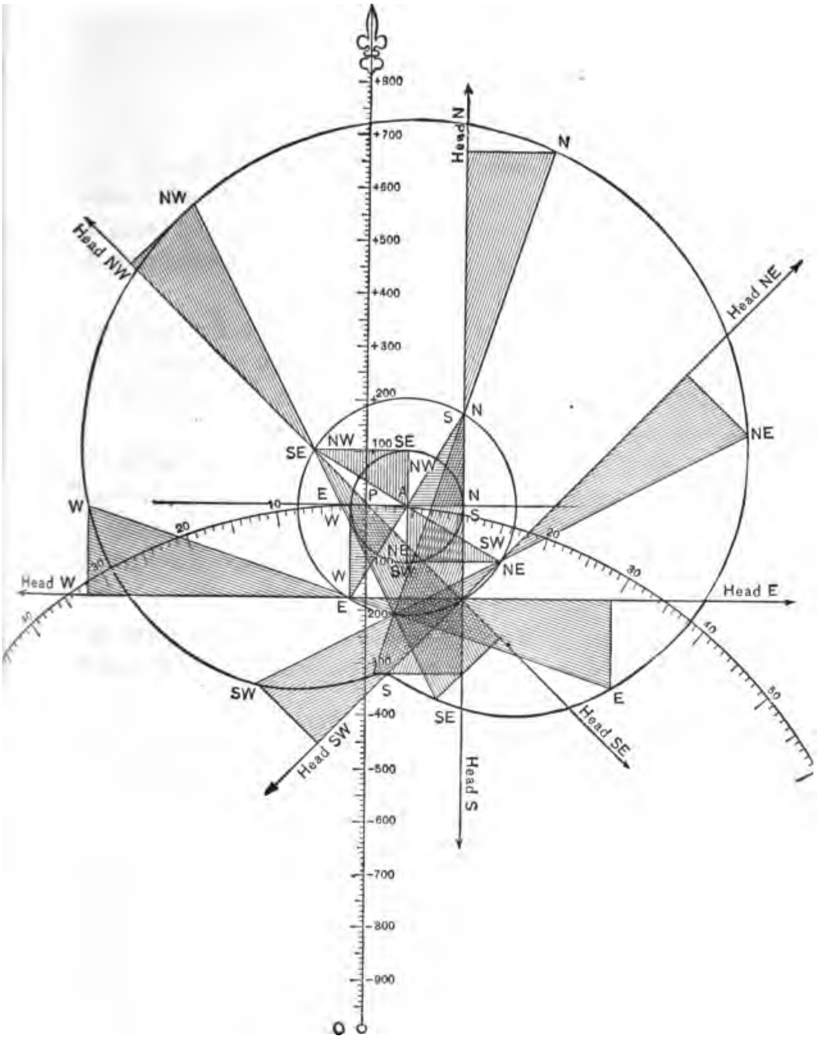
+900
+700
+600
+500
+400
+300
200
100
Head N
N
NW
Head NW
Head NE
NE
N
S
NW
SE
SE
NW
E
P
A
N
W
S
W
NE
SW
SW
NE
W
E
10
20
30
40
20
30
40
50
Head W
Head E
200
100
SW
S
SE
E
Head SE
Head SW
Head S
-400
-500
-600
-700
-800
-900
O

FIG. 55.

The small shaded triangle is the triangle of induced forces producing the quadrantal deviation, and revolves around A as center, the rate of revolution being double that of the ship in swinging.

The large shaded triangle is the triangle of polar forces producing the semicircular deviation; it revolves on the circumference of the quadrantal circle, its apex continually touching that of the inner triangle, the center of revolution being at the point D (see Figs. 53 and 54) and its rate of revolution being the same as that of the ship in swinging.

The final curve, that traced out by the outer corner of the triangle of polar forces, is the curve of the dygogram.

If both $\mathfrak{A}$ and $\mathfrak{E}$ are zero, the center of the second circle becomes P and its radius $\mathfrak{D}$.

If $\mathfrak{A}$, $\mathfrak{E}$, and $\mathfrak{D}$ are all zero, the dygogram for the semicircular forces is a circle whose center is P and radius $\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$; if then either $\mathfrak{B}$ or $\mathfrak{C}$ becomes zero, the dygogram of the remaining force will be a circle whose center will be P and whose radius will be the remaining force.

113. Given the deviations and the horizontal force on two courses, regarding $\mathfrak{A}$ and $\mathfrak{E}$ as zero, to find λ, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ by construction.

Let z_1 and z_2 be the two magnetic courses, δ_1 and δ_2 the corresponding deviations, $\frac{H'_1}{H}$ and $\frac{H'_2}{H}$ be the horizontal forces in terms of H.

Referring to Fig. 54, it is seen that if $OR = \frac{1}{\lambda}\,\frac{H'}{H}$, then $OP = 1$, $OL = 1 + \mathfrak{B} \cos z - \mathfrak{C} \sin z + \mathfrak{D} \cos 2z - \mathfrak{E} \sin 2z$, and $LR = \mathfrak{A} + \mathfrak{B} \sin z + \mathfrak{C} \cos z + \mathfrak{D} \sin 2z + \mathfrak{E} \cos 2z$. Now, if OR is taken as $\frac{H'}{H}$, we shall have $OP = \lambda$, $OL = \lambda + \lambda\mathfrak{B} \cos z - \lambda\mathfrak{C} \sin z + \lambda\mathfrak{D} \cos 2z - \lambda\mathfrak{E} \sin 2z$, and $LR = \lambda\mathfrak{A} + \lambda\mathfrak{B} \sin z + \lambda\mathfrak{C} \cos z + \lambda\mathfrak{D} \sin 2z + \lambda\mathfrak{E} \cos 2z$.

Points on a dygogram corresponding to these last values of OL and LR as coordinates may be found thus: For R_1 lay off the angle $POR_1 = \delta_1$ and take $OR_1 = \frac{H'_1}{H}$; for R_2 lay off the angle $POR_2 = \delta_2$ and take $OR_2 = \frac{H'_2}{H}$. Call R_1 and R_2 datum points (Figs. 56 and 57).

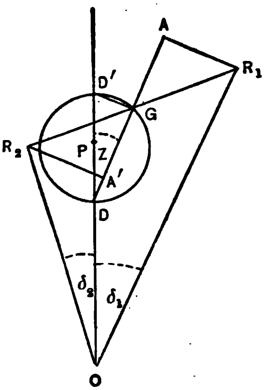

FIG. 56.

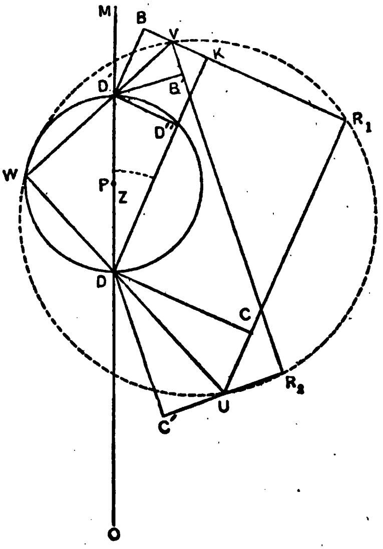

FIG. 57.

There are two different constructions under the above general heading.

(1) **When the two magnetic courses are diametrically opposite** (Fig. 56).

Find the datum points R_1 and R_2 and draw R_1R_2, bisecting it in G, a point of the generating circle. Through G draw AD, parallel to the magnetic direction of the keel line, and intersecting OP in D. Draw GD' perpendicular to AD intersecting OP in D'. Bisect DD' at P and drop a perpendicular from R_1 and R_2 on AD. Then $OP = \lambda$, $GA = GA' = \lambda\mathfrak{B}$, $AR_1 = A'R_2 = \lambda\mathfrak{C}$, and $PD' = \lambda\mathfrak{D}$. By drawing the outline of a ship about G, the signs of $\lambda\mathfrak{B}$ and $\lambda\mathfrak{C}$ become apparent. Having these quantities, find $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ and construct the dygogram.

If the two magnetic courses are N. and S., G will be at D', and we can not determine D, and hence neither λ nor $\mathfrak{D}$. If they are E. and W., G will be at D, and we can not determine D', and hence neither λ nor $\mathfrak{D}$.

(2) **When the two magnetic courses are not diametrically opposite.**—It has been shown in Art. 112 that a point of the dygogram, for example R, Fig. 54, may be found by laying off the angle $z = D'DD''$, then measuring off $D''K = \mathfrak{B}$ and $KR = \mathfrak{C}$. From Fig. 57 it is seen that R_1 may be found by laying off $\mathfrak{B} = D'B$ $(= D''K)$, making the angle $MD'B = z$, and $DC = \mathfrak{C}$ at right angles to the course line $D'B$, and then completing the parallelogram.

Now for another course $MD'B'$ lay off $D'B' = \mathfrak{B}$ in the direction of the course line and $DC' = \mathfrak{C}$ at right angles to it, completing the parallelogram and finding the datum point R_2.

Hence if R_1 is a datum point corresponding to one course $MD'B$, CR_1 is a line parallel to and R_1B a line perpendicular to the course line through that datum point.

In the same way $C'R_2$ and R_2B' are lines through R_2 respectively parallel to and perpendicular to the second course line.

Let u be the intersection of the two lines through R_1 and R_2 parallel to the course lines, that is of CR_1 and $C'R_2$; and let v be the intersection of lines perpendicular to them, that is of R_1B and R_2B'; then from geometry it is plain that if we draw vD' and uD, they will intersect at w, a point of the generating circle DD', that vD' bisects BvB' and uD bisects CuC', and further that w is on the circle passing through u, v, and the datum points R_1 and R_2.

Therefore, to construct a dygogram when the two courses are not diametrically opposite, find the datum points R_1 and R_2 as in case (1). Through R_1 and R_2 draw lines parallel to the keel lines meeting in u and lines perpendicular thereto meeting in v, the keel line through R_1 corresponding to the course z_1 and that through R_2 to the course z_2.

Through v draw a line parallel to one bisecting the angle between course lines, in other words bisecting the angle R_1uR_2. This line cuts the vertical line in D'. From u drop a perpendicular on vD', intersecting it at w and the vertical line at D. Bisect DD' at P. From D drop perpendiculars DC and DC', respectively, on the 1st and 2d course lines; from D', perpendiculars $D'B$ and $D'B'$, respectively, on lines at right angles to said course lines.

Then $OP = \lambda$, and DD' is the diameter of the generating circle.

$$PD = \lambda\mathfrak{D},\ D'B = D'B' = \lambda\mathfrak{B},\ DC = DC' = \lambda\mathfrak{C}.$$

Having found these values, find the coefficients, and construct the dygogram.

The construction fails when the magnetic courses are equally distant from any cardinal point; for, if equally distant from N. or S., $R_1D = R_2D$ and $R_1DR_2 =$ the difference of magnetic azimuths $= R_1wR_2$, the point v will be in the vertical line, w will be at D, and it will be impossible to determine D'; if equally distant from East or West, w will coincide with D', and it will be impossible to determine D.

113a. To find $\mathfrak{B}$ and $\mathfrak{C}$ from observations on one heading (see Art. 94). Having determined δ, z, and $\frac{H'}{H}$, and assuming λ and $\mathfrak{D}$, let $OP =$ unity represent the direction of magnetic North (Fig. 56). Describe a circle, center P, radius $\mathfrak{D}$, cutting OP in D. Draw DA, making $PDA = z$ and cutting the circle in G. Lay off $POR_1 = \delta_1$ (the given deviation δ), to right if $+$, to left if $(-)$; take $OR_1 = \frac{1}{\lambda}\cdot\frac{H'}{H}$; drop R_1A perpendicular to DA; then $GA = \mathfrak{B}$ and $AR_1 = \mathfrak{C}$. An outline of a ship around G, heading properly, will make the signs apparent.

CHAPTER V.

PILOTING.—FIXING SHIP'S POSITION NEAR LAND.—DANGER ANGLE.—DANGER BEARINGS.—FOG SIGNALS.

114. **Piloting** in its broad sense is the act of conducting a ship where navigation is dangerous, as, when coasting, passing through channels, and into harbors. Before reaching pilot waters, a navigator should study the charts and sailing directions of the region, know that they are up-to-date, be conversant with landmarks and aids to navigation, and the state of tides and currents of the locality at the time when he may navigate the waters. After reaching pilot waters, a keen lookout must be kept for dangers as well as aids to navigation, soundings should be taken and depths and character of bottom obtained compared with indications of the chart; in shoal water the hand lead should be kept going. Advantage should be taken of the first opportunity to locate the ship's position by bearings of known objects, and having laid a course clear of all dangers, the ship's position must be frequently plotted by the most convenient of the methods herein explained.

Before proceeding to explain them it will be well to define general terms of frequent use in navigation.

The bearing of an object from a ship is the angle between the meridian and the great circle which passes through the object and the observer on board, and it indicates the direction in which the object is seen from the ship.

It is called true, magnetic, or compass according as the meridian considered is that passing through the geographical poles, the magnetic meridian, or the direction of the compass needle.

On board ship bearings, by compass or pelorus, are measured from North, to the right, from 0° to 360°.

A line of position is any line, straight or curved, on which the ship's position is known to be. It is obtained from observations of either celestial bodies, or terrestrial objects.

A line of bearing.—When a line of position is obtained from a bearing, to make it more distinctive and to indicate its origin, it is called a line of bearing.

Position point.—Any point on either a line of position or a line of bearing at which the ship's position may be assumed is a position point; the actual position of the ship, or a fix, is determined by the intersection of two lines of bearing, two lines of position, or one of each.

115. To fix the position of the ship near land when two or more landmarks of known position are in sight.

(1) **By sextant angles.**—Select three objects so as to give well-conditioned circles (see Art. 34). Generally speaking, the angles should, if possible, be over 30° and the objects in line, or the middle one nearest the observer. Observe by a sextant, whose I. C. is known, the angle between the middle and right objects, and at the same time the angle between the middle and left objects, which are known respectively as the right and left angles; set the right and left arms of a station pointer, or 3-arm protractor, for their respective angles, place protractor on the chart, move it over chart till the beveled edge of each arm passes simultaneously each through its own object. The center of the instrument locates the ship's position on the chart.

A special application of this method is when two of the objects are in range and but one angle is taken.

This is by far the preferable method from the standpoint of accuracy, as the position is independent of compass errors, speed errors, or current effect. Angles can be taken from any part of the ship whilst bearings must be taken from the standard compass or pelorus.

The geometrical theory of the method will be understood from Fig. 58 which, by construction, embodies a method that can be used in the absence of a 3-arm protractor or tracing paper on which the angles could be ruled. Let *A, B,* and *C,* Fig. 58, be the three known positions. The observer at *P* measures angle x between *A* and *B,* and angle y between *B* and *C.* Using the two objects *A* and *B* and angle x alone, the observer may be anywhere on the segment *APB* which becomes a line of position. If x is $< 90°$, the line of position is greater than a semicircumference, if $x = 90°$, the line of position is 180°, if x is $> 90°$, the line of position is $< 180°$, and the position point may be anywhere on the arc, since all angles on the same segment of a circle equal each other.

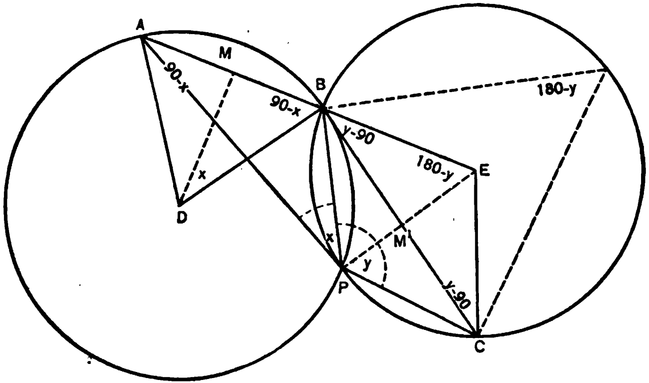

FIG. 58.

Using the two objects *B* and *C* and the angle y alone, the observer may be anywhere on the segment *BPC* which becomes a second line of position, the length of which is governed by rules as already explained for the first line of position *ABP,* and the position point is somewhere on the arc *BPC,* and, being also on *ABP,* is at their intersection *P;* in other words, *P* is the fix.

There are three cases in this method: (1) Both angles $< 90°$, (2) both $> 90°$, (3) one $< 90°$ and one $> 90°$. When an angle is $< 90°$, the center of the circle passing through the observer's position and the two objects is on the same side of the line joining the two points as the observer; in this case lay off *AD* and *BD* making angles $90° - x$ with *AB*, *D* will be the center of the first circle. When an angle is $> 90°$, the center of the circle passing through the observer's position and the two objects is on that side of the line joining the two objects remote from the observer; in this case lay off *BE* and *CE* making angles $y - 90°$ with *BC, E* will be the center of the second circle. Hence, we have the general rule: take the complement of the observed angle; if +, the center of circle will lie on the same side of line joining observed objects as the observer; if (—), the center of circle will be on the opposite side.

The indeterminate case is when the observer and the three objects are on the same circle, or nearly so, or when the two centers nearly coincide. To avoid such a condition see Art. 34.

In case no protractor is at hand it is only necessary to measure the distance *AB,* erect a perpendicular at its middle point *M,* and lay off $MD = MB \tan (90° - x)$ to find the center *D,* then describe the circle with radius *DB.* In a similar way find *E* and with radius *EB* describe the second circle, the intersection giving the fix *P.*

(2) **By cross bearings.**—This consists in finding the fix by two or more lines of bearing. The bearings per standard compass of two points of land, or objects, whose positions are projected on the chart, having been obtained, are corrected for the deviation due to the ship's head at the instant the bearings were taken. Lay the parallel rulers on the nearest com-

pass rose, the edge passing through the center and the degree on the circumference representing the magnetic bearing of a given object, transfer the edge of rulers parallel to itself till it passes through the given object. Draw a light line along the edge. This is a line of bearing and the position of the ship is somewhere on it. In a similar way draw a second line of bearing through the second object, and the ship being also somewhere on this line, the fix is at the intersection of the two. If the compass rose had been true instead of magnetic, the compass bearing would have been corrected for variation as well as for the deviation of the compass. The difference of bearings should be as near 90° as possible for best results; if the difference is small, 15° to 20°, a small error in the bearing will make the fix uncertain. The position determined from the bearings of only two objects may be in error, even when the angles of intersection are good, due to an error in the assumed deviation or even an error of the chart; for these reasons a third line of bearing should be obtained, if a third object is available. Should these three lines of bearing form at their intersection only a small equilateral triangle, its center may be regarded as the fix.

In finding a fix by cross bearings, take first the bearing of that object nearest the fore-and-aft line, ahead or astern, since such an object will change its bearing less than one more nearly abeam, in the interval between bearings. If one object is ahead or abeam, knowing the course, its bearing is taken mentally, and it is only necessary to get a bearing of the second object. A bearing of one object and a bearing of a range, or a bearing of one object in connection with a sextant angle to another object, provided the angle to the second object is sufficiently large, will give a good fix.

116. (*B*) **When one object only is available.**—Having taken a compass bearing of the single object, we have a line of bearing which is true or magnetic according as the compass bearing is corrected for compass error or for deviation alone. If true, it is laid down from a true rose; if magnetic, from a magnetic rose. The ship is somewhere on this line. A fix on this line may be gotten from its intersection with a line of position found at the instant, or from one brought up to the instant of getting a bearing by the run, or by knowing latitude or longitude, or by knowing the distance of the object.

The distance of the object may be estimated; or gotten from its angular altitude, if its height is known; or by Buckner's method when available; or by an accurate range-finder.* Knowing the distance, the ship is somewhere on a line of position whose center is the object and whose radius is the distance, and, being also on the line of bearing, the ship is at their intersection. An estimated distance may be often verified by a cast of the lead where soundings vary considerably.

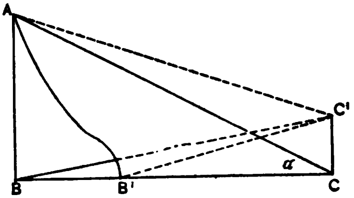

FIG. 59.

Suppose in Fig. 59, $AB = h =$ the height in feet of an object whose angular altitude at $C = \alpha$. Then if $ABC = 90°$ and $BC = d$, $d = \frac{h}{\tan a}$. Now as α is very small and expressed in minutes of arc, $\tan \alpha = \alpha \sin 1'$; therefore, $d = \frac{h}{a \sin 1'}$, and to express d in nautical miles, divide second member by 6080.27; substituting value of sin 1′ we have d (in sea miles)

$$= \frac{h \text{ (in feet)}}{a\,(6080.27) \times .00029} = \frac{h}{a} \cdot \frac{1}{1.76328} = .567\,\frac{h}{a}.$$

Ex. 21.—A lighthouse 140 feet high subtends an angle of 16′; find the distance in nautical miles.

$$d = \frac{140}{16} \times .567 = 4.96 \text{ nautical miles.}$$

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* The Barr and Stroud range-finder is sufficiently accurate for navigational purposes at distances varying between 800 and 7000 yards, and its use is practicable on board ship.

Table 33 of Bowditch gives the distance, by vertical angle, at intervals of one-tenth of a mile up to 5 sea miles for objects whose heights vary from 40 to 2000 feet.

In this method *B*, the foot of the object, should be seen and the angle *ABC* should be 90°; for this reason the observer's eye should be as low down as possible. The error due, however, to a slight height of the eye is inappreciable, but that due to the visible shore line *B′*, not being at *B* the foot of the object, might be material. In other words, *B′* should be at *B* and *C′* at *C*.

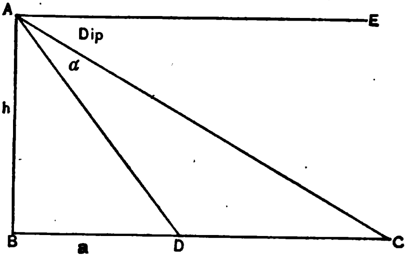

FIG. 60.

Buckner's method is one often used for finding the distance of a target, and may be used for finding the distance of an isolated object beyond which the sea horizon can be seen. In Fig. 60, *AB* is the height of eye above sea-level; *EAC*, the dip of sea horizon *C* due to height $h = AB$; *D*, isolated object; α, the angle between the object and sea horizon beyond; and

$$\text{a (in yards)} = \frac{h \text{ (in feet)}}{3 \tan (\alpha + \text{Dip})}.$$

Table 34 of Bowditch gives the distance in yards for various values of angle α observed at heights of observer's eye varying from 20 to 120 feet. Lecky's off-shore distance tables are more extended in their application, likewise Von Bayer's diagram.

To find the distance of an object of known height, just visible on the sea horizon to an observer's eye of a given height, it is sometimes useful to use Table 6 of Bowditch which gives the distance of visibility of objects at sea for different heights; the distance, owing to the uniform curvature of the sea, depending on the heights both of object and observer's eye.

Ex. 22.—From a height of 45 feet, a light, whose height from the light list is 160 feet, is seen to disappear below the horizon; find its distance in nautical miles.

For 45 feet, distance 7.7 miles.
For 160 feet, distance 14.5 miles.

Required distance = 22.2 miles.

Table 6 of Bowditch is calculated from the formula (to be deduced later on)

$$d = 1.148 \sqrt{h},$$

h = height of object in feet, d = distance in nautical miles of the object just visible in the horizon, the observer's eye being at the surface of the earth.

Now, if h' = the height of eye in feet,

$$d = 1.148 (\sqrt{h} + \sqrt{h'}).$$

Ex. 23.—An observer, height of eye 36 feet, sees in the sea horizon the top of a lighthouse known to be 121 feet high. What is the distance in nautical miles?

$$d = 1.148 (\sqrt{36} + \sqrt{121}) = 1.148 \times 17 = 19.55 \text{ miles.}$$

117. (*C*) By two bearings of a single object and the course and distance run in the interval.—(1) This involves the solution of a plane triangle, given one side and the two adjacent angles. The ship is steering the course AB (Fig. 61). At first observation, ship is on the line of bearing AC and the patent log is read. At second bearing, ship is on the line of bearing BC and patent log is again read. The difference of readings of patent log gives the distance AB, from the course and bearings the angles A and B are known, and $C = 180 - (B + A)$.

(1) **By plane trigonometry.—**

$$AC = \frac{\sin B}{\sin C} \times AB \text{ and } BC = \frac{\sin A}{\sin C} \times AB.$$

The distance of passing abeam $CD = BC \sin B$.

Tables have been compiled giving factors by inspection; the factor of the first column multiplied by distance run between bearing lines gives the distance of object at second bearing, the factor of the second column multiplied by the distance run gives the distance when the object was abeam. The arguments in these tables are the difference between course and first bearing, difference between course and second bearing; the difference of bearings in Table 5A, Bowditch, being at intervals of quarter points, in Table 5B at intervals of two degrees; the factors in each table being for a distance run of one mile.

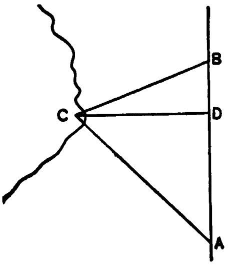

FIG. 61.

So far as the factors are concerned, it is immaterial whether the course and bearings are per compass, magnetic, or true, provided they are all from the same meridian.

Ex. 24.—A ship heading 292° (p. c.), var. + 12°, dev. + 9°, had a lighthouse bearing 234° (p. c.); after a run of 10 miles, the same light bore 166° (p. c.).

Find the distance at second bearing, also when light was abeam.

Difference between course and first bearing 58°.

Difference between course and second bearing 126°.

Table 5B, factor from first column = 0.91 and distance at second bearing = 9.1 miles.

Table 5B, factor from second column = 0.74 and distance abeam = 7.4 miles.

It is to be understood that the course and distance are those over the ground, and due allowance must be made for currents.

A simple application of this method is what is known as the "bow and beam bearing." The bearing of an object is taken when it is 45° on the bow, and patent log noted; another bearing is taken when the object is abeam and patent log again read. The distance run between the times of the two bearings is the distance of the object when abeam, provided the course and distance have not been influenced by currents or bad steerage.

If the first bearing was taken abeam and the second when the object was on the quarter, the distance run multiplied by 1.4 will give the distance of the object at second bearing, the distance run being the distance of object when abeam.

(2) **The graphic solution of this method** is easier and, certainly, more general than the factor solution.

By means of parallel rulers, draw the first line of bearing *AC* (Fig. 61) on the chart. When the bearing has changed sufficiently draw the second line of bearing on the chart. Cut these lines by one representing the course, transferred from the compass rose by parallel rulers, so that the distance intercepted between the two lines of bearing shall equal the distance run. If the course used was the course made good, and no current affected the run, the points of intersection of the course line with the lines of bearing will be the positions of the ship at the times of taking the bearings.

When an accurate reckoning has been kept and a correct estimation of the tidal stream or current experienced in the interval between the bearings can be obtained, the ship's position may be found by noting the intersection with the second line of bearing of the first line of bearing transferred to the time of the second bearing. This method is known as the "running fix." Having laid down the first bearing on the chart, from any point of this line lay off the course and distance run between the times of the two bearings and, from the extremity of this line, the estimated amount of the current or tidal stream in the interval. Through the point so found draw a line parallel to the first line of bearing: this will be the first line of bearing transferred. Then draw the second line of bearing on the chart, and note its intersection with the first line of bearing transferred. This intersection will mark the position of the ship at the time of the second bearing.

(3) **Doubling the angle on the bow.**—In this case note the patent log when the object has a certain bearing on the bow, or, in other words, when there is a certain angle between the ship's head and the object; again note the patent log when this bearing on the bow, or angle, has doubled; the difference between the two readings of the patent log, or the distance run, is the distance of the object at the second bearing.

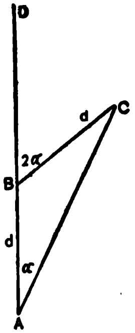

FIG. 62.

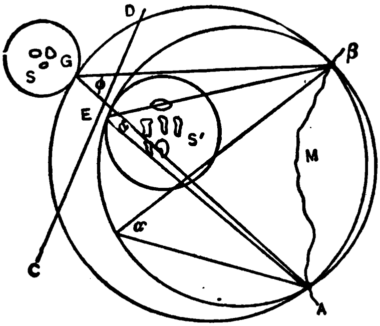

FIG. 63.

This is shown graphically in Fig. 62. Since $DBC = 2DAC$ and also equals $DAC + BCA$, $BCA = DAC$ and $BC = AB =$ distance run.

Hence, the rule "when the angle on the bow is doubled, the distance of the object at the second bearing equals the distance run"; *provided there is no current.*

A special application of this method, and one universally used, is the four-point bearing, the double angle being 90°, and the distance run the distance when abeam. A common application is when the first bearing is two points on the bow, giving the distance on the bow as distance run; again when the first bearing is 60° on the bow, the distance run will be the distance of object when it bears 30° abaft the beam.

Distance of passing an object abeam.—In case the first angle on the bow is 26½° and the second angle is 45°, the distance run between the two bearings will be the distance of passing the object abeam, if course and distance are unaffected by current. A knowledge of this distance is of importance as the point is approached.

118. Danger angle.—When sailing along a coast and it is desired to avoid sunken rocks, or shoals, or dangerous obstructions at or below the surface of the water, and which are marked on the chart, the navigator may pass these at any desired distance by using what is known as a danger angle, of which there are two kinds, a horizontal and a vertical danger angle; the former requires two well-marked objects projected on the chart, lying in the direction of the coast, and sufficiently distant from each other to give a fair-sized horizontal angle; the latter requires a well-charted object of known height.

In Fig. 63, let AMB be a portion of a coast along which a vessel is sailing on course CD, A and B two prominent objects projected on the chart; S and S' are two outlying shoals, reefs, or dangers.

To pass at a given distance outside the center of danger S'.—With the middle point of danger as a center, and the given distance as a radius, describe a circle; pass a circle through A and B tangent to the seaward side of the first circle. To do this practically, it is only necessary to join A and B, and draw a line perpendicular to the center of AB, then ascertain by trial the location of the center of circle EAB. Measure the angle $\alpha = AEB$, set sextant to this angle, and remembering that AB subtends the same angle at all points of the arc AEB, the ship will be outside the arc AEB, and clear of danger S', as long as AB does not subtend an angle greater than α to which the sextant is set.

Now, should it be desired to pass a certain distance inside of a danger S—with the middle point of danger as a center and the desired distance as a radius describe a circle; pass a second circle through A and B tangent to this circle at G. Measure the angle $BGA = \phi$ with a protractor. Then as long as the chord AB subtends an angle greater than ϕ, the ship will be inside the circle AGB and clear of danger S.

Should both dangers exist and it be desired to pass between them with a margin of safety already referred to, steer on course *CD* so that the angle subtended by *AB* shall be $< \alpha$ but $> \phi$.

119. Vertical danger angle.—Practically the same general principle is involved in this as in the horizontal danger angle; however, only one object is used and that one must be of known height.

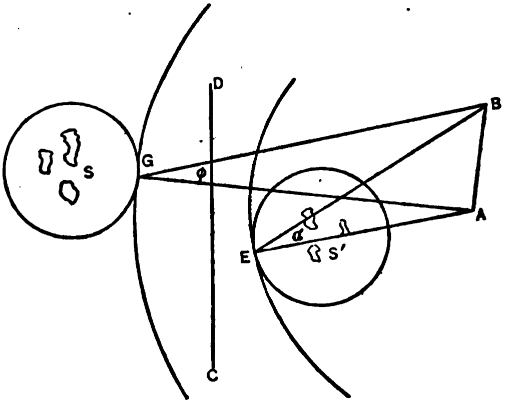

FIG. 64.

In Fig. 64, draw circles around the dangers *S* and *S′* with radii representing a safe margin of safety. Let *AB* be an object of known height. With *A* as a center draw circles tangent at *E* and *G*. Measure the distances *AE* and *AG*; find from Table 33, Bowditch, or by computation, the angular altitude of *AB* of known height for distance *AE*, let it be α; also for distance *AG*, let it be ϕ. To pass outside of and clear of *S′* the angular height of *AB* must be $< \alpha$. To pass inside and clear of *S* the angular altitude of *AB* must be $> \phi$. These are the limits for ensuring a safe passage between *S* and *S′*. In observing a vertical danger angle, the observer should be as near the water line as possible to minimize errors due to height of eye, and the angle at *A* should be 90°.

120. Danger bearing.—A bearing, properly taken and used, may often be of great use in keeping a ship out of danger. Suppose *C* and *C′* to be shoals or rocks near the coast (Fig. 65). A ship is passing on course *efg*. Lay down on the chart the tangents *AD* and *Ag* with any desired margin of safety. With *A* as a center describe the arcs of circles to include dangers *C* and *C′*. Note the magnetic bearing *DA* and *gA* and find what should be the compass bearings for the given course. Now before the distance of *A* is reduced to the radius of the circle's arc enclosing *C*, the bearing of *A* must be to the left of the danger bearing *DA* and kept so to avoid danger. The bearing of *A* must not get to the left of *gA* till the distance of *A* is greater than the radius of the arc enclosing *C′*. It may often be possible to find a danger bearing on a range; for instance, if *A* and *B* are in range on the danger bearing *DA*, the object *B* must be kept open to the right to ensure safety, and the guarantee is better than a compass bearing would give.

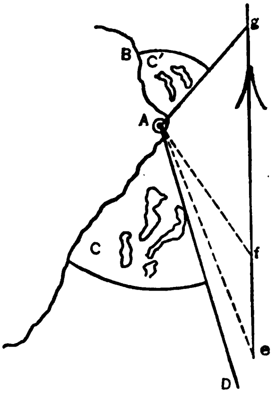

FIG. 65.

Lights as danger guides.—Lights of lighthouses may be used to give warning of danger as the object *A* was used above. Many lights, showing white over safe waters, show red over sectors embracing areas of rocks, shoals, or depths over which the approaches would be dangerous, and it is the duty of the navigator to keep out of the danger sectors. The magnetic bearings showing the limits of the sectors are given from seaward, and the bearing of the light, taken frequently, should not be allowed to get within the danger sector unless the distance of the light is known to be greater than the radius enclosing the dangers.

A navigator is furnished with charts, light lists, and sailing directions, all of which give details as to color, character, and visibility of navigational lights, and the navy regulations make it his duty to become thoroughly conversant with these details before coming within their range of visibility.

It may sometimes, in fact, has often happened, that one light is sighted when the run indicates the ship to be in the region of another light for which a lookout has been kept; hence, the rule, on sighting a light, is to compare its visible characteristics with those laid down in the light lists, and, if apparently not a fixed light, the duration of its periods must be noted by watch.

It must not be forgotten, however, that abnormal atmospheric conditions may increase the range of visibility of a light, whilst mist may decrease it and is often found to make white lights appear red. When a fixed light is first sighted, especially under fair conditions, and there is any question as to whether it is a lighthouse or a vessel's light, simply descend a short distance, and again look for the light. A vessel's light is of limited intensity and, if seen at all, can be seen at any height; a navigation light can be seen, as a rule, as far as the curvature of the earth will permit and this distance, depending on the height of eye and of the light, will be lessened by the observer going lower down. A descent of a few feet may cause a navigation light to disappear.

121. Fog signals.—A navigator should make himself familiar with all fog signals of the locality in which he is cruising; in foggy weather and in the neighborhood of signals, the closest attention must be paid in an effort to hear them and locate their direction. When heard, their periods should be timed and a comparison made with those recorded in the light lists to ensure identification. However, it must not be forgotten that atmospheric conditions affect the transmission of sound and at times cut it off entirely, producing a silent zone in a locality where the signal could be heard distinctly at other times; hence, if dependent on aërial signals, in a fog, slow the ship, navigate with extreme caution, keep the lead going on soundings, make every effort to guard against overlogging, and, as a last resort, if the depth of water will permit, anchor the ship. Ordinarily the sound of an aërial fog signal, if it is to leeward, will be heard sooner from aloft; if to windward, from a point nearer the water.

Fortunately, the cases are now rare in which the navigator has to depend on aërial signals, for the submarine-bell has been generally adopted by maritime nations as a means of making fog signals (see Appendix A).

In using this system, listen with the starboard receiver when the bell is known to be on the starboard side, otherwise use the port receiver; at all events if the bell is heard in both receivers its direction will be shown as being to starboard or port by the greater intensity of sound in the starboard or port receiver, and the correct bearing of the bell may be obtained by so changing the course as to bring it directly ahead, at which position the intensity of sound becomes the same in both receivers, provided the listener can hear equally well in both ears; otherwise there will be an error of direction, the amount being dependent on the difference of hearing in the listener's ears.

The submarine signal is more reliable than the aërial signal since it can be heard on board vessels fitted with receivers at greater distances, these distances varying according to condition of instruments, attention, and delicacy of hearing of listener; its direction can be ascertained with fair accuracy, and the sound is not subject to the silent zone, though there is always a possibility that the bell's mechanism may be deranged.

An approximate fix may be obtained from the bearing of the bell combined with a sounding or with a line of position (if recent and reliable) brought up to the instant of locating the direction of the bell; also from two bearings of the bell and the course and distance run in the interval.

However, in a fog, even when within hearing of a submarine-bell whose direction may be fairly well determined, the navigator must be watchful and cautious, should reduce speed and make a judicious use of both log and lead, bearing in mind the fact that the effect of cross currents encountered will be increased as speed is reduced.

CHAPTER VI.

THE SAILINGS.

122. The position of a ship at sea, at a given moment, is defined by its latitude and longitude. This position is connected with one left, or with one to which the vessel is bound, by the true course and distance between them.

A course and distance can be resolved into difference of latitude and departure, and this departure converted into difference of longitude; so that knowing the course and distance sailed from a given position, the latitude and longitude of the position arrived at can be found; or, when desired, the course and distance between two given positions may be found. The various methods of solution of these problems are termed Sailings, and include, Plane, Parallel, Middle Latitude, and Mercator Sailings.

The term "Dead Reckoning" includes all calculations to determine a ship's position, given only the true courses and distances run from a given point of departure. It involves the principles of the various sailings explained below.

The latitude and longitude determined by "Dead Reckoning" are noted thus, "Lat. by D. R.," "Long. by D. R."

The position by D. R. is liable to error due to bad steering, improper logging of the distances run, faulty allowance for leeway, effects of wind, currents, etc., and the exact position of the ship, out of sight of land, can be determined only by celestial observations.

The term "Day's Work," though frequently applied so as to embody only dead reckoning, the finding of the *C* and *d* made by D. R. from the point of departure, and the *C* and *d* by D. R. from position arrived at to destination, should properly include results arising from a knowledge of the ship's true position obtained either by bearings or celestial observations as indicated in Art. 310.

Particular attention must be paid to plane and parallel sailings as a full understanding of their principles is essential to an understanding of middle latitude sailing which is generally employed for short distances, as in a day's run; for longer distances it will be better to use Mercator sailing, which is a method based on the principles already explained in the articles on the Mercator chart.

The methods of laying down a ship's run and finding the position graphically have been explained in Chapter II.

123. Taking the departure.—On leaving port, at the beginning of a voyage, the ship's position is fixed by some one of the methods explained in the chapter on "Fixing positions near land," the best method available at the time, of course, being used; and from a last position thus obtained, the succeeding traverse, as laid down graphically on the chart, takes its commencement. This final position may be taken from the chart and be considered the point of departure from which future positions may be calculated in the navigator's work book. However, it is frequently the custom to take from the last position at which objects can be distinctly seen, the bearing and distance of some fixed point of land, lighthouse, light vessel, or beacon, whose latitude and longitude are known, and to consider its reversed true bearing and distance as a true course and distance sailed by the vessel, thus taking its position as the point of departure. If the distance is not known, it must be estimated. This is what is known as taking a departure.

The navigator must be careful to correct the compass bearing of this point for the variation, and the deviation of the compass due to the ship's heading at the time of taking the departure, then to reverse the true bearing to get the true course the ship is assumed to have sailed. *This reversed true bearing is called the departure course, and it equals the true bearing ± 180°.*

When a departure is thus taken the departure course and distance appear in the record of the first day's work, in the proper columns of the tabulated form, and are treated like any other course and distance.

Noon position as a point of departure.—In the succeeding part of the voyage, each noon position by observation is taken as a new point of departure.

Latitude left and longitude left.—In dead reckoning, these terms refer to the latitude and longitude of the point of departure.

Latitude in and longitude in.—These terms in dead reckoning apply to the latitude and longitude arrived at, and are marked "by D. R." when by account, or "by obs." when by observation.

Course and distance made good.—For a given interval of time the course and distance from the last point of departure to the position by observation at the end of that time are the course and distance made good.

124. The following notation will be followed in this book whenever it may be necessary to represent the quantities referred to below:

C	will	represent	the	course measured from the North or South towards East or West.
C_N	"	"	"	course measured from North around to the right from 0° to 360°.
L	"	"	"	latitude.
λ	"	"	"	longitude.
L_1	"	"	"	latitude of place left.
λ_1	"	"	"	longitude of place left.
L_2	"	"	"	latitude of place arrived at.
λ_2	"	"	"	longitude of place arrived at.
$l=L_2 \sim L_1$	"	"	"	difference of latitude.
$D=\lambda_2 \sim \lambda_1$	"	"	"	difference of longitude.
p	"	"	"	departure.
d	"	"	"	distance sailed on course C.
L_0	"	"	"	the middle latitude $=\frac{L_1+L_2}{2}$.

Latitude is North or South according as the place is in the Northern or Southern hemisphere. Longitude is East or West according as the place is on a meridian East or West of Greenwich.

In the solutions by *computation* of the triangles of plane, middle latitude and Mercator sailings, the course C is an interior angle of the triangle and is not greater than 90°. Its general direction is determined by l and p or m and D. Having been found, C should be expressed as C_N for practical purposes (Exs. 39, 52, and 53).

If the data includes the course as C_N, express it as C and indicate its proper direction before proceeding with the computation (Ex. 38).

In solutions by *inspection,* as in dead reckoning, the course should be retained in the form of C_N as the traverse tables are tabulated for courses up to 360° (Ex. 25).

PLANE SAILING.

125. For small distances at sea, the curvature of the earth may be neglected, and the small portion of the earth passed over may be regarded as a plane surface, on which the meridians are parallel right lines perpendicular to the equator, the parallels of latitude are right lines parallel to the equator, and the length of a degree is assumed the same whether measured on the equator, meridian, or parallel.

Though this assumption is not strictly correct, the results obtained by plane sailing may be considered sufficiently exact for any ordinary day's run.

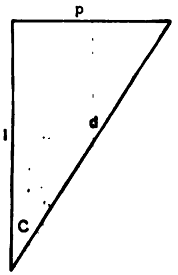

FIG. 66.

The relations existing between the parts that enter into plane sailing are indicated in a right triangle in which C is the course, l the difference of latitude, p the departure in the latitude left or that arrived at. From an application of

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the principles of plane trigonometry, the relations are from Fig 66,

$$\left.\begin{aligned} l &= d \cos C, \\ p &= d \sin C, \\ p &= l \tan C. \end{aligned}\right\} \qquad (115)$$

The solution of the above equations is facilitated by the use of Tables 1 and 2, Bowditch (which are tables for the solution of any right triangle), calling d the hypothenuse, l the side adjacent, and p the side opposite the course C.

Table 1 gives the courses in quarter points and distances up to 300 for each unit; Table 2 gives courses in degrees and distances to 600.

Should the distance for which l and p are desired be greater than the limit of the table, subdivide the distance into two or more parts, finding the l and p for these separate parts and adding; thus the diff. of lat. for 1340 miles, course 10°, will be the diff. of lat. for 600 + diff. of lat. for 600 + diff. of lat. for 140, course 10°, and the departure may be found in the same way; or 1340 may be divided by 4, giving 335, then l and p may be found for 335, course 10°, and multiplied by 4; similarly any other factor, as 5 or 10, might be used.

Since $l = d \cos C$ and $p = d \sin C$, it is apparent that the l and p for any course are respectively the p and l for the complement of the course, as shown in the tabulated form.

So it is apparent that all questions involving C, d, l, and p can be solved by inspection by using Tables 1 and 2, Bowditch. Any expression involving sines,

Table 2.

Diff. of Lat. and dep. for 10° (170°, 190°, 350°)		
Dist.	**Lat.**	**Dep.**
1	1.0	0.2
2	2.0	0.3
3	3.0	0.5
4	3.9	0.7
5	4.9	0.9
6	5.9	1.0
7	6.9	1.2
8	7.9	1.4
9	8.9	1.6
10	9.8	1.7
Dist.	**Dep.**	**Lat.**
80° (100°, 260°. 280°)		

cosines, secants, tangents, or cotangents of the following forms may be referred to the traverse table.

$$\left.\begin{array}{l}\text{Thus } x = 30 \sin 60°,\\ \text{If } p = d \sin 60°,\\ \text{When } d = 30, \text{ find } p.\end{array}\right\} \qquad \left.\begin{array}{l}\text{Also } 35 = x \tan 50°,\\ \text{If } p = l \tan 50°,\\ \text{When } p = 35, \text{ find } l.\end{array}\right\}$$

A triangle may be solved by the "Rule of Sines" in the same way.

When the distance sailed is so great that the curvature of the earth cannot be neglected.—In Fig. 67, let P be the elevated pole of the earth. C and A two places on the surface connected by the loxodrome CA. Let $C'A'$ be an arc of the equator intercepted between the meridians of C and A.

Consider the distance CA to be divided into a very large number of equal distances; each distance forming with its corresponding difference of latitude and departure a right triangle. All these triangles are similar, two angles of each triangle, the right angle and the course C, being equal to the corresponding angles in the other triangles. Each triangle is so small that it may be taken as a plane right triangle. Such would be the triangle abc for the small distance ca.

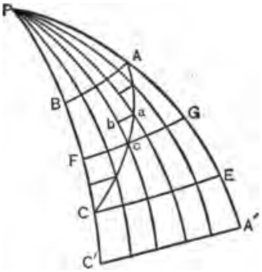


FIG. 67.

Now letting d_1, d_2, d_3 .... d_n be the small distances into which CA is divided;

l_1, l_2, l_3 ..... l_n, their corresponding differences of latitude for the course C;

p_1, p_2, p_3 .... p_n, the corresponding departures for the same course, each departure being measured in the latitude of its own triangle;

we have

$$l_1 = d_1 \cos C,\ \ l_2 = d_2 \cos C \ldots . l_n = d_n \cos C;$$
$$p_1 = d_1 \sin C,\ \ p_2 = d_2 \sin C \ldots . p_n = d_n \sin C.$$

Therefore,

$$l_1 + l_2 + l_3 \ldots . l_n = (d_1 + d_2 + d_3 \ldots . d_n) \cos C.$$
$$p_1 + p_2 + p_3 \ldots . p_n = (d_1 + d_2 + d_3 \ldots . d_n) \sin C.$$

Now since the parallels of latitude through C and A are the same distance apart on all meridians, the difference of latitude of C and A is the sum of the partial differences of latitude, and, as the total distance is the sum of the partial distances, we have

$$\left.\begin{array}{c} l = l_1 + l_2 + l_3 \ldots . l_n = (d_1 + d_2 + d_3 \ldots . d_n) \cos C, \\ \text{or } l = d \cos C. \end{array}\right\} \quad (116)$$

And if each partial departure is measured in the latitude of its own triangle, the sum of these partial departures will represent the true departure in the triangle CAB, and hence,

$$\left.\begin{array}{c} p = p_1 + p_2 + p_3 \ldots . p_n = (d_1 + d_2 + d_3 \ldots . d_n) \sin C, \\ \text{or } p = d \sin C. \end{array}\right\} \quad (117)$$

So that l and p are calculated by the same formulæ whether the curvature of the earth is, or is not, considered. However, the sum of the partial departures is less than the distance between the meridian left and the meridian arrived at measured in the lower latitude, and greater than that measured in the higher latitude, and is approximately equal to the departure of the parallel midway between the two.

When both points of departure and arrival are on the same side of the equator the latitude of the parallel midway between is known as the middle latitude, and is equal to the half sum of the two latitudes; in other words,

$$L_0 = \frac{L_1 + L_2}{2}. \quad (118)$$

From what has been said above, it is evident that a ship sailing due North or South (true) remains on the meridian, changes her latitude only, and the distance sailed is simply a difference of latitude, is either "Northing" or "Southing," and must be so entered in the tabulated form of work. When a ship sails due East or West (true), she remains on her parallel, does not change her latitude, and the distance sailed is either "Easting" or "Westing," and must be so entered in the form for work, this departure to be later converted into its proper difference of longitude.

When a ship sails due East or West (true), on the equator, the distance East or West is itself difference of longitude.

When a ship sails on a loxodrome, at an acute angle with the meridian, she alters both her latitude and longitude.

TRAVERSE SAILING.

126. If a ship sails on several courses instead of a single course, she makes an irregular track, called a traverse, and it is the function of traverse sailing to find the single course and distance that would have taken the ship to the position arrived at, in other words, the resultant course and distance as well as the corresponding difference of latitude and departure.

If C_1, $C_2 \ldots . C_n$ be the different courses, and d_1, $d_2 \ldots . d_n$ the corresponding distances, then

$$l_1 = d_1 \cos C_1, \quad l_2 = d_2 \cos C_2 \ldots . \; l_n = d_n \cos C_n;$$
$$p_1 = d_1 \sin C_1, \quad p_2 = d_2 \sin C_2 \ldots . p_n = d_n \sin C_n;$$

and, as before,

$$l = l_1 + l_2 + l_3 \ldots . l_n,$$
$$p = p_1 + p_2 + p_3 \ldots . p_n,$$

p being measured along the parallel $\dfrac{L_1 + L_2}{2}$, or, as in the case of a single course, "the whole difference of latitude is equal to the sum of the partial differences of latitude, and the whole departure is equal to the sum of the partial departures."

The word sum is used in its algebraic sense, that is to say, if l_N is the sum of the northerly differences of latitude and l_S the sum of the southerly differences of latitude, then $l = l_N \sim l_S$ and is of the same name as the greater; and if p_W is the sum of westerly and p_E of easterly departures, $p = p_W \sim p_E$ and is of the same name as the greater.

The traverse table referred to under plane sailing greatly facilitates the computation.

Having found the resultant l and p, the course and distance made good, or the resultant course and distance, are gotten from the formulæ

$$\tan C = \frac{p}{l}, \quad d = l \sec C,$$

or by inspection from Table 2.

The traverse may run irregularly, and into higher latitudes than the latitudes of the extremities of the distance made good, so that the departure of this course and distance made good may not be the same as the sum of the partial departures; hence, if necessary, separate the traverse into two or more parts, and calculate for each part separately. However, if the traverse does not go into too high latitudes, the error, likely to arise, may be considered immaterial in an ordinary day's run.

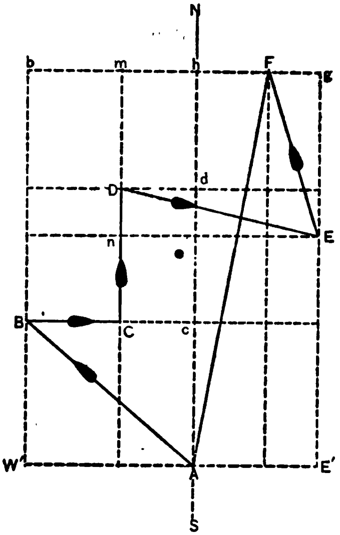


FIG. 68.

Graphic explanation of traverse sailing.—To further explain the principles of traverse sailing, let $W'E'gb$ represent a portion of a Mercator chart; let A, located on a meridian NS and a parallel of latitude $W'E'$, be a place sailed from, and F a place arrived at, after sailing successively from A to B, to C (directly East), to D (directly North), to E, and to F (Fig 68). Let meridians and parallels be drawn through each point of the traverse; the triangles thus formed and the difference of latitude and departure corresponding to each distance are indicated in the figure.

$$\text{For distance } AB\left\{\begin{array}{l}\text{Diff. of Lat.} = Ac.\\ \text{Dep.} = cB = hb.\end{array}\right.$$

$$\text{For distance } BC\left\{\begin{array}{l}\text{Diff. of Lat.} = 0.\\ \text{Dep.} = BC = bm.\end{array}\right.$$

$$\text{For distance } CD\left\{\begin{array}{l}\text{Diff. of Lat.} = CD = cd.\\ \text{Dep.} = 0.\end{array}\right.$$

$$\text{For distance } DE\left\{\begin{array}{l}\text{Diff. of Lat.} = Dn = de.\\ \text{Dep.} = nE = mg.\end{array}\right.$$

$$\text{For distance } EF\left\{\begin{array}{l}\text{Diff. of Lat.} = Eg = eh.\\ \text{Dep.} = gF.\end{array}\right.$$

The course made good is *hAF*, distance made good *AF*, the corresponding difference of latitude *Ah*, and departure *hF*.

Regarding directions towards the top of page and to the right hand as positive, differences of latitude towards the top of page (in this case North) are +, towards the bottom (or South) are −, departures to the right (or East) are +, to the left (or West) are −.

It will be seen, by examining the figure, that *Ah* and *hF* are respectively the algebraic sum of all the differences of latitude and departures corresponding to the several courses and distances sailed. Proof:

$$l = +Ac + (cd - de) + eh = +Ac + ce + eh = +Ah.$$
$$p = -hb + bm + mg - gF = -hm + mF = +hF.$$

Sources of data.—The navigator will find in the ship's log book the latitude and longitude of the point of departure, the compass courses and distances sailed, and correction for leeway, if any; then taking from the chart or tables the variation of the locality, and from the deviation table the deviations for the various compass courses steered, he will have the data for working the traverse.

If in a region of known currents, he must allow for the set and drift as explained later (see Arts. 129-131).

The preparation of the traverse form and data.—(1) In case a departure course enters into the computation, the compass bearing of the point of departure is corrected for variation, and for the deviation due to ship's head when the bearing was taken, and the reversed true bearing thus obtained is entered in the column of true courses, the distance in the column of distances, thus forming the first course and distance of the tabulated form.

(2) Each compass course is corrected for variation, deviation, and leeway, and the result entered in the form under the head of true course, the sum total of distances run on each true course being placed opposite that course in the distance column.

(3) Enter Table 2, Bowditch, find each true course C_N at top or bottom of page, and for each true course and distance find the corresponding differences of latitude and departure, placing them in their respective columns, opposite the courses;

the difference of latitude being placed in the "N." column when the course is northerly, in the "S." column when southerly; the departure being placed in the "E." column when the course is easterly, in the "W." column when westerly. When distances are in miles and decimals, multiply by 10 or by 100, take out for the new whole number the desired quantities, and divide them by the multiplier just used. Thus, for 29.3 take out *l* and *p* for 293 and divide by 10. Where the course is between two given degrees, first find *l* and *p* for each and interpolate.

Add up the "diff. of lat." and "departure" columns; take the difference between the northings and southings to which give the name of the greater; do the same for the E. and W. departures; these resulting differences are, respectively, the difference of latitude and departure of the resultant course.

(4) Then look in Table 2 and turn to that page on which can be found coincidently in the lat. and dep. columns the above-mentioned resultant difference of latitude and departure. The angle at the top or bottom of page, as the case may be, will be the course made by D. R., estimated from the North point to the right, the particular quadrant being determined by a consideration of the resulting differences of latitude and departure. The course C_N will be taken out in the 1st, 2d, 3d, or 4th quadrant according as the co-ordinates *l* and *p* show it to be in the general direction of NE., SE., SW., or NW., respectively.

On the same line with the difference of latitude and departure, in the Distance Column, will be found the distance made by D. R.

(5) The compass being graduated from 0° at North, around to the right through 360°, attention is called to the fact that the 1st, 2d, 3d, and 4th quadrants are respectively the NE., SE., SW., and NW. quadrants; and, for the proper markings of *l* and *p*, that the course C_N is northerly in the 1st and 4th quadrants, southerly in the 2d and 3d, easterly in the 1st and 2d, and westerly in the 3d and 4th.

Interpolating in the Traverse Tables.—When the exact values of l and p are not found together on any page of the traverse tables, interpolation as illustrated in the following examples is resorted to:

Given $l=290.6$ and $p=191.3$, to find C and d.

Write down the given quantities in the column of "Arguments and Interpolations" in the form shown. Then an inspection of the tables with l and p as arguments shows that the required *course* falls between 33° and 34°. Using l as an argument, since in this case its value changes more slowly than that of p for any value of d under $C=33°$ or 34°, interpolate by inspection in these tables for d and p and write the quantities obtained in the columns headed "Base 1" and "Base 2" with the given l common to both. (If the value of p changed more slowly than l, shown by p being larger than l, it would be used to find d and l in the two tables.)

The required values of C and d will then be found by proportioning the differences between the values of C and d under "Base 1" and "Base 2" in the same ratio as that of the given value of p between the values of p in "Base 1" and "Base 2."

	Base 1	Arguments and Interpolations	Base 2
C	33°	Required Course 33°.4	34°
d	346.5	Required Distance 347.9	350.5
l	290.6	290.6 (*given*)	290.6
p	188.7	191.3 (*given*)	196.0

When the values of C and d are given the procedure is similar for finding l and p.

Given $C=33°.4$ and $d=347.9$.

From tables $C=33°$ and 34° find by inspection the values of l and p corresponding to $d=347.9$. Arrange in columns as before and find the required values of l and p by proportion according to the given value of C.

	Base 1	Arguments and Interpolations	Base 2
C	33°	33°.4 (*given*)	34°
d	347.9	347.9 (*given*)	347.9
l	291.8	Required l 290.4	288.4
p	189.5	Required p 191.5	194.5

PARALLEL SAILING.

127. Plane sailing has dealt only with courses and distances sailed, found the resulting differences of latitude and departure, and given to the navigator the course and distance made good and the latitude arrived at, without, however, giving him his longitude. To find the longitude, a relation must be established between it and the departure.

Now departure is measured on a small circle of the sphere paralled to the equator, is expressed in units of a sea mile, and represents the distance between two meridians in the same latitude. The arc of the equator, intercepted between these meridians, is a measure of the angle between them and expresses the difference of longitude.

Before the days of accurate chronometers, ship masters were often uncertain as to their longitude. They would "run down their latitude," that is, go N. or S. till they reached the latitude of the port to which they were bound, then steer for the port, along a parallel E. or W. The distance made along a parallel was the departure, and it was then, as now, the function of parallel sailing to connect departure with its corresponding difference of longitude. Knowing the longitude left, and finding by an application of the principles of parallel sailing the difference of longitude made, we are enabled to find the longitude arrived at.

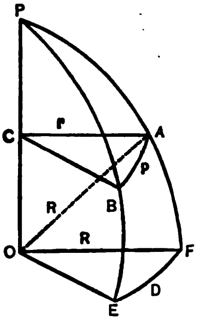


FIG. 69.

In Fig. 69, *EF* is the difference of longitude between meridians *PE* and *PF* and is called *D*. *BA* is the corresponding departure in the latitude of *B* and *A* and is called *p*. Now *D* and p are similar arcs of two circles and therefore are proportional to their radii, hence, $D : p = R : r$, and $p = \frac{rD}{R}$.

Now $\angle AOF = \angle OAC$ is the latitude of B or A, and $\cos L = \frac{r}{R}$; therefore,

$$p = D \cos L; \text{ or } D = p \sec L. \qquad (119)$$

Hence, by this formula, is found the difference of longitude corresponding to a given departure in a given latitude.

The relation of parts involved in parallel sailing are shown in the triangle, Fig. 70, in which L is the latitude, p the departure, D the corresponding difference of longitude, the departure being in sea miles and longitude in minutes of arc.

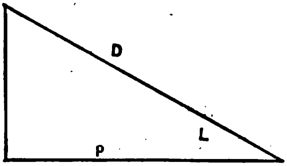


FIG. 70.

The most usual cases that arise under parallel sailing are those in which the departure between two places in the same latitude is given to find the difference of longitude, or, given the difference of longitude between two places in the same latitude, to find the departure; though sometimes the latitude of the parallel may be required, the difference of longitude and corresponding departure between the two places being known.

CASE I.—Given p, to find D.

Ex. 26.—A ship in latitude 49° 35′ N. and longitude 22° 30′ W. sails due South (true) 65 miles, then due East (true) 120 miles; find latitude and longitude in.

By computation:

$$\begin{array}{l} L_1 = 49^\circ\ 35'\ \text{N} \\ l = \underline{\ \ 1\ \ 05\ \ \text{S}} \\ L_2 = 48^\circ\ 30'\ \text{N} \end{array} \left\{ \begin{array}{lll} L_2 = 48\ 30 & \sec & 10.17874 \\ p = 120 & \log & \underline{\ 2.07918} \\ D = 181.1 & \log & 2.25792 \end{array} \right\} \begin{array}{l} \lambda_1 = 22^\circ\ 30'\ \text{W} \\ D = \underline{\ 3\ \ 01.1\ \text{E}} \\ \lambda_2 = 19^\circ\ 28'.9\ \text{W} \end{array}$$

By inspection: Enter table 2 with latitude as a course, find p in the latitude column, and opposite in the distance column is D. It may be necesary to interpolate as in the example. The Lat. of the parallel is 48° 30′ N.

$$\begin{array}{llll}
\text{For } 48^\circ, p=119.8, & D=179 & \text{For } 49^\circ, p=119.4, & D=182 \\
\quad p=120.4, & D=180 & \quad p=120.1, & D=183 \\
\hline
\quad p=120, & D=179.33 & \quad p=120, & D=182.86
\end{array}$$

Hence for Lat. 48° 30′ N., and $p = 120, D = 181.095$.

CASE II.—Given D, to find p.

Ex. 27.—A ship sails on a parallel of latitude 41° 30′ S. from A in longitude 18° 30′ E. to B in longitude 2° 10′ W. Find the distance sailed in nautical miles.

$$\begin{array}{llrll}
\text{Long. of } A & 18^\circ\ 30'\ \text{E} & & L = 41^\circ\ 30' \ldots\ldots \cos & 9.87446 \\
\text{“} \quad B & 2\ \ 10\ \ \text{W} & & D = 1240 \ldots\ldots \log & 3.09342 \\
\hline
 & D = 20^\circ\ 40'\ \text{W} & & p = 928'.7 \ldots\ldots \log & 2.96788 \\
 & \quad = 1240' & & &
\end{array}$$

Ans. 928.7 miles.

By inspection: Enter table 2 with Lat. for the course, find D in the distance column, and, opposite in the latitude column, will be found p.

When D is greater than any tabulated distance, pursue either of the following methods which are given in full to illustrate the use of the traverse tables.

Divide D by 10, find corresponding p in latitude column, then multiply by 10, thus:

$$\left.\begin{array}{l}
\text{For } L = 41^\circ, D = 124;\ p = 93.6 \\
\quad L = 42^\circ, D = 124;\ p = 92.1
\end{array}\right\} \text{By interpolation and multiplication,}$$

$$\text{For } L = 41^\circ\ 30', D = 1240;\ p = 928.5.$$

A closer result may be gotten by inspection by considering D in three parts, $600 + 600 + 40$.

$$\begin{array}{rl}
\text{For Lat. } 41^\circ, D = 600, & p = 452.8 \\
D = 600, & p = 452.8 \\
D = \ \ 40, & p = \ \ 30.2 \\
\hline
D = 1240, & p = 935.8
\end{array}$$

$$\begin{array}{rrl} \text{For Lat. } 42°, & D = 600, & p = 445.9 \\ & D = 600, & p = 445.9 \\ & D = 40, & p = 29.7 \\ \hline & D = 1240, & p = 921.5 \end{array}$$

By interpolation, for Lat. 41° 30′ S., $D = 1240$; $p = 928.65$.

CASE III.—To find the latitude, given p and D.

Ex. 28.—A ship in longitude 45° 10′ W. sails due W. (true) 186.8 miles, and is then in longitude 48° 58′ W. Find the latitude.

By computation: $\cos L = \frac{p}{D}$.

$$\begin{array}{ll} \lambda_2 = 48° \, 58' \, W & p = 186.8 \ldots\ldots \log 2.27138 \\ \lambda_1 = 45 \;\; 10 \;\; W & D = 228 \ldots\ldots\ldots \log 2.35793 \\ \hline D = \;\; 3° \, 48' \, W & L = 34° \, 59' \, N \ldots \cos 9.91345 \\ \;\;\; = \;\; 228 \;\; W & \end{array}$$

By inspection: Turn to that page of table 2 on which $D = 228$ is found in the distance column and $p = 186.8$ in the diff. of lat. column, opposite D. The angle at the top or bottom of page, as the case may be, will be the latitude. If exact coincidence of D and p are not found on a given page, then the angle must be found by interpolation.

In this example Lat. is found to be 35° N.

Examples Under Parallel Sailing.

Ex. 29.—A ship in latitude 38° N. sailed due West till she changed her longitude 5°. What distance did she sail?

Ans. 236.4 miles.

Ex. 30.—A ship in Lat. 40° N., Long. 160° W., sails due East till her longitude is 150° 30′ W. Find by inspection the distance sailed.

Ans. 436.6 miles.

Ex. 31.—How far must a ship sail due East in Lat. 60° N. to change her longitude 5°? *Ans.* 150 miles.

Ex. 32.—From a place in Lat. 30° N., Long. 50° 20′ W., a ship sails due West 240 miles, then due N. 240 miles, and due E. 240 miles. Find Lat. and Long. in by inspection.
Ans. Lat. 34° N.; Long. 50° 07′.6 W.

Ex. 33.—A ship in Lat. 60° N. sails due West 75 miles. How much does she change longitude? *Ans.* 2° 30′ W.

Ex. 34.—A ship in Lat. 38° N., Long. 159° 10′ W. sails due E. 405 miles. Find Lat. and Long. in.
Ans. Lat. 38° N.; Long. 150° 36′ W.

Ex. 35.—Two ships in Lat. 35° N., distant from each other 150.7 miles, sail due North at the same speed for 300 miles. Find by inspection how much closer they are at the end of run. *Ans.* 9.7 miles.

Ex. 36.—Find by inspection in what latitude the length of a degree of longitude will be 46 miles. *Ans.* 40°.

Ex. 37.—Two ships are steaming due East at the same speed. *B* changes longitude twice as fast as *A*, who is in the 20th parallel of N. latitude and to southward of *B*. Find *B*'s latitude by computation. *Ans.* 61° 58′ 31″ N.

MIDDLE LATITUDE SAILING.

128. In plane sailing, the assumption was made that the earth was an extended plane, and, though this assumption was false, the errors for small distances were considered immaterial. Were the earth an extended plane, the departure would be the same in both latitudes left and arrived at. It has been shown that the departure is approximately that of the middle latitude.

In parallel sailing, the earth was regarded as a sphere and a relation was established between departure and difference of longitude.

Now, combining the principles of plane and parallel sailing, we have middle latitude sailing, which finds the difference of longitude corresponding to a departure measured in the middle latitude, and, by partially nullifying the false assumptions of plane sailing, gives a nearer approximation to true results. A still nearer approximation to the truth may be gotten by applying from Bowditch a correction to the middle latitude, and considering the departure measured on this corrected parallel. However, if necessary to do this, it would be better, after finding C and L_2 by plane sailing, to find D by Mercator sailing, explained later on.

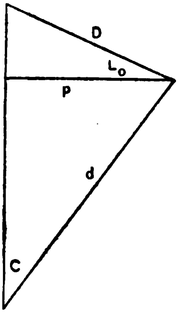


FIG. 71.

The relations of the quantities involved in middle latitude sailing are shown in Fig. 71 by combining the triangles of plane and parallel sailings, regarding the departure as measured in the latitude of the middle parallel.

Let $L_0 = \frac{L_1 + L_2}{2}$ the middle latitude, then

$$\left.\begin{aligned} & l = d \cos C, \ p = d \sin C. \\ & D = p \sec L_0 = d \sin C \sec L_0 = l \tan C \sec L_0. \\ & L_0 = \frac{L_1 + L_2}{2}, \ \tan C = \frac{D \cos L_0}{l}. \\ & L_2 = L_1 + l, \ \lambda_2 = \lambda_1 + D. \end{aligned}\right\} \qquad (120)$$

When not advisable to use M. L. sailing.—The results gotten by using middle latitude sailing are more accurate in low latitudes, less so in high latitudes, and the inaccuracy is greater the greater the difference of latitude, or for a given distance sailed, the smaller the course. Hence when the latitudes are high (over 50° N. or S.), or course small with a large distance producing large differences of latitude, it is better not to use middle latitude sailing, but to use Mercator sailing.

It is not advisable to use middle latitude sailing when the places left and arrived at are on different sides of the equator, unless the two parts of the track on opposite sides of the equator are treated separately, except in the case where the distance each side is so small that the departure is practically equal to the difference of longitude.

In ordinary practice examples under middle latitude sailing come under one of the two following cases. For other variations, however, it is only necessary to draw a figure showing the relation of the parts, and to use that formula which will give the unknown from the known parts.

Case I.—Given the course and distance sailed from a place of known latitude and longitude, to find the latitude and longitude arrived at.

Ex. 38.—A ship in Lat. 36° 40′ S., Long. 48° 40′ W. sailed 38° (true) 150 miles. Find Lat. and Long. in. Solution by computation:

$d = 150$ ....log 2.17609.... log 2.17609
$C =$ N 38° E........cos 9.89653.... sin 9.78934
$l = 118'.2$ N........log 2.07262
p ....log 1.96543
$L_0 = 35°\ 40'\ 54''$ S.................sec 0.09030
$D = 113'.69$ E.....................log 2.05573

	°	′	″		
$L_1 =$	36	40		S	
$l =$	1	58	12	N	
$L_2 =$	34	41	48	S	✱
$L_0 =$	35	40	54	S	
$\lambda_1 =$	48	40		W	
$D =$	1	53	41	E	
$\lambda_2 =$	46	46	19	W	✱

By inspection: Enter Table 2 with course 38°, opposite 150 in distance column, find l=118.2 in Lat. column and p=92.3 in Dep. column. Then with the middle latitude 35⅔° as a course find, by interpolation, opposite p in Lat. column, the D in distance column.

$$\left.\begin{array}{l}\text{For Lat. } 35^\circ,\ p = 92.3,\ D = 112.7 \\ \text{“}\quad 36^\circ,\ p = 92.3,\ D = 114.1\end{array}\right\} \therefore \text{ for Lat. } 35\tfrac{5}{8}^\circ,\ p = 92.3,\ D = 113.7$$

	° ′ ″		° ′ ″
$L_1 =$	36 40 S	$\lambda_1 =$	48 40 00 W
$l =$	1 58 12 N	$D =$	1 53 42 E
$L_2 =$	34 41 48 S	$\lambda_2 =$	46 46 18 W
$L_0 =$	$35\frac{5}{8}$° S		

Case II.—To find the course and distance between two positions of known latitude and longitude.

Ex. 39.—Find the course and distance from Lat. 43° 03′ 24″ N., Long. 5° 56′ 30″ E. to Lat. 39° 26′ 42″ N., Long. 0° 23′ 00″ W.

By computation:

	° ′ ″		° ′ ″	° ′ ″
$L_1 =$	43 03 24 N	$\lambda_1 =$	5 56 30 E	$L_1 =$ 43 03 24 N
$L_2 =$	39 26 42 N	$\lambda_2 =$	0 23 00 W	$L_2 =$ 39 26 42
				82 30 06
$l =$ 216′.7 $=$	3 36 42 S	$D =$ 379′.5 $=$	6 19 30 W	$L_0 =$ 41 15 03 N

$D = 379.5$ ....log 2.57921

$L_0 = 41^\circ\ 15'\ 03''$.. ......cos 9.87612

p.......................log 2.45533 .. ..log 2.45533

$l = 216.7$....................................log 2.33586..........log 2.33586

$C = \text{S } 52^\circ\ 47'\ \text{W } (C_N = 232^\circ\ 47')$...............tan 0.11947 ....sec 10.21837

$d = 358.28$ miles.. ....log 2.55423

By inspection.—Enter Table 2 with $L_0 = 41\frac{1}{4}^\circ$ as C, find corresponding to D 379.5 in distance column, $p = 285.3$ in Lat. column, thus

$$\left.\begin{array}{l}\text{For Lat. } 41^\circ,\ D = 379.5;\ p = 286.4 \\ \text{For Lat. } 42,\ \ D = 379.5;\ p = 282.\end{array}\right\} \begin{array}{l}\text{Therefore} \\ L_0 = 41\tfrac{1}{4}^\circ,\ D = 379.5,\ p = 285.3\end{array}$$

Then find corresponding to $l = 216.7$ and $p = 285.3$ the course and distance thus:

$$\left.\begin{array}{llll}\text{For } p = 285.3, & l = 222.9; & d = 362. & C_N = 232^\circ \\ \phantom{\text{For }} p = 285.3, & l = 215.; & d = 357.2 & C_N = 233^\circ\end{array}\right\} \begin{array}{l}\text{Therefore by} \\ \text{interpolation}\end{array}$$

$$\begin{array}{llll}\text{For } p = 285.3, & l = 216.7; & d = 358.4 & C_N = 232\tfrac{3}{4}^\circ\end{array}$$

Again attention is called to the fact that, if the difference in latitude is large, the assumption that the departure is properly measured in the middle latitude is not strictly correct; and, if greater accuracy is desired, a correction from Bowditch must be applied to the middle latitude to obtain the proper parallel on which to take the departure (see Art. 133). However, it is just as easy and more correct to use Mercator sailing.

When the two places considered are on opposite sides of the equator, no sensible error will be made in the case of an ordinary day's run, which will seldom exceed 400 miles, by taking the difference of longitude equal to the departure. If the distance is great, use Mercator sailing, except when the course is large (more nearly East or West), in which case use middle latitude sailing (see Art. 132).

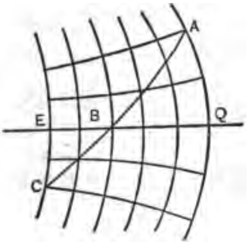


Fig. 72.

However, when the distance between two places, one in North latitude and the other in South latitude, is great, and it is desired to use middle latitude sailing in finding the difference of longitude, the two portions of the track on different sides of the equator may be treated separately. Thus in Fig. 72, let the coordinates of the place A be L_1, λ_1, and those of the place C in the opposite hemisphere be L_2, λ_2.

The track AC is divided by the equator EQ into two parts, AB and BC.

For AB we have

$$AQ = l_1 = L_1,$$

$$p_1 = L_1 \tan C, \text{ and neglecting } \Delta L_1,$$

$$QB = D_1 = p_1 \sec \frac{L_1}{2} = L_1 \tan C \sec \frac{L_1}{2}.$$

For BC we have

$$EC = l_2 = (-)\ L_2,$$

$$p_2 = (-)\ L_2 \tan C, \text{ and neglecting } \Delta L_2,$$

$$BE = D_2 = p_2 \sec \frac{L_2}{2} = (-)\ L_2 \tan C \sec \frac{L_2}{2},$$

whence QE or $D = D_1 + D_2$.

Therefore, for this case we have the following formulæ:

$$\left.\begin{aligned} l &= d \cos C, \\ L_2 &= L_1 + l, \\ D_1 &= L_1 \tan C \sec \frac{L_1}{2}, \\ D_2 &= (-)\ L_2 \tan C \sec \frac{L_2}{2}, \\ D &= D_1 + D_2, \\ \lambda_2 &= \lambda_1 + D. \end{aligned}\right\} \qquad (121)$$

Instead of the middle latitude $\frac{1}{2}\ L_1$ and $\frac{1}{2}\ L_2$, we may for greater precision use $(\frac{1}{2}\ L_1 + \Delta\ L_1)$ and $(\frac{1}{2}\ L_2 + \Delta\ L_2)$.

Examples in Middle Latitude Sailing.

(*By inspection.*)

Ex. 40.—From L_1 49° 28′ 30″ N., λ_1 0° 03′ 15″ E., sailed 312° (p. s. c.) 36 miles, variation —20°, deviation —2°. Find by D. R. L_2 and λ_2.

Ans. $L_2 =$ 49° 40′ 48″ N.
$\lambda_2 =$ 0 48 51 W.

Ex. 41.—From L_1 48° 20′ 29″ N., λ_1 5° 07′ 48″ W., sailed 257° (p. s. c.), 22.2 miles, variation —20°, deviation —3°, thence 232° (p. s. c.), 216.5 miles, variation —20°, deviation —1°. Find L_2 and λ_2.

Ans. $L_2 =$ 45° 01′ 55″ N.
$\lambda_2 =$ 8 16 30 W.

Ex. 42.—A ship leaving Lat. 49° 50′ N., Long. 10° 16′ W., sails to the southward and westward till her departure is 188 miles and the latitude reached is 47° 28′ N. Find the course, distance, and longitude in.

$$\textit{Ans.}\begin{cases}\text{Course } C_N = 233°.\\ \text{Distance } 236.\\ \lambda_2 = 15°\ 00'\ 42''\ \text{W.}\end{cases}$$

Ex. 43.—A ship sails from L_1 24° 23′ S., λ_1 100° 30′ E., and, by observations the next day, finds her position to be 25° 43′ 12″ S., 104° 52′ 38″ E. What was her true course and distance? *Ans.* $C_N = 108.°6$.

Distance 251.9 miles.

Ex. 44.—Find by computation the true C and d from L_1 23° 00′ N., λ_1 109° 55′ W., to L_2 35° 30′ N., λ_2 139° 45′ E. (without correcting middle latitude).

Ans. $C_N = 277°\ 25'\ 12''$.

Distance 5807.5 miles.

CURRENT SAILING.

129. A current may be defined as a body of water moving steadily in one direction.

The set of a current is its course, or the direction in which it is moving.

The drift is the distance the current sets a ship in the time considered. Thus the drift in 20 hours being 10 miles, the drift per hour, ½ mile, would be more properly called the rate. When the rate per hour is known, the drift for any given time is easily found.

When a ship sails directly with or directly against a current, her motion is increased or retarded by the amount of the drift in the interval.

When a ship sails obliquely to a current, her motion may be accelerated, or retarded, according to the angle between the course of the ship and the set of current; and the distance made good is the diagonal of a parallelogram of which one side is the distance made in the direction of the keel and the other side the distance the ship is carried by the current in the direction of the set, in the same interval of time. This resultant direction is in accordance with Newton's first and second laws of motion.

Current sailing.—Current sailing may, therefore, be defined as the means of finding the course and distance made good when a ship's motion is affected by tides or currents, or a course to be steered to make good a given course.

Problems in current sailing.—There are two general cases in practice.

Case I.—Given a course steered and distance run, to find course and distance made good through a current of known set and rate.

Ex. 45.—In Fig. 73, let MM' be the meridian of a place A in North latitude and let it be assumed that a ship, leaving A, steers 210° (true) 8 knots per hour, through a current setting her to the eastward (true) 2 miles per hour. Lay off $AB = 8$ miles in the direction 210°, the speed per hour of the ship on her course. Lay off $AC = 2$ miles in the direction East, the drift and set of current in the same interval of time. Complete the parallelogram by drawing BD and CD, and join AD. By the principle of "the composition of forces," the ship at the expiration of one hour will be at D, having been moved along the diagonal AD under the joint action of two forces, her own propelling force and that of the current. The result under the joint forces is the same as if each force had acted in succession, that is, as if the ship had gone from A to B

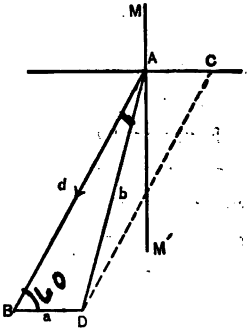


FIG. 73.

under her own propelling force uninfluenced by current, had then stopped, and been swept from *B* to *D* by the current.

In the particular diagram (Fig. 73) *AD* is the distance made good; *MAD*, the course made good (from N. to right).

Solution by construction.—Having a Mercator chart, it is only necessary to lay off *AB* and *AC* from the known position *A* in the proper directions, complete the parallelogram as above explained, then measure the angle *MAD* and the distance *AD*.

In navigating a tideway, since it is customary to state from place to place the direction and rate of tidal streams from hour to hour, it is desirable to plot the estimated position of the ship hourly (and also at every change of course) by laying off from the point of departure a line to represent the course and distance run during the first hour and from its extremity a second line to represent the set and drift of the current during this hour. From the point thus reached, this operation is continued for the second hour with the course and distance run and the set and drift of the current experienced during the second hour; and so on, for the hours or intervals in succession.

Solution by trigonometry.—To solve by trigonometry, make a rough sketch to show the conditions. Referring to Fig. 73, we have the angle $ABD=60°$, $AB=8$, $BD=2$, and it is required to find $\angle BAD$ and AD. Then the course made good or $\angle MAD=210°-BAD=210°-A$. Since $\angle ABD=60$,

$$\angle s.\ \frac{A+D}{2}=\frac{180°-60°}{2}=\frac{120°}{2}=60°.$$

From plane trigonometry we have, as $\begin{cases} a=BD, \\ b=AD, \\ d=AB, \end{cases}$

$$\tan\frac{D-A}{2}=\frac{d-a}{d+a}\cot\frac{B}{2}=\frac{6}{10}\cot 30°.$$

			° ′ ″
6 ....log	0.77815	$\frac{D+A}{2} =$	60
30°.............log cot	10.23856	$\frac{D-A}{2} =$	46 06 07
10 ....colog	9.00000	$D =$	106° 06′ 07″
$\frac{D-A}{2}$ 46° 06′ 07″ tan	0.01671	$A =$	13 53 53

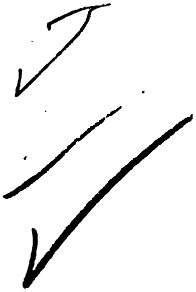
To find $AD = b$, $b = d \cdot \frac{\sin B}{\sin D} = 8 \sin 60° \operatorname{cosec} 106° \ 06' \ 07''$

			° ′ ″
8 ....log	0.90309		210
60°.........log sin	9.93753	$A =$	13 53 53
106° 06′ 07″ log cosec	10.01738	$MAD =$	196 06 07
$b = 7.211$ ....log	0.85800		

The course made good is $C_N = 196° \ 06' \ 07''$. Distance made good per hour = 7.211 miles.

Solution by the traverse table.—It is a simpler plan, however, to consider the course sailed and the set of the current as two separate courses in a traverse as below. Though approximate, results will be sufficiently correct.

True Courses.	Dist.	Diff. lat.		Departure.	
		N	S	E	W
210°	8	....	6.9	....	4.0
90°	2	....	....	2.0	....
			6.9	2.0	4.0
			....	....	2.0
					2.0



With $l = 6.9$ } Course made good $C_N = 196°$
$p = 2.0$ } Distance made good = 7.2 miles.

In the example worked above, the course and distance of the ship, and set and drift of the current were given for one hour only, but the principle holds good for any example in

which the set and drift of current for a given time may be taken as a course and distance in a traverse.

Ex. 46.—A ship in Lat. 40° 30′ N., Long. 48° 05′ W., at noon on Jan. 10, sailed till noon Jan. 11, 240° (p. s. c.) 223 miles. Var. −24°, Dev. −2°. A current sets 95° (true) 0.75 of a mile per hour. Find C_N and d made good, Lat. and Long. in.

Course (p.c.)	Var.	Dev.	True Course	Dist.	S	E	W
240°	−24°	−2°	214°	223	184.9	....	124.7
Current			95	18	1.6	17.9	.....
					186.5	17.9	124.7
					l=186.5	......	17.9
							p=106.8

Hoisting L 40

$$\begin{array}{rl} & \circ \quad \prime \quad \prime\prime \\ L_1 = & 40\ 30\ 00\ \text{N} \\ l = & \underline{\ 8\ 06\ 30\ \text{S}} \\ L_2 = & 37\ 23\ 30\ \text{N} \\ L_0 = & 38\ 56\ 45\ \text{N} \end{array} \qquad \begin{array}{rl} & \circ \quad \prime \quad \prime\prime \\ \lambda_1 = & 48\ 05\ 00\ \text{W} \\ D = & \underline{\ 2\ 17\ 30\ \text{W}} \\ \lambda_2 = & 50\ 22\ 30\ \text{W} \end{array} \qquad D = 137.5\ \text{W}$$

By inspection { Course made good $C_N = 209°.8$.
Distance made good 214.9 miles.

Having found l and p, the course and distance might be gotten from the formulæ

$$\tan C = \frac{p}{l}.$$

$$d = l \sec C.$$

However, the result by inspection is sufficiently close.

Case II.—Given a course and distance to be made good, to find the course it will be necessary to steer through a current of known set and drift, and the distance per hour that will be made good toward destination, and hence the time it will take to make the run.

Ex. 47.—A ship in a given position is to make a port bearing 242° true, distant 180 miles. Her speed is 12 knots per hour. A current is setting 96° true, drift 3 miles per hour. What true course must be steered, and how long will it take to make the run?

Solution by Construction.

Take a point A as the point of departure (Fig. 74). Through A draw the lines NS and WE. Lay off the angle NAB equal to the course to destination, 242°. Lay off the angle NAC equal to the set of the current, 96°. ~~Take ¼ inch = 1 mile as~~ scale. Lay off $AC = \frac{3}{4}$ inch = 3 miles (drift of current). With ~~C as a~~ center and a radius of 3 inches = 12 miles (speed of ship per hour), draw the arc cutting AB at B. Then B is the point of arrival at the end of one hour's run. Draw AB' parallel to CB. Then the angle NAB' is the course to be steered, and AB is the distance made good per hour along the course to destination.

Measuring the angle NAB', it is found to be

Course to be steered = 250°.

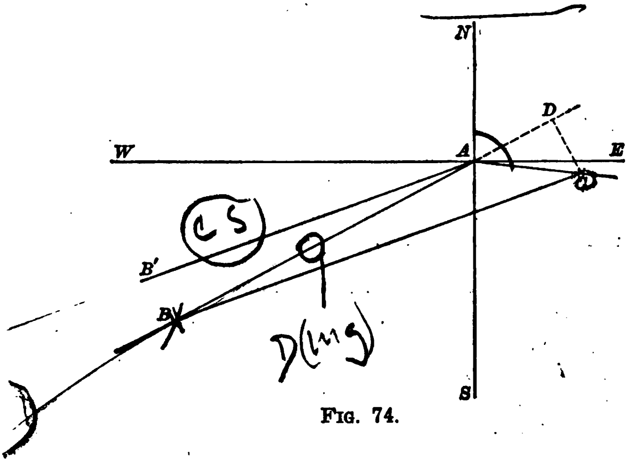


FIG. 74.

Measuring the line AB, it is found to be, distance made good along given course =

2.35 inches = 9.4 miles per hour.

Time to make run = $180 \div 9.4 = 19^h.15 = 19^h\ 09^m$.

Trigonometric Solution.

Draw CD perpendicular to BA produced (Fig. 74). Then in triangle ADC

Angle $DAC = 34°$

$AC = 3$

$DC = AC \sin 34° = 3 \sin 34°$

$AD = AC \cos 34° = 3 \cos 34°$

In triangle BDC

$$BC=12$$
$$DC=3\sin 34°$$
$$\sin DBC=\sin x=\frac{DC}{BC}=\frac{3\sin 34°}{12}$$
$$BD=BC\cos x=12\cos x$$
$$AB=BD-AD$$

	34°	sin	9.74756	cos	9.91857		
	3	log	.47712	log	.47712		
$DC=$	1.6676	log	.22468				
$AD=$	2.487			log	.39569		
8° 2' 11"	12	log	1.07918			log	1.07918
$x=$ ~~7° 57′ 28″~~		sin	9.14550			cos	9.99580
$DB=$	11.885					log	1.07498
$AD=$	2.487						
$AB=$	9.398						

250 2''

Course to be steered = 242° + 7° 57′ 28″ = 249° 57′ 28″.
Distance per hour made good along given course = 9.398 mi.
Time to make run = 180 ÷ 9.398 = $19^h.15 = 19^h\ 09^m$.

Solution by Traverse Tables.

In triangle ADC (Fig. 74).	In triangle DBC.
DAC = course = 34°	DBC = course = x
AC = dist. = 3	BC = dist. = 12
AD = diff. lat.	DB = diff. lat.
DC = departure	DC = departure
	$AB=BD-AD$

Co

Rc 90

For the purpose of the solution, consider DB to be the meridian.

Taking triangle ADC, enter Traverse Tables with 34° as course and 3 as distance, and find l and p.

Then in triangle DBC, enter Traverse Tables with 12 as distance and the value of p found above, and find C and l.

The value of the course so found is the correction to be applied to the course to destination to find the course to be steered. The algebraic sum of the two values of l found is the distance made good along the given course. The departures are equal and opposite.

Then l is the distance per hour made good toward destination, and the time to make the run = 180 ÷ 9.4 = $19^h.15 = 19^h\ 09^m$.

The second course found is the correction to be applied to the course to destination, and the figure will indicate the direction in which it is to be applied. In this example it is additive, and course to be steered$=242°+8°=250°$.

TRAVERSE TABLE.

	Course.	Distance.	Diff. of Lat.		Departure.	
			N	S	E	W
Diff. between current and bearing of port.	34°	8	2.5	....	1.7	....
Diff. between course to be steered and bearing of port.	8°	12	....	11.9	....	1.7
			2.5	11.9	1.7	1.7
			....	2.5	....	1.7
			$l=9.4$		$p=0.0$	

Ex. 48.—Steaming at the rate of 8.5 knots per hour, one wishes to make good a course 76° magnetic, through a current setting 12° magnetic 3.5 miles per hour. What must be the magnetic course? *Ans.* $C_N=97°\ 43'\ 16''$.

Ex. 49.—A port bears from a ship 359° (mag.) distant 127 miles. Steaming at 16 knots per hour, through a current that sets 320° (mag.) at the rate of 3 miles per hour, find the compass course to make the port, deviation −2°, var. −7°. Find also the time occupied in making the voyage.

Ans. Compass course $C_N=7°\ 46'\ 34''$.

Distance made good per hour, 18.22 miles. Time. $6^h.97$.

130. Current from noon positions.—To current is usually attributed the discrepancy between the noon positions at sea by observation and by dead reckoning, or, at any instant, the difference between the position by dead reckoning and one obtained by bearings of known landmarks.

The distance between the two positions divided by the number of hours elapsed since leaving a position, assumed to be correct, will give the hourly rate of the current; the bearing

✓

of the position by observation from that by dead reckoning being the set, or direction of the current.

It must not be forgotten, however, that the current, thus computed and so called, may be due to careless steering, improper logging or determination of the speed, or to errors of observation, rather than to any real motion of the waters of the sea.

Ex. 50.—On April 10, a vessel's noon position by observation was Lat. 40° 44′ N., Long. 47° 12′ 30″ W.; by D. R. Lat. 40° 37′ N., Long. 46° 51′ 48″ W. Find set and drift of current since preceding noon.

Lat. by obs. 40° 44′ 00″ N	Long. by obs. 47° 12′ 30″ W
" D. R. 40 37 00 N	" D. R. 46 51 48 W
$l =$ 7′ N	$D = 20'.7 = 20'\ 42''$ W
$L_0 = 40\frac{2}{3}°$ N	$p = 15'.7$ W

$$\text{Current}\begin{cases}\text{Set, } 294°.\\ \text{Drift, 17.2 miles in 24 hours.}\end{cases}$$

In the above example, since the position by observation is to the northward and westward of that by dead reckoning, or account, it is evident that the ship was set to the northward and westward by the current, therefore mark l as N., and D and p as W. For the middle latitude $40\frac{2}{3}°$, considered as a course, find D in the distance column of Table 2, and opposite D, take p out of the latitude column. With l and p find the corresponding C and d, or, in other words, the set and drift of the current.

Ex. 51.—At noon on Jan. 10, the ship's position by observation was Lat. 25° 43′ 12″ S., Long. 104° 52′ 38″ E. The position by D. R. from previous noon was Lat. 25° 52′ 48″ S., Long. 104° 30′ 24″ E. Find the set and drift of current.

Lat. by obs. 25° 43′ 12″ S	Long. by obs. 104° 52′ 38″ E
" D. R. 25 52 48 S	" D. R. 104 30 24 E
$l = 9'.6 = 9'\ 36''$ N	$D = 22'.23 = 22'\ 14''$ E
$L_0 = 25\frac{3}{4}°$ S	$p = 20'.02$ E

$$\text{Current}\begin{cases}\text{Set, } 64°.4.\\ \text{Drift, 22.2 miles in 24 hours.}\end{cases}$$

In this example the true position is to the northward and eastward of that by account, therefore the ship was set to northward and eastward, and we must mark l, N.; D and p, E.

131. Tidal currents.—The navigator should pay careful attention to the subject of tidal currents, and shape his course, or work his reckoning, to make due allowance for the possible set and drift, in all localities where such currents have been investigated. Much information may be found on charts, in sailing directions and in tide tables.

Finding from the tide tables the times of high and low waters at places along a coast, it may often be possible to make allowance, during a run at such times, for a set towards or from that coast.

When the wind has been strong and steady from one direction for any length of time, a current may be produced setting directly to leeward, or, if already existing, its rate may be greatly increased. The navigator should anticipate and endeavor to allow for its effect.

Current caused by wind.

MERCATOR SAILING.

132. It has been shown that the methods of middle latitude sailing are sufficiently exact for short distances, a day's run for instance, but for finding the difference of longitude between two places widely separated in latitude, or for finding the course between two such places, it is liable to great error. To avoid such errors resort is had to Mercator's sailing, which is based on principles fully explained in Art. 26, and applied in the construction of the Mercator chart, and which furnish the formula that gives practically correct results.

On the Mercator chart, the meridians are drawn parallel to each other and perpendicular to the equator and parallels of latitude, so arcs on parallels are represented as equal to the corresponding arcs of the equator, or differences of longitude; in other words, expanded in a certain ratio. In order that the rhumb line on the chart may make the same angle with each meridian, each infinitesimal element of latitude must be expanded in the same ratio in which each infinitesimal element of the parallel has been expanded. If the earth were a perfect sphere, this ratio would be as the secant of the latitude, but as the earth is a spheroid, its eccentricity must be considered.

The formula from Art. 27, $D = M \tan C$, or

$$D = \tan C \left[7915'.704 \left(\log_{10} \tan\left(\frac{\pi}{4} + \frac{L}{2}\right) - e \log_{10} \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \right) \right]$$

gives the relation existing in Mercator sailing between the constant course C, the latitude L of a point on the loxodrome, and the difference of longitude of that point and the longitude in which the loxodrome crosses the equator. M in the equation is the augmented latitude, or the length of the line on the Mercator chart indicating the latitude, expressed in nautical miles, according to the scale of the chart.

If it is desired to find the difference of longitude between two places in two different latitudes L_1 and L_2, substitute in the equation successively the values L_1 and L_2,

letting M_1 be the augmented latitude corresponding to L_1;
" M_2 be the augmented latitude corresponding to L_2;
" D_1 be the difference of longitude from A (Fig. 76), where track crosses the equator, to the first point in latitude L_1;
" D_2 be the same to second point L_2.

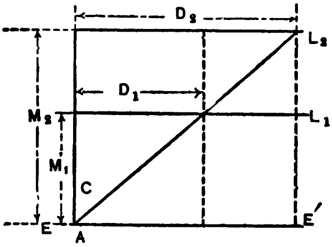


FIG. 76.

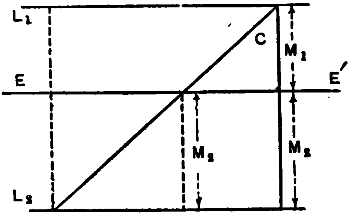


FIG. 77.

In Fig. 76, let EE' be the equator, L_1 the parallel of 1st latitude, L_2 the parallel of 2d latitude, C the constant course,

$$\text{then, } D_1 = M_1 \tan C.$$
$$\underline{D_2 = M_2 \tan C.}$$
$$D = D_2 - D_1 = (M_2 - M_1) \tan C = m \tan C. \qquad (122)$$

m equals the meridional difference, or augmented difference of latitude between L_1 and L_2, and is the length of the line on the Mercator chart which represents the true difference of latitude between L_1 and L_2, expressed in nautical miles, according to the scale of the chart.

Table 3 of Bowditch is a table of meridional parts at intervals of one minute of arc up to 80°, compression having been taken as $\frac{1}{293.465}$. In case L_1 and L_2 are of different names, as in Fig. 77, where EE' equals the equator, M_1 and M_2 are of different names, and the algebraic difference $M_2 - M_1$ becomes $M_2 + M_1$. Therefore,

$$D = D_2 + D_1 = (M_2 + M_1) \tan C.$$

Graphic illustration of the theory of Mercator sailing.—Let $C'E'$ represent an arc of the equator, and CA represent a distance sailed on a rhumb line from C in Lat. L_1 to A in Lat. L_2, shown on the spheroid in Fig. 78, and on the Mercator chart of the corresponding limits in Fig. 79.

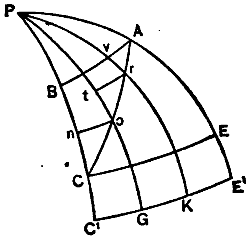


FIG. 78.

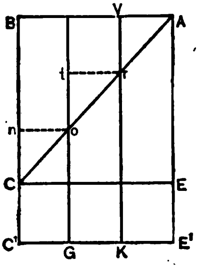


FIG. 79.

Conceive this distance to be subdivided into a large number of small parts, and the elementary triangles to be formed of which the corresponding differences of latitude l_1, l_2, etc., are represented in Fig. 78 by *Cn, ot,* etc., and the departures by *no, tr,* etc.

Each partial departure of Fig. 78 is represented in Fig. 79, a section of a Mercator chart, as an expanded arc equal to the corresponding arc of the equator, *no* equal to *C'G*, *tr* to *GK*, etc.; so that the departure on the spheroid, being equal to the sum of the partial departures, is expanded into the corresponding difference of longitude on the Mercator chart.

But, in order that the angle C on the chart shall remain constant and equal to that on the spheroid, and, that the similarity of the corresponding elementary triangles may be maintained, the ratio of increase of each partial difference of latitude must be the ratio of expansion of each partial departure, and the true difference of latitude CB (Fig. 78) be represented by CB on the chart (Fig. 79).

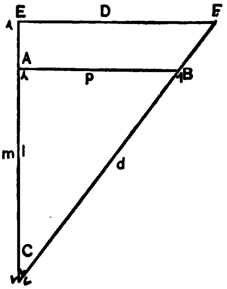


FIG. 80.

The triangles of Mercator and plane sailing.—The parts involved in Mercator sailing may be represented by a right triangle CEF, CE being the augmented difference of latitude m, representing the true difference of Lat. $CA = l$; if AB is drawn parallel to EF, ABC will be the triangle of plane sailing, AB the departure, and CB the true distance of which the expansion on the Mercator chart is CF, since the ratio between l and m is the same as that between p and D. It is thus seen that the triangle CEF furnishes the formula for converting departure into difference of longitude without making the false assumptions of middle latitude sailing.

From Fig. 80 all the formulæ necessary for Mercator sailing can be deduced.

$$\left.\begin{array}{l}\text{From triangle } CEF,\ D = m \tan C.\\ \text{From triangle } ABC,\ d = l \sec C.\end{array}\right\} \qquad (123)$$

Various problems under Mercator sailing may be solved by the above formulæ, but those of actual practice may be said to be:

Case I.—Required the C and d between two places of known position.

$$\text{Formulæ: } \tan C = \frac{D}{m},\ d = l \sec C.$$

Case II.—Required the latitude and longitude in, after sailing a true C and d from a place of known position.

$$\text{Formulæ: } l = d \cos C,\ L_2 = L_1 + l,$$
$$D = m \tan C,\ \lambda_2 = \lambda_1 + D.$$

As the results by Mercator sailing and by middle latitude sailing do not differ sensibly for small distances, the use of Mercator sailing comes principally under Case I, when the two places are far apart.

When not to use Mercator sailing.—Since in Mercator sailing the difference of longitude is found from a formula involving tan C, and tangents of angles near 90° change very rapidly, it is seen that any error in m, the meridional difference of latitude, is greatly increased as an error in difference of longitude when the course approaches 90° or 270°. In such cases use middle latitude sailing.

Use of traverse table.—Problems in Mercator sailing can be solved by the traverse table; the difference of longitude and meridional difference of latitude, being respectively the sides opposite and adjacent in a right triangle, should be looked for in the dep. and diff. of lat. columns, respectively. In using this table where long distances are involved, the quantities given may all be reduced by a common divisor till within the limits of d, l, and p as tabulated, and the results afterwards correspondingly enlarged. This, however, will involve some error in results.

Graphic solution of problems in Mercator sailing are made in every-day navigation, when the reckoning is kept by construction on the Mercator chart as fully explained in Art. 31.

Ex. 52.—Find the true course and distance by Mercator sailing from

$$\left.\begin{array}{l} L_1 = 10^\circ\ 36'\ \text{N} \\ \lambda_1 = 56\ \ \ 34\ \ \text{W} \end{array}\right\} \text{to} \left.\begin{array}{l} L_2 = 36^\circ\ 30'\ \text{N} \\ \lambda_2 = 15\ \ \ 22\ \ \text{W} \end{array}\right\}$$

By computation.—

$$\begin{array}{lll} \quad\ \ \ {}^\circ\ \ \ {}' & & \quad\ \ \ {}^\circ\ \ \ {}' \\ L_1 = 10\ 36\ \text{N} & M_1 = \ \ 635.4 & \lambda_1 = 56\ 34\ \text{W} \\ L_2 = \underline{36\ 30}\ \text{N} & M_2 = \underline{2341.3} & \lambda_2 = \underline{15\ 22}\ \text{W} \\ \ l = 25\ 54\ \text{N} & \ m = 1705.9 & D = 41\ 12\ \text{E} \\ \quad = 1554'\ \text{N} & & \quad = 2472'\ \text{E} \end{array}$$

$$\begin{array}{lll} l = 1554'\ \text{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \text{log}\ \ 3.19145 \\ D = 2472'\ \text{E} \ldots\ldots\ldots\ \text{log}\ \ 3.39305 & & \\ m = 1705.9 \ldots\ldots\ldots\ \text{log}\ \ \underline{3.23195} & & \\ C = \text{N}\ 55^\circ\ 23'\ 28''\ \text{E} \ldots\ \text{tan}\ 10.16110 & \ldots\ldots\ldots & \text{sec}\ \underline{10.24568} \\ d = 2736.1\ \text{miles} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \text{log}\ \ 3.43713 \end{array}$$

$$\textit{Ans.}\left\{\begin{array}{l} \text{Course, } C_\text{N} = 55^\circ\ 23'\ 28''. \\ \text{Distance, } 2736.1 \text{ miles.} \end{array}\right.$$

By inspection.—Enter Table 2. Turn to that page where will be found the nearest coincidence, D in dep. col. and m in diff. lat. col. Now, by interpolation,

$$\begin{array}{lll} D = 247.2, & m = 173.05; & C = 55^\circ \\ D = 247.2, & m = 166.66; & C = 56^\circ \\ \hline D = 247.2, & m = 170.59; & C = 55^\circ.38 \end{array}$$

Therefore, by inspection, course is $C_\text{N} = 55^\circ.38$.

Now with the course and $l = 155.4$ find d.

$$\begin{array}{lll} \text{For } C_\text{N} = 55^\circ, & l = 155.4; & d = 271 \\ C_\text{N} = \underline{56^\circ}, & l = \underline{155.4}; & d = \underline{277.8} \\ \text{Therefore, for } C_\text{N} = 55^\circ.38, & l = 155.4; & d = 273.58. \end{array}$$

Having used a divisor of 10 originally, the true distance by inspection is 2735.8 miles.

Of course, the above interpolation, recorded for illustration, is supposed to be done mentally.

Ex. 53.—Find true C_N and d from Brisbane to Acapulco.

$$
\begin{array}{lll}
\qquad\quad {}^\circ\ \ {}'\ \ {}'' & & \qquad\quad {}^\circ\ \ {}'\ \ {}'' \\
L_1 = 27\ 27\ 32\ \text{S} & M_1 = 1703.7\ \text{S} & \lambda_1 = 153\ 01\ 48\ \text{E} \\
L_2 = \underline{16\ 49\ 10\ \text{N}} & M_2 = \underline{1017.2\ \text{N}} & \lambda_2 = \underline{\ 99\ 55\ 50}\ \text{W} \\
l = 44\ 16\ 42\ \text{N} & m = 2720.9\ \text{N} & D = 107\ 02\ 22\ \text{E} \\
\ \ = 2656'.7\ \text{N} & & \ \ = 6422'.37
\end{array}
$$

$$
\begin{array}{llll}
l = 2656.7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \log\ 3.42434 \\
D = 6422'.37 \ldots\ldots\ldots\ldots & \log\ 3.80770 & & \\
m = 2720.9 \ldots\ldots\ldots\ldots & \log\ \underline{3.43471} & & \\
C = \text{N}\ 67^\circ\ 02'\ 24''\text{E} \ldots & \tan\ 10.37299 \ldots\ldots\ldots & & \sec\ \underline{10.40884} \\
d = 6810.5\ \text{miles} \ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \log\ 3.83318
\end{array}
$$

Ans. $\begin{cases} \text{Course } C_N = 67^\circ\ 02'\ 24''. \\ \text{Distance } 6810.5 \text{ miles.} \end{cases}$

133. To find the value of the correction to the middle latitude.—In middle latitude sailing, it was stated that the formula $D = p \sec L_0 = l \tan C \sec L_0$ was not strictly correct, but that it would be correct, if, to L_0, was applied a correction ΔL, such that the formula

$$D = l \tan C \sec (L_0 + \Delta L)$$

would give the same result as $D = m \tan C$.

From these two may be gotten

$$\frac{l}{m} = \cos (L_0 + \Delta L) = 1 - 2 \sin^2 \left(\frac{L_0 + \Delta L}{2}\right),$$

$$\text{and, } \sin \left(\frac{L_0 + \Delta L}{2}\right) = \sqrt{\frac{m - l}{2m}},$$

$$L_0 + \Delta L = 2 \sin^{-1} \sqrt{\frac{m - l}{2m}},$$

$$\Delta L = 2 \sin^{-1} \sqrt{\frac{m - l}{2m}} - L_0. \tag{124}$$

Values of this correction have been tabulated where the arguments are the middle latitude and the difference of latitude. *It has already been stated that for small values of l, it is unimportant; and that in those cases where its use might be desirable, it would be better to use Mercator sailing.*

Examples in Mercator Sailing.
(By computation.)

Ex. 54.—Find true course and distance from L_1 50° 53′ N., λ 156° 46′ E., to L_2 12° 04′ S., λ_2 77° 14′ W.

Ans. $\begin{cases}\text{Course } C_N = 119° \ 25' \ 28''. \\ \text{Distance 7688.35 miles.}\end{cases}$

Ex. 55.—Find true course and distance from L_1 42° 20′ N., λ_1 31° 30′ W., to L_2 56° 40′ N., λ_2 20° 40′ W.

Ans. $\begin{cases}\text{Course } C_N = 25° \ 59' \ 04''. \\ \text{Distance 956.7 miles.}\end{cases}$

Ex. 56.—Find true course and distance from L_1 45° 02′ S., λ_1 20° 19′ W., to L_2 65° 20′ S., λ_2 18° 37′ W.

Ans. $\begin{cases}\text{Course } C_N = 177° \ 19' \ 46''. \\ \text{Distance 1219.4 miles.}\end{cases}$

Ex. 57.—A ship sails from Lat. 15° 20′ N., Long. 24° 20′ W., 135° (true) a distance of 2500 miles. Find Lat. and Long. in.

Ans. $\begin{cases}L_2 = 14° \ 07' \ 48'' \text{ S.} \\ \lambda_2 = \ \ 5 \ \ \ 15 \ \ \ 48 \text{ E.}\end{cases}$

Ex. 58.—Find the true course and distance by Mercator sailing from a point in Lat. 35° 30′ N., Long. 140° 52′ E. (off Cape Inaboye, Japan), to a point in Lat. 33° S., Long. 71° 49′ W. (off Valparaiso). See Ex. 63 and Plate V.

Ans. $\begin{cases}\text{Course } C_N = 116° \ 13' \ 32''. \\ \text{Distance 9300.55 miles.}\end{cases}$

DAY'S WORK BY D. R.

134. In most works on navigation, the subject of "Day's Work" follows the sailings, and is considered without reference to positions by observation; as these are an essential part of the data used in the daily work of a navigator, this general subject is reserved till after the chapters on latitude and longitude by observations have been studied and understood.

However, it must be recalled that all the calculations entering into the daily dead reckoning itself have been made, and the methods used have been treated, under the head of pilotage, or, of the several sailings. Such are the various methods of fixing the ship's position near land after leaving port; taking departure; use of departure course and distance as a course and distance of the traverse; correction for leeway, variation, and deviation, of the various courses indicated in the ship's log book; entry of the true courses and the distances sailed on each in the proper columns of the tabulated form; consideration of the set and drift of a known current as a separate course and distance of the traverse; finding the resultant difference of latitude and departure; the resultant course and distance; conversion of departure into difference of longitude; finding by D. R. the latitude and longitude at end of run, and the course and distance to port of destination.

It has been shown that the noon position by observation is the true place from which to begin the dead reckoning of the following day; and, in case of a discrepancy between it and the position by D. R., that this discrepancy, if not due to incidental errors of navigation, may be attributed to current, the set and drift of which correspond to the course and distance from the noon position by D. R. to that by observation.

For the solution of a day's work in which positions by observation are used, see Chapter XXI.

In the following example, a day's work by D. R. is illustrated.

Ex. 59.—Making passage from Hampton Roads to Block I. Sound, January 15, 1918, about 1 p. m., took departure from Cape Charles light ship (Lat. 37° 05′ 17″ N., Long. 75° 43′ 29″ W.), bearing (p. s. c.) 271°, distant 8 miles, ship's head East (p. s. c.); dev. —4°, var. —5°. Sailed thence till noon next day the following courses and distances which, with other data, will be found in the form. Required the Lat. and Long. in by D. R., C_N and *d* from the light ship by D. R., and C_N and *d* from the noon position to a point in Block I. Sound, in Lat. 41° 15′ N., Long. 71° 10′ W., by middle latitude sailing.

Wind.	Course (p. s. c.).	Var.	Dev.	Leeway.	True Courses.	Distances.	Diff. of Lat.		Departure.	
							N	S	E	W
	True	bearing L	t. Ship rev	ersed =	82°	8	1.1	....	7.9	....
Nly and Wly	84°	—5°	—4°	0°	75	10	2.6	....	9.7	....
do	79	—5	—4	0	70	15	5.1	....	14.1	....
do	67	—5	—3	0	59	40	20.6	....	34.3	....
do	20	—6	+1	5	20	48.5	45.6	....	16.6	....
Nly	286	—6	+3	5	278	12	1.7	....	....	11.9
Nly and Wly	57	—7	—2	5	53	12	7.2	....	9.6	....
Nly and Ely	87	—7	—4	8	84	20	2.1	....	19.9	....
do	311	—7	+2	5	301	10	5.2	....	....	8.6
							l=91.2	N	112.1 20.5	20.5
								p=	91.6	E

Course by D. R. C_N=45°. Distance 129 miles.

$$\begin{array}{rl} & \;\;\circ\;\;\;\prime\;\;\;\prime\prime \\ L_1 = & 37\ 05\ 17\ \text{N} \\ l = & 1\ 31\ 12\ \text{N} \\ \hline L_2 = & 38\ 36\ 29\ \text{N} \\ L_0 = & 37\ 50\ 53\ \text{N} \end{array} \qquad \begin{array}{rl} & \;\;\circ\;\;\;\prime\;\;\;\prime\prime \\ \lambda_1 = & 75\ 43\ 29\ \text{W} \\ D = & 1\ 56\ 01\ \text{E} \\ \hline \lambda_2 = & 73\ 47\ 28\ \text{W} \end{array}$$

$$\begin{array}{lrl} & & \;\;\circ\;\;\;\prime\;\;\;\prime\prime \\ \text{Lat. of point} & = & 41\ 15\ 00\ \text{N} \\ \text{Lat. in} & = & 38\ 36\ 29\ \text{N} \\ \hline \text{Diff. lat.} & = & 2\ 38\ 31\ \text{N} \\ & = & 158.52\ \text{N} \\ & L_0 = & 39\ 55\ 44\ \text{N} \end{array} \qquad \begin{array}{lrl} & & \;\;\circ\;\;\;\prime\;\;\;\prime\prime \\ \text{Long. of point} & = & 71\ 10\ 00\ \text{W} \\ \text{Long. in} & = & 73\ 47\ 28\ \text{W} \\ \hline \text{Diff. long.} & = & 2\ 37\ 28\ \text{E} \\ & = & 157.47\ \text{E} \\ & p = & 120.9\ \text{E} \end{array}$$

True course to point $C_N = 87\tfrac{1}{3}°$.

Distance 199.4 miles.

CHAPTER VII.

GREAT CIRCLE SAILING.—COMPOSITE SAILING.—GRAPHIC METHODS.

135. **Great circle sailing** is the method of solving problems of navigation that arise from a ship following a great circle track from point of departure to destination. The arc of the great circle passing through two places is the shortest distance between these places, so that a rhumb line is always longer than the great circle distance, except when it coincides with a meridian or the equator.

The rhumb line has long been used by navigators because of the constancy of the course, the ease with which it can be laid down on or taken from a Mercator chart, and the simplicity of the calculations it involves. However, steam vessels, unlike sailing vessels, are independent of winds and currents, and are capable of following that route which means a saving of distance and of time; so it is fair to presume that in the future the great circle will be followed as closely as possible unless it goes into latitudes too high, meets lands, or passes through regions of ice and dangerous navigation.

Comparison of tracks.—The difference in distance and, hence, the saving of time is less when the loxodrome approaches a great circle, as is the case between places near the equator, or near the same meridian; the contrary holds, however, for places in high latitudes, especially when differing much in longitude; a case most remarkable for saving of distance and great divergence between the two tracks occurs when the two places are on the same parallel, but differ 180° in longitude. The Mercator course is either East or West, while the great circle course is North or South, across the elevated pole and 90° away from the former.

A rhumb line laid down on a Mercator chart passes directly through the point of destination; the great circle track plotted on the same chart will be a circuitous path, nearer the pole than the Mercator track, and often going into higher latitudes than is practicable for safe navigation. The great circle track between two places in different hemispheres has a double curvature when plotted on the Mercator chart, the curve in each hemisphere being the same in its entirety. However, a ship following the rhumb line, steers the same course, makes the same angle with each successive meridian, but never heads directly for the port till it is in sight, or till the end of the voyage; another ship following the great circle track always heads for the port, but does so by steering a constantly changing course.

Definitions.—The great circle course is the angle which the great circle passing through a place makes with the meridian of that place. The initial course is the angle which the great circle through points of departure and destination makes with the meridian of point of departure; the final course is the angle which it makes with the meridian of destination.

The distance is the length of the arc of the great circle which forms the path of the ship between the two points, expressed in nautical miles.

Vertices.—In accordance with geometrical principles, the equator bisects the great circle passing through the two places, and that point of the circle in each hemisphere which is farthest from the equator is the vertex in that hemisphere; in other words, the vertices are the points of highest latitude.

Only one vertex is considered, and that is in the hemisphere whose pole determines the course. The vertex may or may not be between the two places. If the initial and final courses are both less than 90°, the vertex falls between the two places; if one is greater than 90°, the vertex falls on the arc produced 180° from this course.

Points of maximum separation.—Since all points of the great circle track betwen two places in the same hemisphere, except those of departure and destination, are nearer the pole than the rhumb line, it follows that there must be some one point where the distance along a meridian between the two tracks is greatest, and this is the point of maximum separation. It is apparent that at this point the courses on the two tracks are equal. Hence, knowing the rhumb course, it is only necessary to find that point of the arc where the great circle course would equal it.

Finding great circle course and distance.—There are four general methods for solving the great circle problem:

(1) By computation.
(2) By azimuth tables.
(3) By great circle charts.
(4) By graphic approximation.

By computation.—The problem consists in the solution by spherical trigonometry of a spherical triangle, formed by the meridians passing through the two places and the great circle arc forming the ship's track. The lengths of the two sides are known, being equal to the co. latitudes of the two places; the included angle at the pole is the difference of longitude of the two places; so the triangle, having two sides and the included angle given, may be solved by Napier's Analogies, but preferably by Napier's Rules.

To plot the curve, not only the vertex, but a series of points along the curve should be determined by their coordinates; and, having been plotted on the Mercator chart, the curve may be traced through them.

In Fig. 81, let *A* be the point of departure, *B* the point of destination, *AB* the great circle passing through them, *V* its

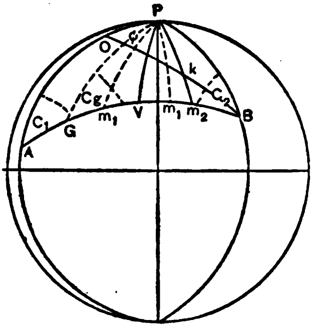


FIG. 81.

vertex, m_1, m_2, etc., points along the curve, P the elevated pole. Position of A, Lat. L_1, Long. λ_1; therefore, $AP = 90° - L_1$. Position of B, Lat. L_2, Long. λ_2; therefore, $PB = 90° - L_2$. APB = difference of longitude of A and $B = \lambda_2 \sim \lambda_1$. C_1 is the initial course; C_2, the final course. d = distance AB. Drop a perpendicular $BO (= k)$ on AP, so that one triangle PBO shall include two of the known parts, and the triangle ABO shall include the required C_1 and d. Also, dividing AP into two parts, $PO = \phi$ and $OA = 90° - (L_1 + \phi)$.

To find the initial course C_1.—Applying Napier's Rules to triangle POB, $\cos(\lambda_2 \sim \lambda_1) = \tan \phi \tan L_2$,

or, $$\tan \phi = \cos(\lambda_2 \sim \lambda_1) \cot L_2, \qquad (125)$$

also, $$\left.\begin{array}{r}\sin \phi = \cot(\lambda_2 \sim \lambda_1) \tan k;\\ \text{to triangle } AOB,\ \cos(L_1 + \phi) = \cot C_1 \tan k;\end{array}\right\}$$

therefore, $$\cot C_1 = \cot(\lambda_2 \sim \lambda_1) \cos(L_1 + \phi) \operatorname{cosec} \phi. \qquad (126)$$

To find the distance $d = AB$.—Proceeding as above,

$$\sin L_2 = \cos \phi \cos k,$$

$$\cos d = \sin(L_1 + \phi) \cos k,$$

therefore, $$\cos d = \sin(L_1 + \phi) \sin L_2 \sec \phi. \qquad (127)$$

The distance is found in degrees, minutes, etc., of a great circle which will be reduced to minutes for distance in nautical miles.

PV is the arc of a meridian perpendicular to the great circle track; therefore PVA and PVB are right angles.

V is the vertex; Lat. L_v, Long. λ_v.

The vertex lies between A and B, unless either C_1 or C_2 is $> 90°$.

Alternative formulæ for finding the distance and course.—The following formula, obtained from (240) by putting $z = d$ (the distance), $L = L_1$, d (the declination) $= L_2$, and $t = \lambda_1 - \lambda_2$, is often preferred for computing the great circle distance, being a general expression for finding the third side of a spherical triangle in which two sides and the included angle are given:

$$\text{hav } d = \text{hav}(L_1 \sim L_2) + \cos L_1 \cos L_2 \text{ hav}(\lambda_1 - \lambda_2).$$

The distance AB having been found, the three sides of the spherical triangle APB become known, as well as the angle at P, hence the initial and final courses may be found from the proportionality between the sines of the angles of the triangle and the sines of the opposite sides,

$$\sin C_1 = \frac{\sin P \times \sin PB}{\sin AB} = \frac{\sin(\lambda_1 - \lambda_2)\cos L_2}{\sin d} = \sin(\lambda_1 - \lambda_2)\cos L_2 \operatorname{cosec} d,$$

and

$$\sin C_2 = \frac{\sin P \times \sin PA}{\sin AB} = \frac{\sin(\lambda_1 - \lambda_2)\cos L_1}{\sin d} = \sin(\lambda_1 - \lambda_2)\cos L_1 \operatorname{cosec} d.$$

To find the latitude and longitude of the vertex.—In the right triangle APV, $PV = 90° - L_v$ and $APV = \lambda_v \sim \lambda_1$, therefore, by Napier's Rules,

$$\cos L_v = \cos L_1 \sin C_1, \tag{128}$$

$$\left.\begin{aligned} \sin L_1 &= \cot C_1 \cot(\lambda_v \sim \lambda_1), \\ \cot(\lambda_v \sim \lambda_1) &= \sin L_1 \tan C_1. \end{aligned}\right\} \tag{129}$$

To find the latitude and longitude of other points of the curve.—Assume meridians Pm_1, Pm_2, etc., differing in longitude θ_1, θ_2, etc. (say 5° or 10°, if desired), from the longitude of vertex, and solve the right triangles thus formed by Napier's Rules.

Therefore,
$$\left.\begin{aligned} \tan L_{m_1} &= \cos\theta_1 \tan L_v, \\ \tan L_{m_2} &= \cos\theta_2 \tan L_v, \text{ etc.} \end{aligned}\right\} \tag{130}$$

Each of these last formulæ will give a position each side of the vertex, or two points of the curve; thus, from first equation,

Lat. m_1, Long. $(\lambda_v - \theta_1)$ and Long. $(\lambda_v + \theta_1)$,

from second equation,

Lat. m_2, Long. $(\lambda_v - \theta_2)$ and Long. $(\lambda_v + \theta_2)$.

It must not be forgotten that the course is ever varying, and that the course to be steered at any meridian is the angle which that meridian makes with the track. By the solution, the angle will be found from the elevated pole towards the East or West, but, if it is found to be greater than 90°, as when the vertex is, or has been, left behind, it may be convenient to name the course as from the depressed pole, or the supplement of the angle found by computation in the triangle of which the elevated pole is one point.

To find the course in any longitude λ_g.—In Fig. 81, PVG is a right triangle, $PV = 90° - L_v$, $VPG = \lambda_v \sim \lambda_g$. Let C_g be the course in Long. λ_g, and C_q be the course at the equator; then, by Napier's Rules,

$$\cos C_g = \sin L_v \sin (\lambda_v \sim \lambda_g), \tag{131}$$

and for the final course C_2 in Lat. L_2, Long. λ_2,

$$\cos C_2 = \sin L_v \sin (\lambda_v \sim \lambda_2). \tag{132}$$

At the point of crossing the equator $\sin (\lambda_v \sim \lambda_q) = \sin 90°$, therefore, at the equator,

$$C_q = \text{Co.}\, L_v, \tag{133}$$

and

$$\lambda_q = \lambda_v \pm 90°. \tag{134}$$

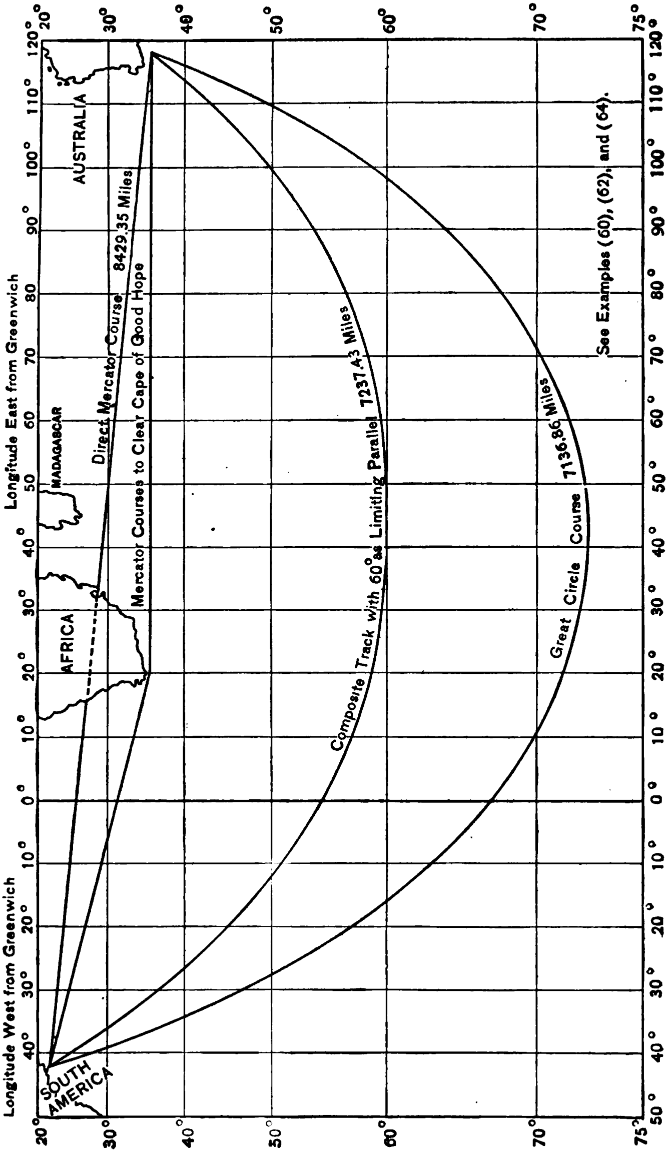
Precautions.—In solving the triangle, let the elevated pole (the pole of that hemisphere in which lies the position $L_1\lambda_1$) be at one angle of the triangle, regard L_1 as positive and L_2, when of a different name from L_1, as negative. Strict regard must be had to the signs of the functions. ϕ may be taken out as positive up to 180°; or, if $\tan \phi$ is negative, instead of taking ϕ in the second quadrant, it may be regarded as negative, the foot of the perpendicular falling the other side of the pole. The angle C_1 will be found with its correct value, if attention is paid to the signs. The course is from the elevated pole, East or West, as B is East or West of A.

C_g and C_2 are reckoned from the **elevated pole when approaching the vertex and from the depressed pole when going away from it,** toward East or West according as the ship is proceeding eastward or westward.

The fact of the vertex being ahead or astern is determined by a comparison of its longitude with that of the meridian from which the course is taken.

Though, when considering the above mentioned courses as angles of a spherical triangle, the method given is the proper one to pursue in the solution of that triangle; still, as soon as found, the course, for practical purposes, should be expressed in the more convenient form of C_N which is measured from North, around to the right, from 0° to 360°. C_N will be simply $x°$, $180° - x°$, $180° + x°$, or $360° - x°$, according as the course by solution is N. $x°$ E., S. $x°$ E., S. $x°$ W., or N. $x°$ W., respectively.

PLATE IV.



Longitude East from Greenwich
Longitude West from Greenwich
120° 110° 100° 90° 80 70 60 50 40° 30° 20° 10° 0° 10° 20° 30° 40°
20° 30° 40° 50° 60° 70° 75°
AUSTRALIA
MADAGASCAR
AFRICA
SOUTH AMERICA
Direct Mercator Course 8429.35 Miles
Mercator Courses to Clear Cape of Good Hope
Composite Track with 60° as Limiting Parallel 7237.43 Miles
Great Circle Course 7136.86 Miles
See Examples (60), (62), and (64).

Ex. 60.—Find the great circle initial course and distance from a point off the SW. coast of Australia in Lat. 35° 40′ S., Long. 118° 06′ 07″ E., to a point off the Brazilian coast in Lat. 22° 15′ S., Long. 41° 30′ W. Also Lat. and Long. of the vertex.

In this example both L_1 and L_2 are +, $\lambda_2 \sim \lambda_1$ = 159° 36′ 07″.

$\lambda_2 \sim \lambda_1$	= 159° 36′ 07″	cos	−9.97188	cot	−10.42961				
L_2	= 22 15 S	cot	10.38816			sin	9.57824		
ϕ	= 113 34 48	tan	−0.36004	cosec	10.03786	sec	−10.39791		
L_1	= 35 40 S							cos 9.90978	sin 9.76572
$L_1 + \phi$	= 149 14 48			cos	− 9.93418	sin	9.70871		
C_1	= S 21 38 00 W			cot	+10.40165			sin 9.56663	tan 9.59835
d	= 118 56 51.5 = 7136.86 miles					cos	−9.68486		
L_v	= 72 34 20 S							cos 9.47641	
$\lambda_v \sim \lambda_1$	= 76 58 49								cot 9.36407
λ_1	= 118 06 07 E								
λ_v	= 41 07 18 E								
C_N	= 201 38 00								

See plate (IV).

The solution of this example, for the required distance and course, by the alternative formulæ is as follows:

Distance.						Course.	
$\lambda_1 - \lambda_2$ =	159°	36′	07″	log hav	= 9.98616	log sin	= 9.54225
L_1 =	35	40	00 S	log cos	= 9.90978		
L_2 =	22	15	00 S	log cos	= 9.96640	log cos	= 9.96640
				θ log hav	= 9.86234		
				θ nat hav	= 0.72835		
$L_1 \sim L_2$ =	13	25	00	nat hav	= 0.01365		
Distance, d =	118	56	50	nat hav	= 0.74200	log cosec	= 10.05796
= 7136.8						log sin C_1	= 9.56661
						Course C_1	= S. 21° 38′ W.

Ex. 61.—In the above example find the coordinates of two points of the curve, one in a longitude 40° to eastward of the vertex, the other 40° to the westward. Find also the course at this western point and the final course (see Fig. 81).

Here $\theta = 40°$, $\lambda_v \sim \lambda_\theta = 40°$, $\lambda_v \sim \lambda_2 = 82° \; 37' \; 18''$, $\lambda_v \sim \lambda_g = 40°$.

$$\text{Formulæ} \begin{cases} \tan L_m = \tan L_v \cos \theta. \\ \cos C_g = \sin L_v \sin (\lambda_v \sim \lambda_g). \\ \cos C_2 = \sin L_v \sin (\lambda_v \sim \lambda_2). \end{cases}$$

$\sim \lambda_2$	$= 82°$	$37'$	$18''$	....	....	sin 9.99639
$- \lambda_g = \theta$	$= 40$			cos 9.88425	sin 9.80807	
	$= 72$	34	20 S	tan 10.50319	sin 9.97959	sin 9.97959
	$= 67$	43	00 S	tan 10.38744		
= N	52	10	22 W		cos 9.78766	
= N	18	52	54 W			cos 9.97598

rdinates of Points { East of vertex Lat. 67° 43′ 00″ S., Long. 81° 07′ 18″ E.
West of vertex Lat. 67 43 00 S., Long. 1 07 18 E.

Course at Western Point $C_N = 307° \; 49' \; 38''$.

Final course $C_N = 341° \; 07' \; 06''$.

Ex. 62.—In the above example, given $L_1 = 35° 40'$ S., $\lambda_1 = 118° 06' 07''$ E., $L_2 = 22° 15'$ S., $\lambda_2 = 41° 30'$ W., find the coordinates of points of the curve at intervals in longitude each side of the vertex of 10° up to 70°. Lat. of vertex 72° 34′ 20″ S., Long. of vertex 41° 07′ 18″ E.

Value of θ.	L' tan L.	L' cos θ.	L' tan L_m.	Latitude.	Longitudes.		
					East of Vertex.	West of Vertex.	
°				° ′ ″	° ′ ″	° ′ ″	
0	0.50319	0.00000	0.50319	72 34 20 S	41 07 18 E	41 07 18 E	(Vertex).
±10	0.50319	9.99335	0.49654	72 19 12 S	51 07 18 E	31 07 18 E	
±20	0.50319	9.97299	0.47618	71 31 40 S	61 07 18 E	21 07 18 E	
±30	0.50319	9.93753	0.44072	70 04 32 S	71 07 18 E	11 07 18 E	
±40	0.50319	9.88425	0.38744	67 43 00 S	81 07 18 E	1 07 18 E	
±50	0.50319	9.80807	0.31126	63 58 16 S	91 07 18 E	8 52 42 W	
±60	0.50319	9.69897	0.20216	57 52 43 S	101 07 18 E	18 52 42 W	
±70	0.50319	9.53405	0.03724	47 27 12 S	111 07 18 E	28 52 42 W	

To find the point of maximum separation.—The point of maximum separation between the great circle and Mercator track is the point where the two courses are the same.

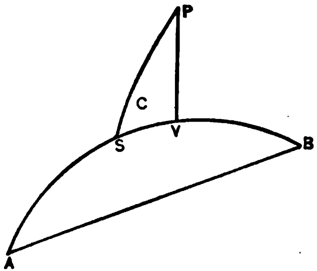
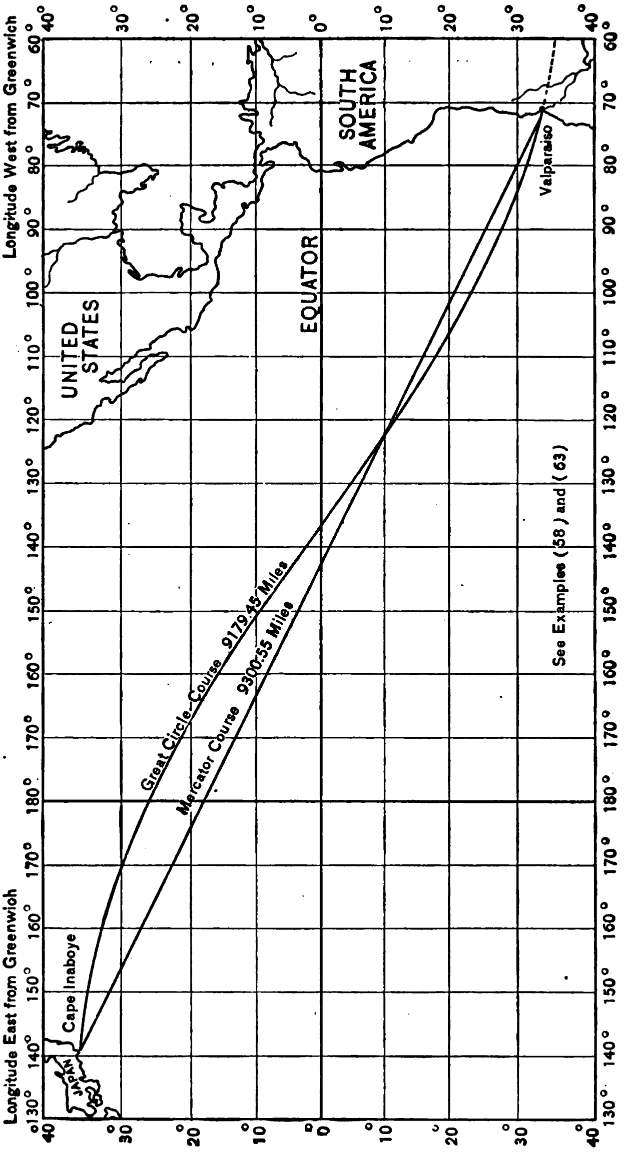


FIG. 82.

In Fig. 82, *AB* is the Mercator track, *ASB* the great circle route, on which *V* is the vertex and *S* the point of maximum separation. To find this latter point it is only necessary to solve the right triangle *VPS* for *PS* ($= 90° - L_{ms}$) and $\angle VPS$ ($= \lambda_v \sim \lambda_{ms}$), where L_{ms} is the Lat. and λ_{ms} the longitude of the point, having given $PV = 90° - L_v$ and $\angle PSV =$ the Mercator course. The point is one of but little practical value.

PLATE V.


Longitude West from Greenwich
60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180° 170° 160° 150° 140° 130°
Longitude East from Greenwich
40° 30° 20° 10° 0° 10° 20° 30° 40°
UNITED STATES
SOUTH AMERICA
EQUATOR
JAPAN
Cape Inaboye
Valparaiso
Great Circle Course 9179.45 Miles
Mercator Course 9300.55 Miles
See Examples (58) and (63)

Ex. 63.—**Find the great circle initial course and distance from a point in Lat. 35° 30′ N., Long. 140° 52′ E. (off Cape Inaboye, Japan), to a point in Lat. 33° S., Long. 71° 49′ W. (off Valparaiso). (b) Given the Mercator course between the above points $C_N = 243° 46′ 28″$, find the points of maximum separation. (c) Find the coordinates of points on the curve at intervals of 10° of longitude up to 80°, along the track, working from the vertices, and longitude in which the circle crosses the equator. (See example 58 and plate V).**

$\lambda_2 \sim \lambda_1$	= 147° 19′ 00″	$\overline{\cos}$ 9.92514	$\overline{\cot}$ 10.19275			
L_2	= 33 S	$\overline{\cot}$ 10.18748		$\overline{\sin}$ 9.73611		
ϕ	= 52 20 48	$\overset{+}{\tan}$ 10.11262	cosec 10.10143	sec 10.21405		
L_1	= 35 30 N				cos 9.91069	sin 9.76395
$L_1 + \phi$	= 87 50 48		cos 8.57491	sin 9.99969		
$C_1 =$	N 94 13 50 E		$\overline{\cot}$ 8.86909		sin 9.99881	$\overline{\tan}$ 11.13093
d	= 152 59 27 = 9179.45 miles			$\overline{\cos}$ 9.94985		
L_v	= 35 43 07 N				cos 9.90950	
$\lambda_v \sim \lambda_1$	=(−)7 15 35					$\overline{\cot}$ 10.89488
λ_1	140 52 E	$L_v = 35° 43′ 07″$ S				
λ_v	133 36 25 E	$\lambda_v = 46\ 23\ 35$ W and $C_N = 94° 13′ 50″$.				

(b) To find the points of maximum separation, find the coordinates of the point of the curve in each hemisphere where the great circle course equals the Mercator course $C_N = 243° 46' 28''$.

L_v .... = 35° 43′ 07″......cos. 9.90950......cosec. 10.23373
Mer. C .... = S 63 46 28 W...cosec 10.04718........cos 9.64533

L_{ms} .... = 25 10 ....cos 9.95668

$\lambda_v - \lambda_{ms}$ .... = 49 11 45.......................... sin 9.87906

λ_v in North Lat. = 133° 36′ 25″ E λ_v in South Lat... = 46° 23′ 35″ W
$\lambda_v \sim \lambda_{ms}$...... = 49 11 45 $\lambda_v \sim \lambda_{ms}$ .... = 49 11 45

λ_{ms} in North Lat. = 177 11 50 W λ_{ms} in South Lat. = 95 35 20 W
Lat. .... = 25 10 N   Lat............. = 25 10 S

(c) The results of the solution of this subdivision of the example will be found tabulated on the opposite page.

Finding the great circle course from azimuth tables.—The azimuth of a heavenly body is tabulated in the tables with the arguments *L, d,* and *t*. Now, from the observer's position the azimuth of a heavenly body is the same as the great circle course to that terrestrial position having the same heavenly body in its zenith; so, to use the azimuth tables for finding the great circle course to a place, it is only necessary to substitute for the body's declination the latitude of destination, for the body's hour angle the difference of longitude between the places expressed in time, and to consider the latitude of departure as that of the observer. The rules for marking the azimuth apply for marking the course.

In a similar manner, "Weir's Azimuth Diagram" may be used in place of the azimuth tables.

Solution by gnomonic charts, or great circle sailing charts.—This subject has already been considered under the head of gnomonic charts (Art. 24). Especial reference is made to the gnomonic charts issued by the U. S. Hydrographic Office, on which are provided the means of determining the great circle

COORDINATES OF THE CURVE AT INTERVALS OF 10° OF LONG. FROM THE VERTICES.

Value of θ.	Log tan L_v.	Log cos θ.	Log tan L_m.	Coordinates in N Lat.		Coordinates in S Lat.	
				Latitude.	Longitude.	Latitude.	Longitude.
				° ′ ″	° ′ ″	° ′ ″	° ′ ″
Vertex.	9.85677			35 48 07 N	133 36 25 E	35 48 07 S	46 23 35 W
10°	9.85677	9.99335	9.85012	35 18 14 N	143 36 25 E	35 18 14 S	56 23 35 W
20	9.85677	9.97299	9.82976	34 02 51 N	153 36 25 E	34 02 51 S	66 23 35 W
30	9.85677	9.93753	9.79430	31 54 43 N	163 36 25 E	31 54 43 S	76 23 35 W
40	9.85677	9.88425	9.74102	28 50 50 N	173 36 25 E	28 50 50 S	86 23 35 W
50	9.85677	9.80807	9.66484	24 48 25 N	176 23 35 W	24 48 25 S	96 23 35 W
60	9.85677	9.69897	9.55574	19 46 30 N	166 23 35 W	19 46 30 S	106 23 35 W
70	9.85677	9.53405	9.39082	13 49 00 N	156 23 35 W	13 49 00 S	116 23 35 W
80	9.85677	9.23967	9.09644	7 07 03 N	146 23 35 W	7 07 03 S	126 23 35 W
90	9.85677	Inf. neg.	Inf. neg.	0 00 00	136 23 35 W	0 00 00 S	136 23 35 W

The longitude of the point of crossing the equator is 90° from that of either vertex, the point on the track between the vertices being 136° 23′ 35″ West.

course and distance directly, without transferring positions to a Mercator chart. Reference is also made to the polar chart (Art. 25), which is available for either hemisphere. Any meridian may be taken as that of Greenwich, and the two places, between which the great circle course and distance are desired, having been plotted, join them by a straight line. This line is the great circle track from which any number of coordinates may be transferred to a Mercator chart. The polar chart is especially available in the Southern hemisphere where great circle sailing possesses so many advantages.

Solution by graphic methods; Use of terrestrial globe.—Locate the two places on the globe, move it till both places coincide with the upper edge of the horizon circle. Draw a line between the two points along the edge of the horizon circle. This will be the required great circle distance which can be measured by the scale on the horizon circle, and, when reduced to minutes of arc, will be the distance in nautical miles. Take off the latitudes and longitudes of as many points as may be desired, transfer them to a Mercator chart, and trace in the arc. The courses and distances from point to point on this arc may be gotten directly from the chart; or, by computation, using middle latitude or Mercator sailing.

Graphic chart methods.—Various methods are now used to lay down a great circle track on a Mercator chart. These obviate the calculations which, by some people, may be considered laborious.

Towson's method permits a track to be laid down on a Mercator chart with a great degree of accuracy. His linear index gives the latitude and longitude of the vertex, whilst the accompanying tables give the true course at every degree of longitude from the vertex.

Airy's method.—The following method, proposed by Professor Airy, when Astronomer Royal, lays down a curve which is a very close approximation to the great circle arc:

(1) Join the two places on the chart by a straight line. Erect a perpendicular at its middle point, on the side next to the equator, producing the perpendicular beyond the equator, if necessary.

(2) Find the middle latitude between the two places, and with this middle latitude enter the table below and take out the corresponding parallel. The intersection of this parallel with the perpendicular will be the center of the required arc.

Middle Latitude.	Name.	Corresponding Parallel.	Middle Latitude.	Name.	Corresponding Parallel.
°		° ′	°		° ′
20	Opposite	81 13	52	Opposite	11 33
22	"	78 16	54	"	6 24
24	"	74 59	56	"	1 13
26	"	71 26	58	Same	4 0
28	"	67 38	60	"	9 15
30	"	63 37	62	"	14 32
32	"	59 25	64	"	19 50
34	"	55 05	66	"	25 09
36	"	50 36	68	"	30 80
38	"	46 0	70	"	35 52
40	"	41 18	72	"	41 14
42	"	36 81	74	"	46 37
44	"	31 38	76	"	52 1
46	"	26 42	78	"	57 25
48	"	21 42	80	"	62 51
50	"	16 39			

An approximate great circle track may be thus laid down: Compute the initial and final great circle courses between the two places *A* and *B*. Join *AB* on the chart, erect a perpendicular at its middle point. Find the differences between the Mercator and the two computed great circle courses. Lay off the angles *DAB* and *EAB* equal to these differences. Erect perpendiculars to *AD* and *AE*, cutting the first perpendicular in *p′* and *p*. The point *c* midway between *p* and *p′* will be the center of the required arc whose radius will be *cA* (Fig. 83).

136. Composite sailing.—Whenever the great circle track passes into higher latitudes than it is practicable or desirable to go, some of the advantages may be secured without going into regions of ice and danger, by following a composite track, a form of sailing first proposed by Mr. Towson. Decide on the parallel above which it is inadvisable to go, sail on the arc of a great circle which passes through the point of departure and has its vertex on this limiting parallel, proceed along the parallel till there is met a second great circle which, passing through the point of destination, has its vertex also on the limiting parallel; then follow this arc to destination.

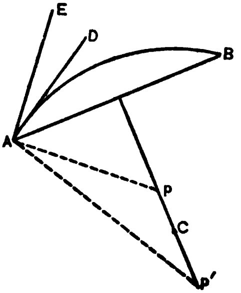


FIG. 83.

There are three general methods used in composite sailing: (1) By gnomonic charts. (2) By computation. (3) By graphic methods.—

By gnomonic charts.—Draw lines from points of departure and destination tangent to the limiting parallel. In the case of the great circle sailing charts of the U. S. Hydrographic Office, find the track from point of departure to point of tangency and from second point of tangency to point of destination, the intervening distance being found along the parallel from a Mercator chart or by parallel sailing. On a polar chart (see Art. 25) tangents are drawn in the same way to the limiting parallel. Suppose it is desired to find the composite track from $L_1 = 45°$ N., $\lambda_1 = 150°$ E., to $L_2 = 47\frac{1}{2}°$ N., $\lambda_2 = 130°$ W., the limiting parallel being 50° N. From the first position draw *CE* (Fig. 10) tangent to the parallel of 50°, and *DF* tangent to the same parallel; *C* and *D* being, respectively, the points of departure and destination. Transfer any desired number of points, including points of tangency, from the track on the gnomonic chart to the Mercator chart, by the coordinates of latitude and longitude. Then sail from point to point of the first great circle till the parallel is reached, along the parallel, and from point to point of the second great circle to destination.

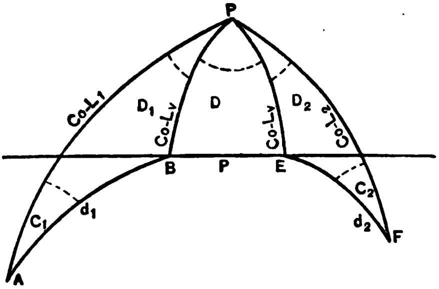


FIG. 84.

By computation.—In Fig. 84, let *BE* be the limiting parallel, *AB* and *EF* the great circles which pass, respectively, through the points of departure *A* and destination *F*, and have their vertices on the limiting parallel. The composite track will be ABEF. Since the limiting parallel furnishes the L_v in each case, $PB = PE = CoL_v$. Letting the different elements that enter be represented as indicated in Fig. 84, by Napier's rules, we have,

$$\left.\begin{array}{rl}
\text{In triangle } ABP, & \sin C_1 = \cos L_v \sec L_1. \\
& \cos D_1 = \cot L_v \tan L_1. \\
& \cos d_1 = \operatorname{cosec} L_v \sin L_1. \\
\text{In triangle } FEP, & \sin C_2 = \cos L_v \sec L_2. \\
& \cos D_2 = \cot L_v \tan L_2. \\
& \cos d_2 = \operatorname{cosec} L_v \sin L_2. \\
\text{In triangle } BEP, & D = (\lambda_2 \sim \lambda_1) - (D_1 + D_2), \\
& p = D \cos L_v. \\
\text{Composite distance} & = d = d_1 + p + d_2.
\end{array}\right\}(135)$$

Ex. 64.—From example 60 (Art. 135), we have L_1 35° 40′ S., λ_1 118° 06′ 07″ E., L_2 22° 15′ S., λ_2 41° 30′ W.; it is required to find the shortest possible route by composite sailing when the 60th parallel of South Lat. is the limiting parallel.

Find the distance, initial and final courses.

Solution of the eastern triangle Fig. 84.

	° ′ ″			
L_1	= 35 40 00 S	sec 10.09022	tan 9.85594	sin 9.76572
L_v	= 60 S	cos 9.69897	cot 9.76144	cosec 0.06247
C_1	= S 37 59 03 W	sin 9.78919		
D_1	= 65 31 15	. . . . .	cos 9.61738	
d_1	= 47 40 48 = 2860′.8	. . .	. . .	cos 9.82819

Solution of the western triangle Fig. 84.

	° ′ ″			
L_2	= 22 15 S	sec 10.03360	tan 9.61184	sin 9.57824
L_v	= 60 S	cos 9.69897	cot 9.76144	cosec 10.06247
C_2	= N 32 41 54 W	sin 9.73257		
D_2	= 76 20 15	. . . .	cos 9.37328	
d_2	= 64 04 20 = 3844′.33	. . .	. . .	cos 9.64071

$D = (\lambda_2 \sim \lambda_1) - (D_1 + D_2)$	$D = 1064.62$	log 3.02719
$D = 17° 44′ 37″$	L_v 60°	cos 9.69897
$D = 1064′.62$	$p = 532.3$	log 2.72616
Initial course $C_N = 217° 59′ 03″$	$d_1 = 2860.8$	
Final course $C_N = 327° 18′ 06″$	$d_2 = 3844.33$	

See plate (IV). $d = p + d_1 + d_2 = 7237.43$ miles.

Comparing the great circle and composite distances in this example, it is seen that the great circle distance is 7136.86 miles, the composite distance 7237.43 miles, or that there is a difference of 100.57 miles in favor of the great circle.

Graphic methods.—In graphic methods, use may be made of the terrestrial globe, or the track may be laid down on a Mercator chart approximately as follows:

Decide on a limiting parallel. Join the two places on the chart by a straight line, at whose center erect a perpendicular and prolong it till it meets the limiting parallel. Through this point of intersection and the two given points pass a circle; then sail from point to point on this circular route, by middle latitude or Mercator sailing, till the limiting parallel is reached; along that parallel to the second point of intersection with the circle; then from point to point of the remainder of the circle, by middle latitude or Mercator sailing, till destination is arrived at.

Examples Under Great Circle Sailing.

Ex. 65.—Find the great circle initial course and distance from Melbourne in Lat. 37° 49′ 53″ S., Long. 144° 58′ 42″ E., to Callao in Lat. 12° 03′ 53″ S., Long. 77° 08′ 20″ W. Also Lat. and Long. of the vertex.

$$\textit{Ans.}\begin{cases} C_N = 132^\circ\ 55'\ 36''. \\ d = 6984.37 \text{ miles.} \\ L_v = 54^\circ\ 40'\ 00''\ \text{S.} \\ \lambda_v = 158^\circ\ 25'\ 27''\ \text{W.} \end{cases}$$

Ex. 66.—Find the great circle initial course and distance from San Francisco in Lat. 37° 47′ 30″ N., Long. 122° 27′ 49″ W., to Sydney in Lat. 33° 51′ 41″ S., Long. 151° 12′ 39″ E. Also position of vertex.

$$\textit{Ans.}\begin{cases} C_N = 240^\circ\ 17'\ 10''. \\ d = 6445.25 \text{ miles.} \\ L_v = 46^\circ\ 39'\ 32''\ \text{S.} \\ \lambda_v = 100^\circ\ 30'\ 01''\ \text{E.} \end{cases}$$

Ex. 67.—(*a*) Find the great circle initial course and distance from Cape Vanderlind, Urup I., Lat. 45° 37′ N., Long. 149° 34′ E., to Pt. Reyes Lt. Ho. on the coast of California, Lat. 37° 59′ 39″ N., Long. 123° 01′ 24″ W. Also Lat. and Long. of vertex.

(*b*) Find the longitudes of intersection of the great circle with the 50th parallel of North latitude, and the course at first intersection.

(*c*) Not wishing to go further North than the 50th parallel, on account of the Aleutian Islands, find the increase of distance by pursuing the 50th parallel from the 1st to the 2d intersection, instead of following the great circle entirely.

$$\textit{Ans.}\quad (a)\begin{cases} C_N = 62^\circ\ 46'\ 15''. \\ \quad d = 3738 \text{ miles.} \\ L_v = \ \ 51^\circ\ 32'\ 26''\text{ N.} \\ \lambda_v = 174^\circ\ 40'\ 45''\text{ W.} \end{cases}$$

$$(b)\begin{cases} \text{Long. of West intersection, } 166^\circ\ 30'\ 46''\text{ E.} \\ \text{Long. of East intersection, } 155^\circ\ 52'\ 16''\text{ W.} \\ \text{Course at West intersection, } C_N = 75^\circ\ 22'\ 19''. \end{cases}$$

(*c*) Increase of distance, 14.7 miles.

Ex. 68.—Find the great circle initial course and distance from Brisbane, Australia, Lat. 27° 27′ 32″ S., Long. 153° 01′ 48″ E., to Acapulco, Lat. 16° 49′ 10″ N., Long. 99° 55′ 50″ W. Also Lat. and Long. of the vertex.

$$\textit{Ans.}\begin{cases} C_N = 82^\circ\ 04'\ 28''. \\ \quad d = 6748.63 \text{ miles.} \\ L_v = \ \ 28^\circ\ 29'\ 44''\text{ S.} \\ \lambda_v = 136^\circ\ 13'\ 45''\text{ E.} \end{cases}$$

Ex. 69.—Find the great circle initial course and distance from a point off Cape Agulhas in Lat. 34° 55′ S., Long. 20° 01′ E., to a point off Java Head in Lat. 6° 55′ S., Long. 105° 02′ E. Also Lat. and Long. of the vertex.

$$\textit{Ans.}\begin{cases} C_N = 92^\circ\ 51'\ 32''. \\ \quad d = 4918.4 \text{ miles.} \\ L_v = 35^\circ\ 01'\ 05''\text{ S.} \\ \lambda_v = 25^\circ\ 00'\ 11''\text{ E.} \end{cases}$$

PART II.

NAUTICAL ASTRONOMY.

CHAPTER VIII.

GENERAL DEFINITIONS.—THE VARIOUS SYSTEMS OF SPHERICAL CO-ORDINATES, AND CORRELATED TERMS.

137. Nautical astronomy is a special application of practical astronomy to the needs of seagoing people, who, by observations of the heavenly bodies, are enabled to determine the latitude and longitude at sea, and the error of their principal navigational instruments, the chronometer, the compass, and the sextant.

The heavenly bodies are the fixed stars, and those bodies constituting what is known as the solar system; namely, the sun, the planets and their satellites, comets, and meteors. The fixed stars, numbering many millions, are situated at immense distances beyond the limits of the solar system. Of all these heavenly bodies, only the following need be considered for navigational purposes: the sun, the moon, four planets (Mars, Venus, Jupiter, and Saturn), and about 30 fixed stars.

Astronomy teaches that the planets revolve about the sun, from West to East in elliptical orbits, at varying rates of speed, according to their positions in their orbits as well as their distances from the sun, and at the same time rotate on their axes. The period of a complete revolution, or time required to move through 360° in its orbit, constitutes the planet's sidereal period or year; and the period of a complete rotation on its axis is a planet's sidereal day.

The earth is one of the planets of the solar system; its orbit is in a plane inclined about 23° 27½′ to the plane of the equinoctial. The form of this orbit is elliptical, the sun being at one of the foci. The advance of the earth in its orbit is irregular, being the most rapid near perihelion, about January 1, and slowest near aphelion, about July 1.

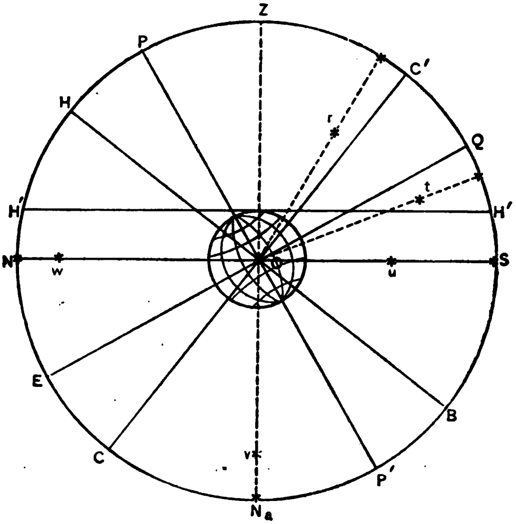


FIG. 85.

138. The celestial sphere.—To an observer on the earth's surface all the heavenly bodies appear to lie upon the concave surface of a sphere of indefinite radius of which only half is visible, the other half being cut off by the horizon. Owing to the insignificant ratio of the earth's radius to that of this assumed sphere, the eye of the observer may be considered as being at *O*, the earth's center. This sphere is called the celestial sphere. However, these bodies, like *r, t, u, v, w,* etc. (Fig. 85), are not at the same distance from the observer, and, being projected on the celestial concave, their apparent positions depend on their directions only and ~~not on linear dis-~~

tances. In like manner, the various fixed points and circles of the terrestrial globe defined in Chapter I may be projected on the celestial sphere, the point of sight considered to be at the center of the earth, O in Fig. 85.

The axis of the celestial sphere is the indefinite prolongation of the earth's axis intersecting the celestial sphere in two points called the celestial poles, corresponding to and named like the North and South poles of the earth. That pole above the horizon at any place is known as the elevated pole, the one below the horizon as the depressed pole. Axis PP' (Figs. 85 and 86).

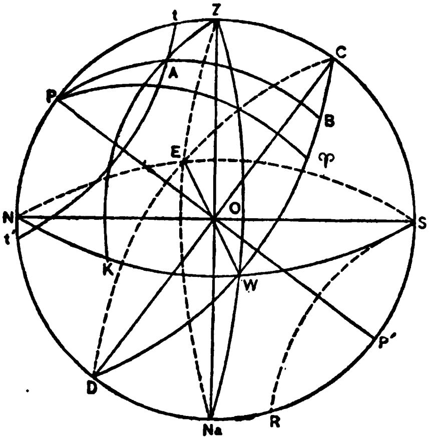


FIG. 86.

The celestial equator, also called the equinoctial, is the great circle of the celestial sphere in which the plane of the terrestrial equator, indefinitely extended, meets the celestial sphere; EQ (Fig. 85), $EDWC$ (Fig. 86).

Horizons.—A plane passed tangent to the earth's surface at the feet of the observer will be his sensible horizon, $H'H'$ (Fig. 85); a second plane parallel to this through the center of the

horizon { sensible
rational

earth will be his rational horizon, *NS* (Fig. 85); and these two planes indefinitely extended intersect the celestial sphere in practically one great circle called the celestial horizon. The point *Z* directly over the observer's head is the **Zenith,** the point N_a, directly opposite and under his feet, is the **Nadir.**

The celestial meridian is the great circle of the celestial sphere passing through the poles of the heavens, the zenith and nadir. It intersects the horizon in the North and South points, the North point being the one nearer the North pole. It is the great circle of the celestial sphere cut out by the indefinite extension of the plane of the terrestrial meridian. That semicircle which lies on the same side of the axis as the zenith is the upper branch; the other semicircle is the lower branch of the meridian. In Fig. 85, *PQP′E* is the meridian (in this particular figure it is also the solstitial colure), *PQP′* is the upper branch, *PEP′* the lower branch of the meridian. In Fig. 86, *PZP′D* is the meridian, *PZP′* being the upper branch.

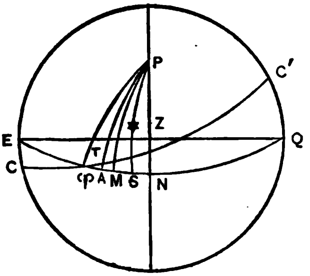


FIG. 87.

Ecliptic.—Though the earth in reality moves around the sun, completing its revolution of 360° in one sidereal year, the sun's center apparently describes a circle in the opposite direction on the celestial sphere, and this great circle is the ecliptic, *CC′* (Fig. 85); also *C*♈*C′* (Fig. 87), a projection on the plane of the horizon.

Angle of planes of ecliptic and celestial equator.—The celestial equator is inclined to the ecliptic at the same angle that the earth's equator is inclined to the earth's orbit, about 23° 27½′.

Equinoctial and ecliptic points.—The two opposite points of intersection of the equinoctial and ecliptic are practically fixed points on the celestial sphere.

The sun's center crosses the equator twice a year, once about March 21, once about September 21, and these being times of equal day and night, are called equinoxes, and the points of crossing, equinoctial points. The point known as the first point of Aries is the point of the equinoctial occupied by the sun in passing from the southern to the northern hemisphere, on or about March 21; hence it is called the vernal equinoctial point. The other point is occupied by the sun's center on or about September 21, and is called the autumnal equinoctial point. Though now about 30° distant respectively from the constellations of Aries and Libra, in early ages they defined the western limits of those signs in which the corresponding constellations lay, and hence were designated as the first points of Aries and Libra.

Owing to the precession of the equinoxes, the constellation Aries has passed from the sign of Aries into that of Taurus, but the vernal equinoctial point, designated by the sign ♈, is still called the "first point of Aries."

The points of the ecliptic 90° from the equinoctial points are called solstitial points, as at these points the sun reaches its greatest declination, occupying the northern one about June 21, and the southern one about December 21; in other words, the obliquity of the ecliptic equals the sun's greatest declination, North or South. The hour circle passing through the solstitial points is called **the solstitial colure,** *PQP'E*, (Fig. 85). The hour circle passing through the equinoctial points is called **the equinoctial colure,** *POP'* (Fig. 85), also *P*♈ (Figs. 86 and 87). The sign ♈ stands for the vernal equinoctial point; the term vernal equinox refers to the time of the sun's passing through that point, but, as custom sanctions its use to represent the point, the term "vernal equinox" will in future be applied to the point, and its symbol will be ♈.

Vernal
equinox

✓

139. Determination of a point of the celestial sphere.—The position of any point on the surface of the celestial sphere is determined when its angular distances are given from any two great circles on that sphere, whose positions are known.

The equinoctial and ecliptic are fixed great circles on the celestial concave, and the vernal equinox is practically a fixed point on the equinoctial, having a motion of only 50″.2 a year to the westward due to precession. Each of these great circles is used as the primary of a system of coordinates in fixed observatories; but at sea altitudes are measured above the visible horizon, and then referred to the celestial horizon, so that for seagoing people a system in which the horizon is the primary becomes necessary. Hence three systems are in use, each named after its primary, (1) Ecliptic, (2) Equinoctial, (3) Horizon Systems.

3 Nautical Systems

not made

The Ecliptic System and Correlated Terms.

140. The ecliptic system.—The primary circle of this system is the ecliptic which has already been defined; the secondaries are great circles passing through the poles of the ecliptic called circles of latitude, the one passing through ♈, the vernal equinox, being the principal one, *H*♈ (Fig. 88).

+ is pole of ecliptic

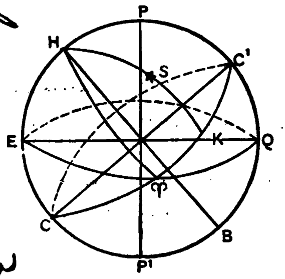


FIG. 88.

Ecliptic

Celestial longitude is the arc of the ecliptic intercepted between the vernal equinox and the circle of latitude passing through the body, reckoned positively towards the East, from 0° to 360°.

The celestial latitude of a body is the angular distance from the plane of the ecliptic measured on a circle of latitude passing through the body. In Fig. 88, *CC′* is the ecliptic; ♈*K*, the celestial longitude; and *KS*, the celestial latitude of the heavenly body *S*.

The coordinates in this system are unaffected by diurnal rotation; hence it is a convenient system at fixed observatories, especially when considering the motions of the sun and bodies composing the solar system. It is not used at sea.

The Equinoctial System and Correlated Terms.

141. The equinoctial system.—In this system, the primary is the equinoctial which has already been defined, and the secondaries are the great circles passing through the poles of the equinoctial. The solstitial colure is a secondary common to this and the ecliptic system. The secondary of this system passing through the zenith of a place is called the celestial meridian, and that one passing through a heavenly body is called a declination circle.

The declination of a heavenly body is its angular distance from the plane of the equinoctial, measured on the declination circle passing through the body. It is given in degrees, minutes, and seconds, and is marked N. or S., according as the body is North or South of the equinoctial (*BA*, Fig. 86).

0° min.' ''

The polar distance of a heavenly body is its angular distance from the pole (usually from the elevated pole), and, being measured on a declination circle, it equals 90° — the declination; but, if the declination is negative (of an opposite name from the latitude), the polar distance equals 90° + the declination.

Parallels of declination are small circles whose planes are parallel to that of the equinoctial.

The rotation of the earth is always performed in the same interval of time, a sidereal day, which is divided into 24 sidereal hours, and gives to the fixed stars an apparent movement in planes parallel to the equinoctial, through 360° in the same interval of time. From the time of apparent rising in

✓

Stars seem
move as
one disc

the East till the time of apparent setting in the West, the stars maintain their relative positions with reference to each other. This apparent motion, being due to the daily rotation of the earth, is called apparent diurnal motion of the heavens, and the path of any one star during its complete revolution is called its diurnal circle.

Right sphere.—To an observer at the equator, stars will rise and set vertically and their diurnal circles will be bisected by the horizon, so that the stars will be 12 hours above and 12 hours below the horizon; the planes of the diurnal circles being at right angles to the observer's horizon, the celestial sphere in this case is called a right sphere.

aspect of Heavens
1) Right Sphere
2) Oblique Sphere

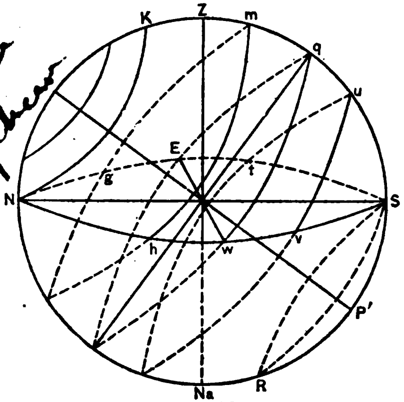


FIG. 89.

Parallel sphere.—Could an observer be at the North pole, he would see the stars of North declination sailing around, maintaining a constant altitude above the horizon, never rising and never setting. Stars of South declination would be invisible. The planes of the diurnal circles being parallel to the horizon, the celestial sphere would in this case be called a parallel sphere.

Oblique sphere.—To an observer at some point between the equator and the pole, say the North pole, the stars will rise and set at an oblique angle with the horizon. This applies to any heavenly body, whose declination will permit of any part of its diurnal circle coming above the horizon. A body of 0° declination will rise in the East point, set in the West point, and be the same length of time above and below the horizon; *EqW* (Fig. 89) is the diurnal circle of such a body.

A body of North declination will rise and set to northward of the East and West points, and be above the horizon more than 12 hours. In North latitude, stars of South declination, if visible at a place in North latitude, rise and set to southward of the East and West points, and will be above the horizon less than 12 hours. Since the declination of the sun in summer time is of the same name as the elevated pole, the sun is then above the horizon more than 12 hours; in other words, summer days are longer than winter days. Those stars whose polar distance is less than the altitude of the elevated pole, which is the radius of the circle of perpetual apparition, *NK* (Fig. 89), never set, but revolve around the elevated pole of the heavens. Those whose diurnal circles lie within the circle of perpetual occultation, *RS* (Fig. 89), never rise, and hence are invisible. This aspect of the heavens is known as the oblique sphere.

Hour circles.—In this apparent revolution of the heavenly bodies around the earth, their declination circles are continuously describing angles around the poles, which are called from the divisions of time hour angles, and, analogously, the declination circles are called hour circles; hence hour circles are defined as great circles passing through the poles of the heavens. *PB* (Fig. 86) is the hour circle of the body *A*.

As a star, for example *A* (Fig. 86), moves in its diurnal path about the pole, a point *B* of its hour circle moves uni-

15° arc in 1 hr.
15' arc in 1 min
15" arc in 1 sec

formly over the equinoctial through 360° of arc in 24 sidereal hours, 15° of arc in one hour, 15′ of arc in one minute, and 15″ of arc in one second of time, thus establishing a relation between arc and time.

What is said here about the apparent movement of a star's hour circle will apply to the movements of the hour circle of any heavenly body whose increase of right ascension is uniform; and, as time in any system used is the angle at the pole, measured by an arc of the equinoctial, all time, however measured, is converted into arc at the rate of 15° of arc to one hour of time. See Art. 178.

Transit or culmination.—The passage of a celestial body across the meridian of a place is called its transit or culmination; the upper transit occurs when it crosses the upper branch of the meridian, and the lower transit when it crosses the lower branch of the meridian. When a body's diurnal path is within the circle of perpetual apparition, both transits occur above the horizon, the upper one above the pole, the lower one below it; whilst those bodies, whose diurnal circles lie within the circle of perpetual occultation, are never visible at the given place.

Hour angle.—The hour angle of a heavenly body, or of any point of the sphere, is the inclination of the hour (or declination) circle passing through the body, or point, to the celestial meridian, and is measured by the arc of the equinoctial intercepted between these two circles. Hour angles are properly reckoned from the upper branch of the meridian, positively toward the West, and are usually expressed in hours, minutes, and seconds of time from 0^h to 24^h. However, for convenience in practical work, it is better, in fact it is usual in the American naval service, to regard the hour angle as minus when the body observed is East of the meridian up to 12^h. In Fig. 86, *ZPA* is the hour angle of the body *A* and it is measured by the arc *CB*.

Solar time.—The hour angle of the sun is called solar time, there being 24 hours of solar time in the interval between two consecutive upper transits of the sun over the same meridian, and this interval is called a solar day.

Sidereal time.—The hour angle of the 1st point of Aries, or vernal equinoctial point, is called sidereal time, there being 24 hours of sidereal time in the interval between two consecutive upper transits of the 1st point of Aries over the same meridian, and this interval is called a sidereal day. Owing to the fact that the 1st point of Aries is practically a fixed point of the equinoctial, the sidereal day is the time of revolution of the earth on its axis, or, in other words, of the apparent revolution of the celestial sphere through 360°.

Relation between solar and sidereal days.—Owing to the angular movement of the sun in its apparent orbit to the eastward (this apparent motion of the sun being due to the movement of the earth in its orbit about the sun), the sun comes to the meridian each day on an average about $3^m\ 56^s.555$ of sidereal time later than on the previous day; therefore, the solar day is longer by that amount than the sidereal day.

Right ascension.—The right ascension of a heavenly body is the inclination of its hour circle to that passing through the vernal equinox, or the arc of the equinoctial intercepted between these two hour circles. It is measured from the vernal equinox positively to the eastward from 0 hours to 24 hours. For body A (Fig. 86), ΥPB, measured by the arc ΥB, is the right ascension.

The fixed stars are at such immense distances as to be unaffected by the earth's change of position in its orbit; the coordinates of this system, however, declination as well as right ascension, are slightly affected by the precession of the equinoxes.

Relation of H. A. and R. A.—From the preceding definitions of hour angle and right ascension it is evident from Fig.

86, in which A is a heavenly body, Υ the 1st point of Aries or vernal equinox, PZN_a the meridian, PB and $P\Upsilon$ hour circles, that the local sidereal time which equals the right ascension of the meridian is the angle $ZP\Upsilon$, measured by ΥC; the hour angle of the heavenly body A is the angle ZPA, measured by CB; its right ascension is ΥPB, measured by the arc ΥB, and $\Upsilon C = CB + \Upsilon B$, or (1) *the local sidereal time at a given instant always equals the algebraic sum of the hour angle and the right ascension of the same body at that instant.*

When the hour angle is zero, the heavenly body is on the meridian, and its right ascension then equals the local sidereal time at that instant, or (2) *the right ascension of the meridian at a given instant equals the local sidereal time.*

These are two facts that must be fully realized and understood by every navigator; and it follows from the first proposition that when two of the angles are given, the third can be easily found.

The right ascension and declination of heavenly bodies are determined at fixed observatories, and tabulated in the Nautical Almanacs; knowing these, the position of a heavenly body is easily determined in this system, the right ascension being reckoned along the equinoctial to the eastward from the vernal equinox in a manner similar to the reckoning of longitude from the prime meridian on the terrestrial sphere; and the declination is reckoned North or South of the equinoctial along the declination circle, as latitude is reckoned North or South of the terrestrial equator along a terrestrial meridian. This system is the most convenient one for representing the motions of the fixed stars, owing to the very slight changes in coordinates.

The Horizon System and Correlated Terms.

142. The horizon system.—The primary circle of this system is the celestial horizon; the secondaries are great circles of the celestial sphere passing through the zenith and nadir; their planes being perpendicular to the horizon, they are called vertical circles. The principal secondary is the celestial meridian which intersects the horizon in the North and South points, each of which is named from the nearest pole. The celestial meridian is the secondary common to both the horizon and equinoctial systems; $SZNN_a$ (Fig. 90).

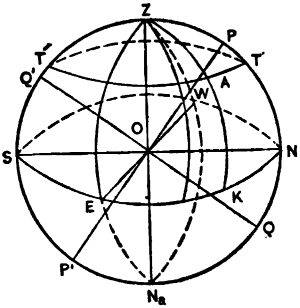


FIG. 90.

The prime vertical is the vertical circle pasing through the E. and W. points of the horizon; its plane is, therefore, perpendicular to that of the celestial meridian. ZWN_aE (Fig. 90), is the prime vertical.

The azimuth of a heavenly body is the angle at the zenith, measured by the arc of the celestial horizon, between the meridian and the vertical circle passing through the body; PZK, (Fig. 90), for body A.

Though the azimuth, as an angle of the astronomical triangle, is reckoned from the elevated pole towards the East or West, according as the body is East or West of the meridian, and though so estimated when tabulated in azimuth tables, still navigators of the present day reckon azimuth in both hemispheres more conveniently from the North point of the horizon, around to the right, from 0° to 360°.

If the angle Z found by solution is $x°$, navigators will consider the azimuth, or Z_N, simply as $x°$, $180° - x°$, $180° + x°$ or $360° - x°$, according as the bearing of the body by solution is N. $x°$ E., S. $x°$ E., S. $x°$ W. or N $x°$ W., respectively.

The amplitude of a heavenly body is the angular distance of the body, when in the horizon, from the prime vertical. It is reckoned from the East point when the body is rising, and from the West point when setting; towards North or South according as the body is North or South of the prime vertical.

The true altitude of a heavenly body is its angular distance from the plane of the celestial horizon, measured on a vertical circle passing through the body from 0° to 90°; KA (Fig. 90), for body A.

The zenith distance is the angular distance of the body from the zenith, measured on its vertical circle, and equals the complement of the altitude; ZA (Fig. 90), for body A.

From what has been said it follows that in this system the position of a body is given by its altitude and azimuth, the coordinates determined by navigators at sea, so observed positions are referred to this system; but as tabulated elements to be used by navigators all over the world must be referred to a system unaffected by the position of the observer, the value of the equinoctial system becomes apparent.

The predicted positions according to this latter system are found in the American Ephemeris and Nautical Almanac, as well as in other publications.

Referring to Fig. 90, let

O be the observer,
Z the zenith,
N_a the nadir,
P the elevated or N. pole,
$SZNN_a$ the celestial meridian,
$NESW$ the celestial horizon,
$EZWN_a$ the prime vertical,
N the North point of the horizon,
S the South point of the horizon,
PP' the axis of the sphere,
QQ' intersection of planes of celestial equator and meridian.
A a heavenly body,
ZAK its vertical circle,
AK its altitude,
AZ its zenith distance,
PZK its azimuth,
PON the altitude of the elevated pole,
Q^1OZ the declination of the zenith.

To prove that latitude equals the altitude of the elevated pole.—The arc of the meridian SQ' (Fig. 90), intercepted between the planes of the celestial equator, QOQ', and celestial horizon, SON, measures the inclination of the planes of these two great circles to each other; this inclination is also measured by the arc ZP intercepted between their poles, but $ZP = 90° - PN$ and $SQ' = 90° - Q'Z$; therefore,

$$PN = Q'Z,$$

or,

$$PON = Q'OZ.$$

Terrestrial latitude has been defined in Art. 1 as the angular distance of a place measured on its meridian N. or S. of the equator. As the zenith is the projection of a place and the equinoctial the projection of the terrestrial equator on the celestial sphere, the latitude of a place is the declination of the zenith; therefore,

$$\text{Lat.} = Q'OZ = PON,$$

or, **latitude equals the altitude of the elevated pole.**

143. The astronomical triangle.—The spherical triangle PZA (Fig. 86), formed by arcs of the celestial meridian, and the vertical and hour circles passing through the body A, is called the astronomical triangle, and it is this triangle that the navigator solves in working for latitude or longitude, remembering that when the observed body is on the meridian the triangle reduces to a straight line. The angles are: ZPA the hour angle, PZA the azimuth, and PAZ the position angle; the sides of the triangle are PZ, the co-latitude of the place of observation, AP the polar distance, and AZ the zenith distance of the body. The position angle is not used, but when any three of the other five parts are given, the remaining two can be found by spherical trigonometry.

By definition the co-latitude and zenith distance can never be greater than 90°. If the declination is of the same name as the latitude, it is regarded as positive and the polar distance equals 90° minus the declination; if of a different name from the latitude, the declination is regarded as minus and the polar distance equals 90° plus the declination.

In studying the astronomical triangle diagrams will be found most useful, and the most appropriate are those found by stereographic projections in which the point of sight is at one pole of the primitive circle. In Figs. 91, 92, and 93, *PZM* is a projection of the astronomical triangle.

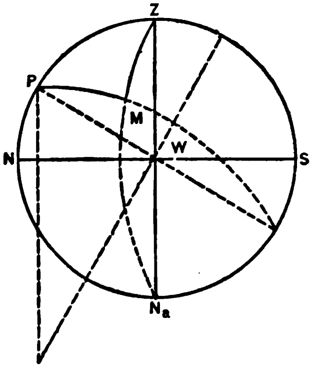


FIG. 91.

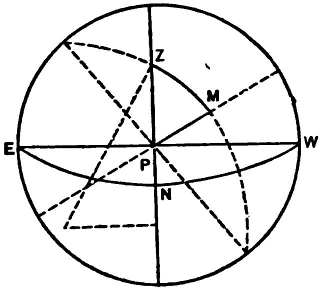


FIG. 92.

On the plane of the meridian, the point of sight is at the E. or W. point, and both *Z* and *P* are on the primitive circle (Fig. 91), in which *M* is a heavenly body West of the meridian.

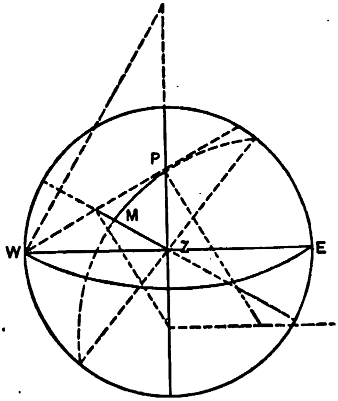


FIG. 93.

On the plane of the equator, the point of sight is at the depressed pole. *P* is at the center of the primitive circle, meridian, and declination or hour circles are projected as straight lines (Fig. 92).

On the plane of the horizon, the point of sight is at the nadir. *Z* is at the center of the primitive circle. All vertical circles, and hence the celestial meridian, are projected as straight lines (Fig. 93).

CHAPTER IX.

THE SEXTANT, THE VERNIER, AND THE ARTIFICIAL HORIZON.—METHODS OF OBSERVING HEAVENLY BODIES.

144. The sextant.—The sextant is a small portable instrument used for measuring the angles between two bodies or objects, whether or not one or both are celestial or terrestrial, and for measuring the altitudes of heavenly bodies or terrestrial objects above the visible horizon. Its principal use is at sea, where the use of fixed instruments would be impossible, in measuring altitudes for finding the latitude and longitude. The octant is a similar instrument, and is used for the same purposes, but the length of its limb is only about one-eighth of a circle.

As the name implies, the arc or limb (*c*) of the sextant (Fig. 94) is equal to about one-sixth of a circle, or 60° of arc, though graduated, as will be explained later on, so that each degree of the limb is really divided into two degrees of graduation, the subdivisions of the degrees being frequently as close as 10′ of arc, on an arc of silver, gold, or platinum. The limb and its supporting frame are of brass. A brass index arm (*o*), pivoted at the center of the circle whose arc forms the limb, is movable, carrying at the movable end a vernier (*d*) and magnifying glass (*g*) to read subdivisions of the graduated arc, and at the pivoted end a silvered mirror (*a*) whose plane must be perpendicular to that of the index arm and frame. This mirror, called the index glass, moves with the index arm. A second glass (*b*), called the horizon glass, one-half transparent and one-half silvered, the dividing line being parallel to the plane of the instrument, is fixed and should also be perpendicular to the plane of the limb. The graduations of limb and vernier should be such that the zero of one will be in coincidence with the zero of the other when the index and horizon glasses are parallel.

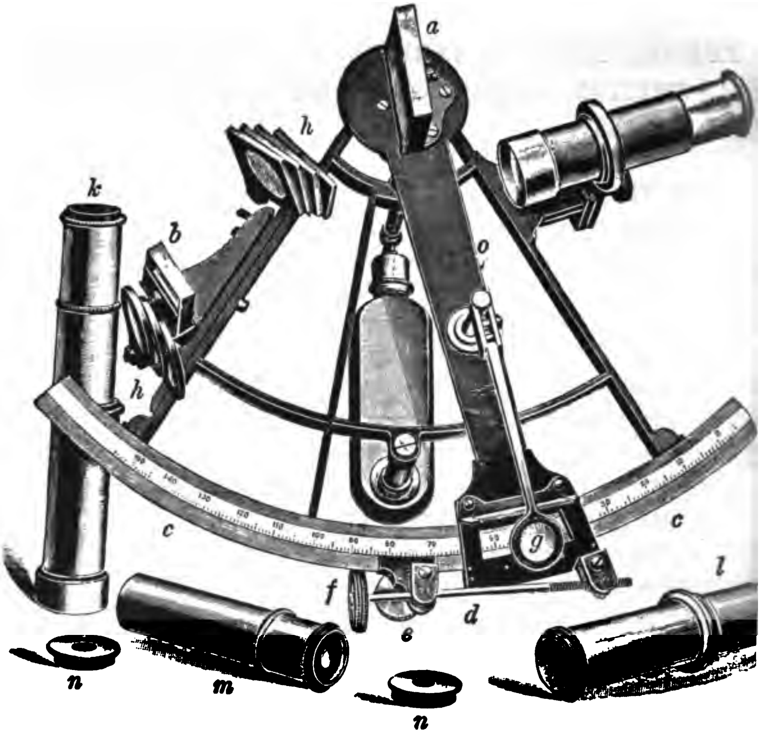


Fig. 94.

A telescope (*i*) which directs the line of sight through the

√

horizon glass and parallel to the plane of the instrument, is carried in a ring capable of movements at right angles to the plane of the instrument, shifting the axis of telescope from the silvered to the transparent part of the horizon glass, or vice versa.

Colored glasses (*h*) of different shades are fitted for use before both index and horizon glasses.

The index arm is fitted with a clamp (*e*) for securing it to the limb, and a tangent screw (*f*) for giving it small motions after clamping.

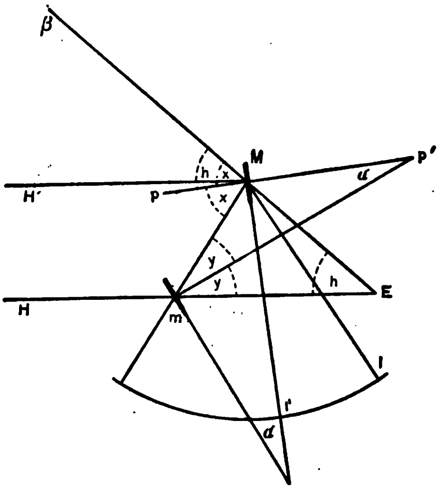


FIG. 95.

Besides the telescope (*i*), the sextant box is usually fitted out with a star or inverting telescope (*k*), a plain or sighting tube (*l*), and neutral glasses or caps (*n*) for the telescopes. The use of these caps obviates the necessity for the use of the colored shade glasses. The box also contains a screw driver, adjusting keys, a magnifying glass, and spare mirrors.

145. The optical principle of the sextant.—The optical principle of the construction of the sextant is thus stated: "The angle between the first and last directions of a ray of light, which has suffered two reflections in the same plane, is equal to twice the angle which the two reflecting surfaces make with each other."

To prove this, let M and m be the two reflecting mirrors of a sextant whose planes are perpendicular to the plane of the sextant, in this case the plane of the paper (Fig. 95). Let B be a body whose ray falling on M is reflected to m and by m to the eye at E; then BEm will be the angle between the first and last directions of ray BM, after having been reflected twice in the same plane. The angle between the mirrors is equal to the angle between lines perpendicular to them, pp' being perpendicular to M and mp' to m, and it is required to prove that BEm or $h = 2a$. Since the angle of incidence equals the angle of reflection, $BMp = pMm$, and $Mmp' = p'mE$; by geometry

$$\begin{array}{ll} \text{from } \Delta Mp'm,\ x = y + a \ \therefore & 2x = 2y + 2a \\ \text{from } \Delta MEm, & 2x = 2y + h \\ \hline \text{therefore} & h = 2a \end{array}$$

146. Application of the principle in measuring angles.—Suppose it is desired to measure the angular distance between two bodies, B and H, H sufficiently distant that the rays $H'M$ and Hm are sensibly parallel. The instrument is held so that its plane passes through both objects, the object H being seen directly through the telescope and horizon glass. Now let the index arm be so placed and clamped that the two glasses are parallel to each other; then will the ray $H'M$ be reflected by the two glasses parallel to itself, and the observer's eye at E will see both direct and reflected images in coincidence. Suppose this position of the index arm is MI, then for the given position of the horizon glass, I should be the zero of graduations of the limb. Now move the index bar, and with it the fixed mirror M, to the position MI', so that a ray from the second object B shall be reflected in the direction mE; the observer looking directly at H through the transparent part of the horizon glass, sees the reflected image of B in coincidence with the direct image of H. The angle h is the angle measured, but h is twice the angle between the mirrors, or $h = 2\alpha$; and, since $\alpha = IMI'$, h is twice the angle through which the index bar has moved, that is, twice the difference of the readings I and I'. To avoid doubling the angle, every half degree of II', and in fact of the whole limb, is marked as a whole degree, and the observer, reading directly from the limb, has only to subtract the reading at I from that at I', to get the angular distance between H and B. If the instrument is in proper adjustment, the reading at I is zero, that is, the limb is graduated from I as an origin. If this point of reference, I, does not coincide with the zero of graduation, the sextant has an error, called index error, which affects all angles observed with it at the time.

The degrees of the limb are further subdivided, those of the finest sextants being divided into six equal parts, each part 10′ of arc, and in order to read fractions of these divisions, recourse is had to the vernier.

147. The vernier.—This is a graduated scale (Fig. 96) to slide along the divisions of a graduated limb to facilitate the readings to fractions of a division of the limb. It is so constructed that the length of the vernier is exactly the length of a certain integral number of divisions of the limb, and is divided into one more or less divisions than that certain number; the fraction of a division of the limb is indicated by the division of the vernier which is in coincidence with a division of the limb, as will be explained later. The most usual method of construction is to make the number of divisions on the vernier one more than on the corresponding arc of the limb, and the explanation of this type follows.

To explain the working of a vernier, let *AB* (Fig. 96) be the arc of a limb, each division 20′; *CD* the vernier, the length of which is taken as 19 times the length of a division of the limb and is divided into 20 equal parts, thus each division of the vernier comprises 19′ of arc or is less by 1′ of arc than any division of the limb. The first line of the vernier is the zero line, and the reading of the limb is determined by the position of this zero. If this zero coincides with any division of the limb, the division line of the vernier marked 1 falls short of the next division of the limb by 1′, the next division line of the vernier marked 2 falls short of the next line of limb by 2′, and so on until the line marked 20 of the vernier coincides with a line of the limb; hence, if the vernier is advanced

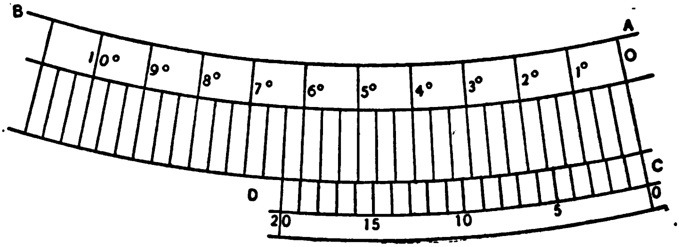


FIG. 96.

through 1′ of arc, the line marked 1 of the vernier will coincide with a division of the limb, if we advance it through 2′ of arc, the line marked 2 will coincide with a division of limb, and so on, and if the *n*th line of the vernier is found to be in coincidence with a division of the limb, it will be evident that the zero of the vernier has advanced *n* minutes.

General rule for navy sextants.—The general rule followed in the construction of verniers for the U. S. Navy is to take the length of the vernier exactly equal to the length of a certain integral number of divisions of the limb and divide the vernier length into equal parts, the number of which must be greater by one than the number of the divisions of the limb.

Let $l =$ value of a division of the limb,
$v =$ value of a division of the vernier,
$n =$ number of parts into which the vernier is divided,
$n - 1 =$ number of parts in the corresponding length of the limb,

$$l(n-1) = vn \therefore v = \frac{n-1}{n} l,\ v - l = \frac{n-1}{n} l - l = -\frac{l}{n},$$

$$\text{Least count of vernier} = l - v = \frac{l}{n}.$$

Of course, the graduations of the limb and the vernier must be in the same unit.

Ex. 70.—The limb of a sextant is divided to 10′ of arc. Construct a vernier to read to 10″ of arc.

The least count being 10″, a division of the limb 10′ = 600″, a division of the vernier is 590″. Therefore,

$$600\ (n - 1) = 590\ n,$$
$$60\ n - 59\ n = 60,\ n = 60.$$

Take 59 divisions of the limb for the length of the vernier, and divide it into 60 equal parts.

Ex. 71.—A sextant limb reads to 15′ of arc; the vernier is taken in length as 44 divisions of the limb. What is the least count of the vernier?

$$l - v = \frac{15' \times 60}{45} = 20''.$$

148. Reading the sextant.—First note the position of the zero of the vernier, then read the limb up to the division line immediately to the right of the zero of the vernier; this will be a certain number of degrees, or a certain number of degrees plus a certain number of the divisions of a degree. Say the sextant limb is graduated to 10′ of arc, and suppose the nearest division referred to is 33° 40′; now follow the arc of the vernier until a vernier line is found in coincidence with any line of the limb. Suppose the least count of the vernier is 10″ of arc, and the reading of the vernier at the coincident line is 2′ 20″, then the angle is 33° 42′ 20″. All angles measured on the limb are spoken of as "on the arc."

Excess of arc.—The limb of a sextant is generally graduated not only to 120°, but the limb is often of such extent as to be graduated up to 150°. This part of the limb is the arc proper, but, in all sextants, the limb and graduated arc are continued to the right beyond the zero for a short distance, and this arc is called the "excess of arc," and angles measured on it are spoken of as "off the arc." These angles are read from zero to the right, or backwards, and the vernier must also be read backwards. If a division of the limb is *n* minutes, then the vernier is marked to read *n* minutes; so read the vernier directly and subtract its reading from *n* minutes to get the vernier reading to be added to the reading of the limb, off the arc, immediately to the left of the zero of the vernier. Thus if a sextant limb reads to 10′, the vernier to 10″, and the zero of the vernier falls between 3° 10′ and 3° 20′ off the arc, and the vernier, if read directly, shows 2′ 50″, then the vernier read backwards would be 7′ 10″, and the angle 3° 17′ 10″.

149. Errors.—Sextants are subject to two general classes of errors. The first class comprises what are known as constant errors, and, though they usually arise from defects in construction, they may at times be occasioned by injuries received in legitimate use, or to abuses due to ignorance or carelessness. These errors should be eliminated, or ascertained and tabulated by the maker. In a high-grade instrument from a maker of good reputation, this class of errors should not exist.

The second class, known as the adjustment class, comprises those errors that should be removed by the navigator himself.

Constant errors.—(1) The centering error is due to the fact that the axis of the index bar is not at the center of the limb nor perpendicular to its plane. No sextant should be bought without careful inspection and not until after tests as to the centering error have been made.* If the eccentricity is found to be greater than .005 of an inch, the instrument should be rejected. (2) Error of graduation. The limb may not be a plane surface, and graduations of both limb and vernier may be inexact. There may be flexure of limb due to varying temperature, or accidental blows, producing great errors in angles. (3) Prismatic effect of mirrors and shade glasses, due to a want of parallelism between the two faces.

The above are all faults of construction.

The combined total errors of eccentricity and graduation can be ascertained together by measuring known angles with the sextant; the error can be found for a number of positions of the index bar, and then for other intermediate angles by interpolation. The known angles referred to may consist of angles laid off by a theodolite at intervals of 10° to 20° to cover the range of the sextant.

The combined error can also be ascertained by a series of artificial horizon observations, observing stars of nearly equal altitudes N. and S. of the zenith. Half the difference of latitudes resulting from each star will be the error for that altitude. The correction will be minus, if the latitude from the star on the polar side of the observer is greater than that from the star on the equatorial side of the observer; and plus, if vice versa. As this error varies on different parts of the arc, and generally increases with the angle, it would require many observations to determine it with any degree of satisfaction.

The determination of this error at sea is an entirely different proposition; theoretically it can be done by measuring the angular distance between two stars, and comparing this with the angular distance ascertained by computation. This, however, is not practicable on account of the complications due to refraction and aberration.

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* Navigating sextants issued to the U. S. Navy should bear a certificate of inspection from the U. S. Naval Observatory, giving the correction for eccentricity at intervals of 10° of arc.

However, since sextants are liable to accidents at sea, it may be desirable to ascertain this error, if only approximately. Now, if we can observe the angular distance between stars on the same vertical circle, the question of refraction will become a very simple matter, as the altitudes may be either observed (the horizon being clear), or computed for the instant of measurement of the arc. Those stars that have practically the same right ascension, or right ascensions differing by 12 hours, will be on the meridian, and hence on a vertical circle at the same time. There are many groups of such stars whose right ascensions do not differ more than either 30 minutes, or $12^h \pm 30^m$, and these might be used without much error. However, the right ascensions of the following groups are practically the same

α Aurigæ	η Ursæ Majoris
β Orionis	α Virginis
α Scorpii	α Pavonis
ζ Ophiuchi	γ Cygni

and the right ascensions of the groups below differ by practically 12 hours,

α Ursæ Minoris	α Tauri	γ Geminorum
α Virginis	α Trianguli Australis	α Lyræ

and the stars of each pair are on the meridian at practically the same time. Now the true distance, within an error of a few seconds of arc only, neglecting aberration, between the stars in each of the first four groups, that is, stars of the same right ascension, is the difference of their polar distances at the instant of meridian passage; for stars of the second groups, that is, for stars whose right ascensions differ by 12 hours, the true distance will be the sum of their polar distances.

If any stars, paired off as above, are visible, measure the arc between them, when on the meridian; either observe or compute their altitudes, and take from tables the corresponding refractions. If both stars are on the same side of the zenith, add the difference of refractions to the observed arc; if the stars are on opposite sides of the zenith, add the sum of refractions to the observed arc; the result is the corrected sextant distance; the difference between this and the true distance obtained from the polar distances is the total error for that angle. Knowing the index error, the error due to eccentricity and graduation may be found.

Knowing the right ascension of a star, it is easy to find the ship's time of its transit, and hence the time for measuring the arc. However, for many apparent reasons, even this method is ordinarily impracticable, so that at sea the sextant should be guarded carefully against all possible injuries.

Graduations.—Examine carefully the graduations of both limb and vernier. If the zero of the vernier is in coincidence with one division of the limb, an inspection of the divisions of the vernier should show an increasing separation between the divisions of vernier and limb in a direction towards the zero of vernier till the last division of the vernier is reached, when it should be coincident with a division of the limb. By shifting the position of the vernier, the divisions of the limb are tested for equality, whilst for magnitude they may be tested by measuring known angles of various sizes. Faults of graduation, if developed on inspection before buying, should cause the rejection of the sextant.

Prismatic effect of index glass.—This can be tested by observing a large angle, say, 120° to 130°, between two objects, and again measuring the angle with the index glass upside down. If measurements agree, the sextant having been adjusted in both cases, there is no prismatic error. If they do not agree, reject the mirror. When a reflected image, the angle being large, is not clearly defined, or there seems to be a fainter outline on a clearer image, it is evident that rays reflected from the inner and outer faces of the index glass are not parallel, and that the glass is prismatic.

Prismatic effect of horizon glass.—The want of parallelism of the two faces of the horizon glass is not a matter of great importance, as all angles and the index correction are affected alike.

Prismatic effect of shade glasses.—A want of parallelism in shade glasses, when used in front of the index glass, will affect the index correction, which should be determined with and without them. The index error should be determined also for each combination of shade.

These shade glasses, when known to be prismatic, should be discarded; and, if thought to be prismatic, colored caps should be put on the telescopes and the use of shade glasses discontinued.

Imperfections of shade glasses between the eye and horizon glass, or in the colored cap of the telescope, affect the object and the reflected image alike, so that the angle between them is unaffected.

150. Adjustment of the sextant.—It is the duty of the navigator, or of the person using a sextant, to keep it in adjustment; in other words, to see that the index and horizon glasses are both perpendicular to the plane of the instrument and parallel to each other when the zeros of the vernier and limb are in coincidence, and that the line of sight of the telescope is parallel to the plane of the instrument.

To adjust the index glass.—Hold the sextant in the left hand, place the index bar near the center of the limb, and, with the index glass nearest the eye, the eye being near the plane of the instrument, look into the mirror so as to see the reflected image of the limb. If the image and the arc as seen direct form a continuous line, the adjustment is correct. If they form a broken line, the mirror inclines forward or backward, according as the image rises or droops. Adjust the glass by means of screws at the back of the glass till the arc and its image appear perfectly continuous.

To perfect the adjustment, it may be necessary in some cases to tilt the mirror so much as to require the use of a liner of blotting paper under one edge.

To adjust the horizon glass.—The horizon glass may produce two kinds of error, a lateral error and an index error.

Place the zeros of vernier and limb in coincidence, and look through the telescope at a star. If the two images coincide, the adjustment is correct. If the reflected image is to the right or left of the direct image, there is lateral error due to the fact that the horizon glass is not perpendicular to the frame; if the reflected image is above or below the direct image, there is index error due to the fact that the mirrors are not parallel. The adjusting screws for this glass are sometimes back of the glass, at other times below and to one side. Move the arm and bring the mirrors parallel so as to have the reflected image on the same line but to right or left of the direct one. By proper screws remove the lateral error, so that, as the arm is moved, the reflected image passes directly over the direct image. Now place the zeros in coincidence, and, by the proper screws, make the two images coincide, thereby eliminating the index error. However, these two errors are so intimately connected that in an effort to remove one, the other is affected; so it is better to adjust by the "halving method." Place the zeros in coincidence, remove half the lateral error, then half the index error, and so on till adjustment is perfect.

The sea horizon may be used to test the adjustment of the horizon glass as follows: hold the instrument vertically and make the reflected and direct image of the horizon a continuous line. Then incline the instrument so that its plane makes but a small angle with the plane of the horizon. If the true and reflected horizons are in perfect continuation, each of the other, the glass is perpendicular to the plane of the instrument, the reading of limb and vernier being the index error. If the reflected horizon appears above the true one, the glass leans too much inward, otherwise outward.

The sun may be used in the same way as the star was used, but, owing to its size and brightness, perfection of adjustment is not so easily reached.

To make the line of sight of telescope parallel to the plane of the instrument.—Screw the inverting telescope into the collar, turn the eye tube till the two wires at its focus are parallel to the plane of the instrument. Place the sextant upon a table in a horizontal position, look along the plane of the limb, and make a mark upon a wall, or other vertical surface, at a distance of about 20 feet; draw another mark above the first at a distance equal to the height of the axis of the telescope above the plane of the limb; then so adjust the telescope that the upper mark, as viewed through the telescope, falls midway between the wires. The adjustment is made by tightening or loosening one of the two adjusting screws on the collars, doing the reverse with the other screw.

Index error.—Before using a sextant to make observations of any kind, the sextant being otherwise in adjustment, it is necessary to find the point of the graduated arc where the zero of the vernier falls when the two mirrors are parallel to each other, or at the time when the reflected image of a distant object is found to be coincident with the direct image of the object itself. If this point is not coincident with the zero of the limb, the sextant has an index error which affects all angles taken at the time. These angles, as measured, will be too small or too large, according as the zero of the vernier falls to the right or to the left of the zero of the arc; in other words, the reading of the vernier is subtractive, if on the arc, additive, if off the arc. The error applied in this way is known as the index correction, and is represented by the letters I. C.

Should it be desired to eliminate the index error, place the zeros of vernier and limb in coincidence, and, by means of the proper adjustment screws, turn the horizon glass about an axis perpendicular to the plane of the instrument till the reflected and direct images of a star or distant object are in coincidence. Be careful, however, not to disturb the perpendicularity of the horizon glass. It is not advisable to try to keep the index error at zero, but it should be determined before every observation under any circumstances; and the knowledge that there is one, even though small, makes its determination necessary at the time of any set of observations.

To determine the index correction.—By a star.—Bring the reflected image of the star into coincidence with its direct image, then read the arc and vernier. The reading is the index correction; + if off the arc, — if on the arc.

By the sea horizon.—Hold the instrument vertical, and make the true and reflected horizons continuous. The reading of the arc and vernier is the correction, + or — as before.

By the sun.—Bring upper limb of reflected image of sun tangent to the lower limb of the sun seen directly, read the sextant, + if off the arc, — if on the arc; then bring the lower limb of the reflected image tangent to the upper limb of the sun seen directly, read the sextant, + if off, — if on the arc. The index correction will be one-half the algebraic sum of the two readings. If well taken, the result can be checked by finding from the Almanac the sun's semidiameter which should equal one-fourth the algebraic difference of the two readings. However, since refraction acts in the vertical plane, and affects the vertical diameter of the sun, the amount of course depending on the altitude, it is preferable at low altitudes to use the horizontal diameter in finding the index correction; but, as the difference of the refractions for the upper and lower limbs of the sun, for altitudes over 30°, amounts to only a few seconds of arc, there is no practical advantage in using the horizontal diameter at the higher altitudes.

151. Using a sextant. To observe at sea an altitude of the sun, in other words, the angular distance along a vertical circle from the sun to the horizon: adjust the telescope to distinct vision by looking through the telescope at the horizon, moving the eye piece in or out till the horizon is clearly and distinctly seen, then screw it into its collar; see the instrument in adjustment and ascertain the index correction; put down the necessary shade glasses before the index glass, place the index bar near the zero of the limb, and see the tangent screw in mid position; hold the instrument in the right hand by the handle so that its plane shall be vertical, and direct the line of sight to a point of the horizon directly below the sun. Now move the index bar with the left hand, and, if the sextant is held properly when its reading is near the altitude of the sun, its reflected image will be seen to descend. Make an approximate contact with the horizon and clamp the index bar. Now rotate the sextant slightly around the line of sight as an axis, making the reflected image skirt along the horizon; and, by means of the tangent screw, find one point at which the sun is just tangent to the horizon; the reading of the sextant at that time is the altitude of the sun's lower limb. Just before the altitude is taken tell the assistant to "stand by," and at the instant when the altitude is taken, say "mark." * The assistant notes the seconds, minutes, and hours of his watch, recording opposite the time the degrees, minutes, and seconds of the altitude. At the time "mark," the sun should be at the lowest point of its arc, just tangent to the horizon and in the center of the field of view. If by inclining the sextant, the sun is moved from this center to points nearer the plane of the instrument, or farther off, the angle, as read on the arc, will be too great. If there is much glare around the horizon, as is frequently the case when the sun is observed, especially at low altitudes, shade glasses should be turned down in front of the horizon glass as may be necessary. The amount of light and, hence, the brightness of the reflected object can be varied by moving the telescope towards or from the plane of the sextant.

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* An experienced observer should be able to note the time of his own observations.

To observe the altitude of a star.—In observing a star, the observer can use the inverting telescope, which has greater magnifying power than the ordinary direct one, or he can use the plain tube. Place the zero of the vernier on the zero of the limb, look at the star, hold the instrument vertical, move the index bar outward, keeping the reflected image in sight till the image of the star is just below the horizon; clamp, and whilst rotating the instrument, use the tangent screw and find the lowest point of the swing just on the horizon. It is advisable not to use any telescope or tube until able to observe well without it. In bringing down and observing a star with a tube or the unassisted eye, keep both eyes open. This method is essential to avoid bringing down the wrong star, and it might be used for the sun with beginners, though it would be very trying on the eye.

Sometimes when latitude is approximately known, and an observation of a star on the meridian is to be made, the instrument can be advantageously set to the approximate altitude.

An observer can determine his personal error in measuring altitudes of stars by taking several altitudes of Polaris for latitude at a place whose latitude is accurately known.

To measure the angle between two visible objects.—Direct the line of sight (or the telescope which should be and can be easily used after a little practice) toward the left hand object, if both are nearly in the same horizontal plane, or toward the lower one, if one is much above the other. With this object in direct view, through the plain part of the horizon glass, move the index arm until the image of the other object, after reflection by the index glass, is seen in the silvered portion of the horizon glass. Having made a partial contact of the two images, clamp the instrument, screw in the telescope (if not already in its collar), perfect the contact with the tangent screw, and then read the limb.

If, for any reason, it should be desirable to point the telescope to the right hand object, hold the sextant upside down, with the handle in the left hand and above.

152. Care of sextant.—Keep your sextant in your own hands, or in its box which should not be put on a table from which it may be thrown off, nor in a roomy drawer wherein it may slide. Do not allow any one else to use it. Never put it away damp, as the moisture will surely cloud the mirrors and rust the metal. Wipe off the mirrors and arc with chamois or silk, but permit no polishing of the arc of limb or vernier. In adjusting the instrument when screws work against each other, be sure to loosen one as the other is tightened. When in adjustment, do not tinker with the screws even to remove an index correction, which, if small, can be allowed for. Keep tangent screw in mid position. Keep the arc clean by occasionally applying a drop of ammonia, and do not use oil except on screw threads.

153. Resilvering mirrors.—It often happens that mirrors are injured by dampness or other causes, especially when doing hydrographic work, and then they require resilvering, which may be done in the following way:

Take an unbroken piece of tin foil about one-quarter of an inch larger in all dimensions than the mirror to be resilvered, lay it on the clean surface of a pane of glass about five inches square, smooth out the foil, being careful not to tear it, put a drop of mercury on the foil spreading it carefully with the finger over the surface, put on another drop and repeat the operation, and continue the process till the surface is fluid, being very careful that no mercury gets under the tin foil.

Lay on the supporting glass a piece of tissue paper so that its edge shall cover the edge of the foil; having cleaned the glass to be silvered, lay it on the tissue paper, and transfer it slowly and carefully to the mercury surface, keeping a gentle pressure on the glass to prevent the formation of bubbles.

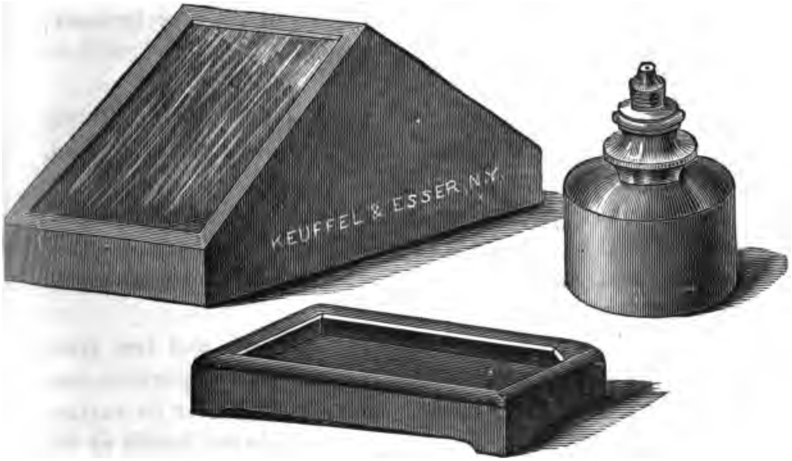


FIG. 97a.
ARTIFICIAL HORIZON.

Place the mirror face downward and slightly inclined, to allow any surplus mercury to run off, and let it remain so till the following day, when the tin foil should be trimmed off flush with the edge of the mirror, and a coating of varnish made from spirits of wine and red sealing wax applied.

154. The artificial horizon (see Fig. 97a).—This consists of a small shallow basin about 8 inches long, 4 inches wide, and $\frac{3}{4}$ inch deep, containing mercury or any other fluid whose surface will reflect a heavenly body. The surface must be horizontal, and free from products of oxidation or other matter diminishing the reflecting power.

The basin or receptacle, made of wood or iron, is covered by a roof consisting of two pieces of plate glass in a frame to protect the surface of the mercury from dust or wind.

An iron bottle is furnished to contain the mercury when not in use; this bottle is provided with a screw stopper and a funnel to prevent loss of mercury in handling; these articles complete the artificial horizon outfit.

In the absence of mercury, molasses or oil may be used; but, with oil, the receptacle or basin must be blackened on the inside.

Reflecting horizons of black glass, plane and accurately ground, made level by levelling screws, are sometimes used, though not recommended.

Care of and preparation of artificial horizons for use.—The artificial horizon finds its principal use on shore in rating chronometers, and then should be at its best and used under the most favorable conditions.

The surface of the mercury must be clean and free from dust and the roof perfectly dry. Scum and impurities may be removed from mercury by gently drawing over its surface the straight edge of a piece of blotter cut to the length of the basin, pressing it below the surface of the mercury, and inclining it so that it may act like a scoop. If the mercury is alloyed, wash it with sulphuric acid, then with water, and filter it through muslin.

Before pouring the mercury into the basin, see the basin well cleaned and dry, remove the funnel and stopper, and screw on the funnel, so that, by passing through a greatly contracted passage, the mercury may be cleansed. Place a finger over the opening, shake up the bottle, then invert it, and let any scum rise to the surface. Hold the bottle inverted over the basin, remove the finger, and let the mercury run into it.

When the basin is full, put the finger over the aperture and reverse the bottle; it is better to do this before all the mercury is out, or nearly so, in order to keep back any scum or impurities. The roof should be placed over the basin for a few moments to allow any moisture in the imprisoned air to be deposited on the glass surface of the roof, which is then lifted, wiped off, and replaced. A piece of cloth for the basin to rest on, and large enough to receive the edges of the roof, will keep out moisture.

A roof should be used whose glass has no prismatic effect, but this can be eliminated, where it exists, by reversing the roof in each set of observations. However, in observing stars on both sides of the zenith, since the mean of results will be taken, and the prismatic effect is of an opposite sign in each case, the observer should keep the same end of the roof towards himself in each set of observations.

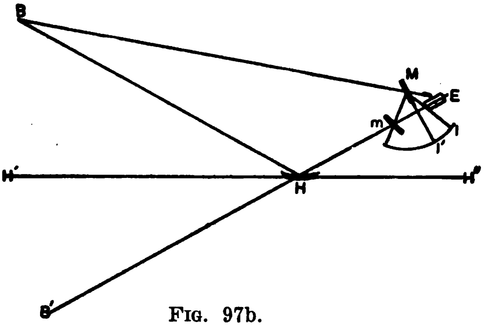


FIG. 97b.

Advantages of the artificial horizon.—By using an artificial horizon, and halving the angle, the errors of observation, whether of instrument or observer, are also halved; and the correction for dip, which depends on both the height of eye and refraction, is obviated, as the artificial horizon furnishes a horizontal plane. Its use, however, is limited to shore observations.

Theory of the artificial horizon (Fig. 97b).—A ray from B is reflected from the basin H to the eye, making $\angle BHH' = \angle EHH'' = \angle H'HB'$. A ray from B is also reflected by the sextant mirrors, making the sextant image coincident with the basin image. Now $\angle BHB' = \angle HEB + \angle HBE$, but the body B is so far off that the ray BH is practically parallel to the ray BM, so that the angle $BHB' = \angle BEH =$ angle read from the arc $= 2BHH'$, or twice the altitude of B.

To take an observation of the sun, using an artificial horizon.—Select an observation spot so that the basin may be evenly placed on a solid foundation, in a sheltered position undisturbed by breezes, or any movements or jars in the vicinity which might ripple its surface. In case equal altitudes are to be observed, the spot should be so selected that, if the sun is observed at one altitude on one side of the meridian, its view may not be shut out by houses, trees, hill tops, or other obstructions, when at the same altitude on the other side of the meridian.

Clean the basin and put it with its length nearly in line with the direction of the sun, but a little in advance; pour the mercury into the basin and see its surface cleared of scum and impurities. Wipe the glass of the roof and cover the basin.

Determine the I. C. and put the tangent screw in mid position.

The observer should sit on a low stool or on the ground, with his back supported, if possible; assuming the most comfortable position possible under the circumstances, so as not to tire himself; and, at the same time, so placing his eye that he may see the image of the sun reflected from the center of the mercury.

Turn down the necessary shade glasses before both the index and horizon glasses, and, without putting in the telescope, direct the line of sight to the sun, and bring it down till the lower limb of the image reflected by the mirrors overlaps the upper limb of the image reflected by the mercury. Screw in the telescope with a colored cap on the eye piece, throw back the colored shade glasses, and proceed to observe the altitude. The sextant reading corrected for I. C. gives double the apparent altitude of the limb observed. Half the result, corrected for S. D. (+ for the lower limb, — for the upper limb), parallax, and refraction gives the true altitude of the center.

If observing in the forenoon, the suns will separate; have the time marker "stand by" and as they separate, the lower limb of the apparent sextant image being just tangent to the upper limb of the horizon image, say "mark." The assistant notes the time, records it, and also the angle. It is usual to take the altitudes at equal intervals in arc, setting the sextant at the next division after one observation, and waiting for tangency or contact. The interval should be sufficient to permit care and accuracy in reading and observing.

If not afraid of the prismatic effect of the shades, different colored shade glasses may be used before the index and horizon glasses, giving different colors to the two images of the sun, and making it easier to distinguish them.

If used, these shade glasses give sufficient protection to the eye, and the colored cap of the telescope is not used.

Determining the limb observed.—If immediately after contact the two images of the sun were observed to separate in the forenoon, or close in on each other in the afternoon, then the limb observed was the lower limb; otherwise, the observed limb was the upper limb.

To take an observation of a star, using an artificial horizon.—The observer places himself so as to see the image of the star reflected in the mercury of the basin; this image will seem as far below the surface of the mercury as the real star seems above it.

Before screwing in the tube or the inverting telescope, place the zero of the vernier on the zero of the limb, direct the line of sight to the star, both eyes open, keep the plane of the sextant vertical, and move the index bar, keeping the star's reflected image in sight, till the image reflected by the mirrors (the sextant image) is in coincidence with that reflected by the mercury and seen directly through the center of the telescope collar and the horizon glass. The sextant reading corrected for I. C. gives double the star's apparent altitude. Half the result, or the apparent altitude, corrected for refraction, gives the true altitude of the star.

If intending to use the tube or inverting telescope, screw it in as soon as the star has been brought down, and proceed with the observations, saying "mark" to the assistant when the two images are in coincidence.

CHAPTER X.

CHRONOMETERS AND TORPEDO-BOAT WATCHES. STOP AND COMPARING WATCHES.

CHRONOMETERS.

155. Definition.—The term chronometer is applied to a portable timepiece of superior workmanship, furnished with special mechanism consisting of compensation balance, balance spring, and escapement, so constructed as to obviate changes in its rate due to expansion or contraction of its mechanism, through effect of heat or cold. The term chronometer, however, is more generally applied to one adapted for use on board ship. A marine chronometer should beat half seconds. Its special function is to furnish the time of the prime meridian, almost universally taken as that of Greenwich.

Mean or sidereal chronometer.—A chronometer may be regulated to keep mean or sidereal time; if to keep mean time, it is called a mean time chronometer and its units are those of mean solar time; if to keep sidereal time, it is called a sidereal chronometer and its units are those of sidereal time. (See Art. 171.)

The mean time day at any place commences when the mean sun is on the upper branch of the meridian at that place, and the theory of a mean time chronometer, keeping the time of that place, is that it shows $0^h\ 0^m\ 0^s$ when the mean sun is on the upper branch of the meridian.

Practically every chronometer has an error and a daily rate, gaining or losing, so that in order to have a chronometer regulated to a local or to Greenwich mean time, its error on that time must be known, and also its daily rate, *i. e.,* the daily gain or loss. Both are plus, if the chronometer is fast and gaining; minus, if slow and losing. It is customary, however, to use the error as a correction, which is the quantity to be applied to a chronometer reading to reduce it to the correct time at that instant. This is plus when the chronometer is slow, and the daily change is plus when the chronometer is losing. If the correction of a given mean

CHRONOMETER.

time chronometer is on G. M. T., the resulting time will be G. M. T. The letters C. C. usually represent this correction.

Since a sidereal day at a place begins when the vernal equinox is on the upper branch of the meridian at that place, the theory of the sidereal chronometer is that it should show $0^h\ 0^m\ 0^s$ when the vernal equinox is so situated, and its error on any sidereal time (the amount it is fast or slow on that time) and its daily rate must be known in order to say the chronometer is regulated to that time.

Rating.—The question of rating a chronometer will be considered elsewhere in this work.

Number for safety.—The U. S. Naval vessels carry at least three good chronometers. In this way, each may be a check on the other, and irregularity on the part of one will be made evident by a comparison of 2d differences as recorded each day, it being assumed that all will not have the same kind of daily rate.

There is no particular advantage in having two chronometers, except for the possibility of injury to one, for, if they begin to differ widely, after a period of regularity, it would be difficult to determine which is the good or faulty one, in the absence of means to check their indications.

Stowage on board.—The chronometer, swung in gimbals in its own case, is placed in the chronometer box as soon as received on board. This box has as many compartments as there are chronometers, and each compartment is well padded with hair and lined with baize cloth to prevent sudden changes of temperature and to reduce shocks and tremors as much as possible. The box is surrounded by a strong casing sufficiently large to admit of a clear space of at least two inches all around. Before reception of the chronometers on board, the box and its casing should be secured to a solid block of wood, bolted to the beams of the deck, in a part of the ship that is to be the permanent abode of the instruments; as low down in the ship as possible where the temperature may be equable, and where gun-fire may have its least effect; amidships and so as near the center of motion as convenient, sufficiently far forward as not to be affected by vibrations of the screw; removed from influences of masses of iron, especially vertical iron, dynamos, electric wiring, or magnets of any description. The chronometers, when stowed, should be allowed to swing freely in the gimbals, should constantly occupy the same relative position, with the XII-hour mark towards the same part of the ship, and should not be removed except for necessity.

Designation.—Instead of designating chronometers by their numbers or maker's name, it is customary to denote them by the letters of the alphabet, A, B, C, D, etc. The standard, called A, made by a maker of well-known reputation, should have a first-class record, a clear, distinct beat to half seconds, and a small stable rate, the stability of rate being of more importance than its amount. It should occupy a central position among the others. Of course, a record should be made in the chronometer journal of the number and maker of chronometers to which the above letters may be assigned.

156. A maximum and minimum thermometer should be kept in the case with the chronometers, and recorded at the time of winding; it should be kept in a vertical or nearly vertical position. A small horse-shoe magnet is used to reset the indices. This magnet should be kept with keeper on outside the box, and never brought near the chronometers. Should the mercury column become divided, move the indices well away from the column, then holding the thermometer vertical, bulb end up, give it a quick movement downward, bringing it to an abrupt rest. Repeat this process, if necessary, till the break in the column disappears, being careful to keep the indices clear of the mercury.

157. Winding.—Some chronometers are made to run 8 days and are wound but once a week; however, it is believed they would run better if wound every day. Our service chronometers will run for 56 hours, and should be wound every 24 hours, at regular and stated times, in order to bring into play each day the same part of the spring, and thereby contribute to regularity and stability of rate. The time for winding chronometers depends on the commanding officer, and may be just before 8 a. m. or just before noon; at all events, they should be reported wound to the captain at 8 a. m. or noon, as the case may be.

To wind.—Place the left hand over the face, turning slowly and carefully the chronometer bowl in its gimbals; then, holding the bowl firmly in the left hand, rotate the valve with forefinger or thumb of left hand, according to direction of rotation, or as most convenient, until the key hole is uncovered; place the key in position with right hand, wind slowly to the left the required number of half turns, usually 7 or 8, counting the half turns as the chronometer is wound, to avoid too much force at the last one, though be careful to wind to a full stop. After winding, remove the key, and the valve should close automatically; if it does not, close it to keep out dust and dampness; then the chronometer is eased to its original position, the XII-hour mark pointing as before. It is well to note after winding that the indicator on face of dial, which shows the number of hours since winding, then reads 0^h.

After winding, compare chronometers, fill in the columns of chronometer comparison book, reset maximum and minimum thermometer, and stow the magnet away so as not to be anywhere near the chronometers.

When run down.—When run down, a chronometer will not start on winding; however, the balance wheel can be set in vibration by a quick, but not a violent, horizontal circular motion.

Resetting hands.—Should it be desirable to reset the hands, unscrew the glass cover from the face, place the winding key on the projecting stem, and turn the hands in one direction only, which should always be ahead. If practicable to do so, avoid altering the position of the hands by starting the chronometer at the time indicated. Never touch the hands nor turn them backwards.

158. Comparison.—The following method of comparing is generally followed, it being better that a single observer should alone make the comparison.

The observer determines upon a certain time by standard for the comparison, and, holding the comparison book in the left hand, enters that time for A. Opening the glass top of A, but only the wooden top of B, so as not to hear the ticking of the latter, he takes a beat from A, say 5 seconds, before the second hand is to reach the comparing mark. Casting his eye upon the dial plate of B, listening intently to the beats of A, he counts by ear the beats which elapse before the second hand reaches the comparing mark. At that instant he reads the seconds, minutes, and hours of B. A second comparison will verify the first, or indicate an error. Enter the reading of B below that of A, note the difference and 2d differences in their proper places in the comparison book. Compare C, and all the other chronometers in the same way with A, recording carefully the hours, minutes, and seconds of each comparison in the proper places.

It is desirable to consider the standard fast of the other chronometers as well as fast on the Greenwich mean time, whatever the actual indications of A may be, so that by subtracting the comparison of any chronometer with the standard from the error of the standard (adding 12 hours to the latter, if necessary), the error of that particular chronometer, fast on Greenwich time, is obtained. To illustrate, suppose A is fast of G. M. T. $0^h\ 10^m\ 10^s$ and $A - B = 11^h\ 10^m\ 50^s$.

The first may be written thus A — G. M. T. = $0^h\ 10^m\ 10^s$
and A — B = $11\ \ 10\ \ 50$

Therefore,
B is fast of G. M. T. $59^m\ 20^s$, or B — G. M. T. = $0^h\ 59^m\ 20^s$

Comparison of a mean and sidereal time chronometer.—In the comparison of a mean time chronometer with a sidereal time chronometer, the difference between the two can be obtained within a very small fraction of a second by watching for the coincidence of their beats. Since the second of sidereal time is shorter than that of mean time, or 1^s of sidereal time = $0^s.99727$ mean time, the sidereal chronometer gains on the mean time chronometer $0^s.00273$ in 1^s, and therefore gains one beat, or $0^s.5$, in 183 seconds. Hence, once in about every three minutes the two chronometers beat together, and, as the observer, when watching one, and counting the beats of the other, fails to note any difference in the beats, he records the corresponding half seconds of the two chronometers and notes the minutes and hours of each.

159. Cleaning and oiling.—Chronometers should be cleaned and oiled every three or four years; when the oil becomes dried, or thickened, the rate will be irregular. Besides, the mechanism should be cleaned occasionally to prevent or remove any rust that may follow exposure to dampness.

160. Transportation.—Whenever chronometers are to be transported, even for short distances, clamp the catch of gimbal ring, and carry them in their transportation boxes, or in a handkerchief which is passed under the box, through handles, and square knotted on top. Great care must be taken to give them no shock or circular movement; when a chronometer is carried in a boat, it should be kept suspended by the hand. Chronometers may be transported while running in the following way: remove the bowl from the gimbal ring, being careful not to let it fall or be jarred when unscrewing the pivot screw of the ring; wrap the bowls in soft paper, place them, dials up, in circular paste-board boxes with corrugated paste-board packing (if these boxes are not on hand, wrap bowls in cotton); place these boxes, tops up, wrapped in cotton, or hair, in a large rectangular basket, as far from the center as possible; put down and secure top of basket, and carry it by its handles, protecting it from jars or jerky movements.

For transportation to a distance.—Chronometers should, whenever possible, be sent by hand; but if necessary to send by express, as when sending to a long distance, and also for repairs, they should be allowed to run down, be dismounted, and the balance stayed with clean cork at diametrically opposite points. Place the gimbals in the bottom of the case, over which put a pad of cotton wrapped in soft paper to form a seating for the bowl. The chronometer bowl is wound around with rolls of cotton in paper, seated on the cotton pad in the bottom of the case, and then covered by a similar pad. See the case tightly packed and closed, put into its transporting box, which is then securely closed and itself wrapped in a thick padded covering, and marked "Delicate Instrument, Carry by Strap."

To stay the balance with cork.—Unscrew the bezel, leaving the chronometer movement free in the bowl. Turn over the bowl on the left hand, supporting it by the fingers around the dial edge; lift the bowl, uncovering the movement; stop the balance by touching it very lightly with a dry piece of paper, and stay it by two dry and clean strips of cork placed gently under the outer rim at points diametrically opposite, so as not to cover oil holes or touch any other parts of the mechanism. Replace the movement in the bowl and screw on the bezel.

161. Effect of change of temperature.—When marked changes of climate are encountered, as when on a long voyage, the chronometer rate will change, and this change is universally recognized as due to the changes of the temperature experienced by the chronometer; Hartnup's law, governing the peculiarities developed under such circumstances, as usually stated, reads as follows: "Every chronometer goes fastest in some certain temperature, called the temperature of compensation, and this can be calculated for each chronometer from rates determined in three fixed temperatures. As the temperature varies either side of this temperature of compensation, the chronometer goes slower, and the rate varies as the square of this variation in degrees."

This should be the mean temperature to which chronometers are subjected; and considering their actual use, for navy chronometers, it is approximately 69° F.

General equation.—If $\theta°$ be the temperature of compensation, r the chronometer rate at that temperature, z the temperature constant or change of rate for one degree of temperature either side of $\theta°$, $\theta_1°$ the temperature for which the corresponding rate is r_1, then the effect of temperature alone is expressed by the general equation:

$$r_1 = r + z\,(\theta° - \theta_1°)^2.$$

This equation serves for temperatures between 45° F. and 90° F., but outside these limits, the change of rate is proportional to a higher power than the square.

The quantities involved in this equation differ for every chronometer. For the same chronometer r will vary, but so long as the temperature compensation is maintained the same, that is, so long as the compensating balance is not changed, $\theta°$ and z will remain practically constant.

The equation is that of a parabola. The requirements of the equation are satisfied by the general equation of a parabola,

$$y^2 = 4ax, \text{ or } x = \frac{y^2}{4a}.$$

Taking the axis of X in the line representing the temperature of compensation, if the ordinate y is the variation of temperature in degrees from the temperature of compensation, the abscissa x is the change in rate for this variation, then $y = \theta^\circ - \theta_1{}^\circ$ and $x = r_1 - r$; and for the value of $y =$ unity, $x = \frac{1}{4a} = z$.

After having found θ°, z, and r, any number of points on the parabola may be found, by solving the general equation, assuming $\theta_1{}^\circ$ at intervals of 5°, and finding the corresponding r_1.

To find θ, z, and r.—At the U. S. Naval Observatory all chronometers are subjected to two tests in the temperature room, the variation being from 90° F. to 50° F. in the first; and from 50° F. to 90° F. in the second. The chronometers are exposed in each test for one week to certain predetermined temperatures under certain fixed rules; the errors being determined at the beginning and end of each week, the daily rates for the several temperatures are obtained from them.

1 The data found at the mean temperatures of 55°, 70°,
2 and 85° F. are used in the general formula for the determina-
tion of θ, z, and r, and from these values, a curve is constructed
3 for each chronometer. This curve with rates plotted up to
4 date, known as Form No. 1, or "Rate Curve for Temperature at Observatory," accompanies a chronometer when issued. The same record sheet contains Form No. 2, or "Rate Curve and Observations on Board Ship," and Form No. 3, or "Observed Errors and Mean Daily Rates and Temperatures." These, with Form No. 4, "Record of Daily Comparisons and Memoranda," should receive the navigator's close attention.

162.—Sea temperature curve.—It is not at all likely that the chronometer will have the same rate on board as at the observatory, though a new curve, if determined, may prove very similar to that found at the observatory; therefore, when a rate has been determined on board, and is found to plot off the curve, the difference between it and the curve rate for that temperature will be a constant to be applied to all rates taken off the curve.

Any navigator can compute and plot the curve of his chronometers, especially after they have been subjected to a wide range of temperature. To do this let *a*, *b*, and *c* each, be the mean of several rates that differ but little at a low, a mean, and a high temperature, and *d*, *e*, and *f*, the means of their corresponding temperatures; then, by substitution in the general equation

$$r_1 = r + z\,(\theta^\circ - \theta_1{}^\circ)^2\text{, we have}$$

$$a = r + z\,(\theta^\circ - d^\circ)^2,$$

$$b = r + z\,(\theta^\circ - e^\circ)^2,$$

$$c = r + z\,(\theta^\circ - f^\circ)^2.$$

$$\text{Whence, } \theta^\circ = \frac{(b-c)\,(d^2-e^2)-(a-b)\,(e^2-f^2)}{2[(a-b)\,(f-e)-(b-c)\,(e-d)]}$$

$$z = \frac{a-b}{(\theta-d)^2-(\theta-e)^2} = \frac{b-c}{(\theta-e)^2-(\theta-f)^2} = \frac{c-a}{(\theta-f)^2-(\theta-d)^2}$$

$$r = a - z(\theta-d)^2 = b - z(\theta-e)^2 = c - z(\theta-f)^2.$$

In the absence of Form No. 2, take a sheet of profile paper evenly ruled both ways. Let the horizontal lines represent degrees numbered at the left hand, and the vertical lines tenths of seconds numbered at the top. Take one vertical line as the zero of rate, depending on the size and sign of rate, let all rates to right be plus or gaining, to the left be minus or losing. Plot the points for every 5°, and trace in the curve. The intersection of this curve with any temperature will be the mean rate for that temperature read from above.

163. Hack chronometer and comparing watch.—It has already been said that the chronometers should not be subject even to occasional removal. This is true, and in order to get the chronometer times of certain desired instants, as the instant of receipt of the noon signal, or the drop of a time ball, etc., use must be made of a less valuable time piece known as the Hack, an inferior grade of chronometer, or of a comparing watch. The watch is in constant use at sea for marking times of observations, and in order to get the chronometer times of the observations, the watch should be compared just before or just after (preferably both) with the standard. In each case the watch reading at comparison should be subtracted from the chronometer reading at comparison, adding 12 hours to the latter, if necessary for the subtraction; so that the difference will always be in the form of $C - W$, and, therefore, ~~additive to a given watch time~~.

Example: Just before taking a sight the watch was compared with the standard chronometer:

$$C = 1^{h}\ 30^{m}\ 22^{s},\ W = 8^{h}\ 20^{m}\ 30^{s}.$$

W. T. of observation $8^{h}\ 31^{m}\ 03^{s}$. Find C. T. of observation.

$$\begin{array}{rl}
C = & 1^{h}\ 30^{m}\ 22^{s} \\
W = & 8\ \ \ 20\ \ \ 30 \\
\hline
C - W = & 5^{h}\ 09^{m}\ 52^{s} \\
W = & 8\ \ \ 31\ \ \ 03 \\
\hline
\text{C. T. of obs.} & 1^{h}\ 40^{m}\ 55^{s}.
\end{array}$$

Should the $C - W$ obtained before and after the observation, or the instant for which the chronometer is desired, differ in value, then the correct $C - W$ at the moment of observation must be obtained by interpolation; as the total change in the $C - W$ occurs between the times of comparisons, the proportional change from first comparison to time of observation must be applied to that time, or the first $C - W$.

164. Torpedo boat watches.—For use in torpedo boats the Department issues, instead of chronometers, stem-winding, lever-escapement watches that run for about 30 hours. An extra crystal and main spring are provided for each watch.

Stowage.—The watch is kept in a wooden inner box and a padded transporting case, and secured where it will be least affected by magnetic fields, boat's vibrations, changes of temperature, or moisture.

Winding.—Watches should be wound daily at a given time, preferably at 8 a. m., care being taken not to press the spring on the rim near the stem that releases the hour and minute hands, and thereby turn them. Should the hands be accidentally turned, reset the watch by turning the hands forward, never backward. In winding be careful not to turn the stem with one hand and the case with the other; the case should be firmly held in one hand, and the watch wound by turning the stem in the other hand with a careful uniform motion, coming gradually to a stop.

General care.—The back should be kept closed, avoiding thereby, as much as possible, injury to the works through dust and moisture. However, if a watch gets wet, it should be opened and drained, then immersed in alcohol for several minutes that all moisture may be absorbed; after removal, allow any remaining alcohol to drain and evaporate, then immerse and keep the watch in good lubricating oil till it can be sent for repairs.

The watch should never be carried into a dynamo room nor allowed ~~to come under the influence of an electric field~~.

It should never be allowed to hang freely as from a hook, nor subjected to sudden jerky vibratory motions.

It should be cleaned and oiled at least once in three years, and then only by an expert watchmaker.

Preparations for shipment by express.—Let the watch almost run down, open the back, stop the balance very gently with a piece of tissue paper, and insert a thin sliver of cork or a piece of folded tissue paper between it and the next wheel with the least possible force; close the watch, put it in its box, pack with cotton in the transporting case, wrap and mark the package, "Delicate Instrument, Handle with Care."

165. Stop and comparing watches.—The above remarks as to winding, care, examination, and cleaning apply to the stop and comparing watches furnished to a navigator.

CHAPTER XI.

COMPARISON OF SIDEREAL AND TROPICAL YEARS.—THE CIVIL YEAR.—THE CALENDAR.—RELATION OF SOLAR AND SIDEREAL TIMES.

166. In considering the apparent motion of the sun in the ecliptic, two methods are used in finding the apparent time of its revolution, giving rise to two different years. In the first method a year is the interval between two successive apparent passages through the same equinoctial point, or the interval between two successive apparent crossings of the plane of the equator at the first point of Aries.

These particular times are most easily and accurately observed at astronomical observatories; and this period, called the tropical or equinoctial year, forms the basis of time in civil life, since the changes of the sun's declination, and, in consequence, the recurrence of the seasons depend upon it.

Repeated determinations show the length of the tropical year to be $365^d\ 5^h\ 48^m\ 47^s.8$, or $365^d.2422$ of mean solar time.

167. The calendar and the civil year.—Like the modern Mohammedan calendar, the Roman calendar up to 45 B. C. was based upon the lunar year of 12 months or 355 days. In early times, many of the religious observances were connected with the times of new and full moon, and for this reason the priesthood made the calendar purely lunar, notwithstanding the fact that, by so doing, the seasons were caused to fall in different months in succeeding years, and much confusion resulted.

In 45 B. C., Julius Cæsar reformed the calendar, introducing what is known as the Julian calendar. Adopting 365¼ days as the proper length of the tropical year, but, recognizing the importance for the ordinary purposes of life of a year containing an exact number of days, he ordered that the civil year should consist of 365 days, except that in every fourth year an extra day should be inserted, making the leap year 366 days; he also ordered that the year should begin on January 1, which in 45 B. C. was the day of the new moon next following the winter solstice. Up to that time the year had begun in March, and this change altered the length of the preceding year 46 B. C., and, in consequence, that year is known as the "year of confusion."

This calendar was adopted by the Council of Nice in 325 A. D., in which year the vernal equinox fell on March 21. The average length of the year being 365¼ days in the Julian calendar, some arrangement should have been made to allow for the difference between it and the tropical year, which, however, was not done.

Owing to accumulated errors, the vernal equinox fell on March 11, in 1582, at which time Pope Gregory XIII reformed the calendar by omitting 10 days and bringing the vernal equinox back to March 21.

To guard against further error he established what is known as the Gregorian correction, which, when made, will prevent any appreciable error even in several thousand years. Regarding the year as 365¼ days, an annual error of about $11^m\ 12^s$ is introduced. This accumulated error amounts to about three days in 400 years, and to eliminate this, three of the inserted days are to be left out every 400 years, and they are to be omitted from those leap years completing a century not divisible by 400. Thus 1700, 1800, 1900 were not leap years.

168. The sidereal year.—The second method of determining a year considers the interval between the sun's apparently leaving and returning to the same position relative to the stars. This is the correct astronomical period of the apparent motion of the sun through an arc of 360° of the ecliptic, or of one complete revolution of the earth around the sun. This period is called a sidereal year, and repeated determinations show the length of this year to be 365 days, 6 hours, 9 minutes, 9.6 seconds or $365^d.25636$ of mean solar time.

The difference in length of the tropical and sidereal years is due to the precession of the equinoxes, which causes an annual movement of the first point of Aries to the westward of $50''.22$ of arc.

169. Relation of solar and sidereal time.—From the definitions already given of a sidereal and a solar day (Art. 141), and from a consideration of the apparent continuous motion of the sun towards the East in the ecliptic, thus causing the sun to move towards the West in the diurnal movement of the heavens more slowly than the vernal equinox, it is plain that the solar day is longer than the sidereal day; and in the interval of time in which the sun makes a complete revolution in its apparent orbit, that is, in a sidereal year, the mean sun, owing to its apparent movement to the eastward, falls behind the stars in diurnal movement 360°, or 24 hours, so that in a sidereal year, the number of daily revolutions which the sun appears to make about the earth is less by one than the number of daily revolutions made by the vernal equinox. The sidereal year, therefore, which contains $365^d\ 6^h\ 9^m\ 9^s.6$ of mean solar time, contains $366^d\ 6^h\ 9^m\ 9^s.6$ of sidereal time.

CHAPTER XII.

TIME.—CONVERSION OF ARC INTO TIME AND VICE VERSA.—RELATION OF THE L. S. T., H. A., AND R. A. OF THE SAME BODY.—FINDING THE EQUATION OF TIME, ASTRONOMICAL TIME, AND GREENWICH TIME AND DATE.—GAIN OR LOSS OF TIME WITH CHANGE OF POSITION.—CROSSING THE 180TH MERIDIAN.—STANDARD TIME.

170. **Time** is the hour angle of some heavenly body, or of a fixed point in the heavens, whose apparent diurnal motion is taken as a measure of duration. The earth's motion on its axis, being perfectly uniform, furnishes the standard of measurement; and hence time is measured by the interval between two successive transits of a heavenly body, or of some fixed point in the heavens, over the same branch of a meridian, this interval being called a day. The apparent revolution of the heavens is due to the rotation of the earth on its axis, and, as this rotation is always performed in the same time, the interval above referred to would be the same, whether measured by the apparent motion of the sun, moon, a star, or a fixed point in the heavens, were it not for the apparent and real movements of these bodies. In navigation and astronomy, three kinds of time are used, depending on the celestial point or body whose successive transits over the same branch of a meridian determine the day.

171. The three days are the sidereal, apparent solar, and mean solar days, each of which days is divided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds; the subdivisions of the sidereal day being sidereal time, of the apparent solar day apparent time, and of the mean solar day mean time.

A sidereal day has already been defined as the interval of time between two successive transits of the vernal equinox, or the first point of Aries, over the upper branch of the same meridian. The sidereal day at a place is regarded as commencing when that point is on the upper branch of the meridian, and the sidereal hour angle is then $0^h\ 0^m\ 0^s$. This should be the reading of sidereal clocks at that instant when their error on local sidereal time is zero. The sidereal time at any instant is the hour angle of the first point of Aries, reckoned as already explained for hour angles.

The apparent solar day is the interval of time between two successive transits of the true sun over the upper branch of the same meridian, and apparent time at any instant at a given place is the true sun's hour angle, reckoned as already explained for hour angles. This is the time to which the deck clocks at sea are regulated. Apparent noon is the instant of the true sun's upper transit, or when its hour angle is $0^h\ 0^m\ 0^s$.

Since the true sun's apparent motion is in the ecliptic and not in the equinoctial, and the motion in the ecliptic is not uniform, its change in right ascension is not uniform, and apparent solar days are of unequal length. For this reason, apparent time cannot be kept by clock mechanism, which requires a standard of time that can be subdivided into unvarying lengths.

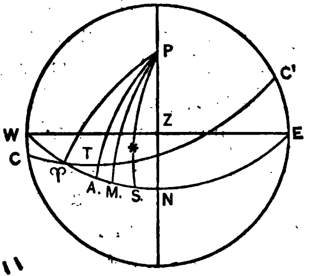
The sidereal days are of uniform length, and sidereal time is kept by sidereal clocks at fixed observatories. Owing, however, to the daily increase of the sun's right ascension, the vernal equinoctial point crosses the meridian approximately $3^m\ 56^s$ earlier each day by solar time, so that, whilst the local sidereal time of apparent noon on March 21 is approximately $0^h\ 0^m\ 0^s$, on September 21 it is approximately 12 hours, and on the next 21st of March it is approximately 24 hours or 0 hours again. As solar time determines the question of light and darkness, which in turn regulates the hours of the business world, it is evident that sidereal time is not suited for the ordinary and practical purposes of life, bearing, as it does, no simple relation to the phenomena of day and night.

The mean sun and mean time.—Since the time for general use must be uniform, and since the true sun is the body which would naturally furnish a measure of time, if its motion were regular, it becomes necessary to adopt, instead of the true sun, a fictitious sun called a mean sun, which is assumed to move in the plane of the equinoctial and to increase its right ascension uniformly, that is, to move in the equinoctial at the mean rate of the true sun in the ecliptic, and the time measured by the motions of this mean sun is called mean time.

A mean solar day is the interval between two successive transits of the mean sun over the upper branch of the same meridian, and mean solar time at a given place is the hour angle of the mean sun at that instant. Mean noon is the instant when the mean sun is on the upper branch of a meridian, the hour angle of the mean sun being then $0^h\ 0^m\ 0^s$.

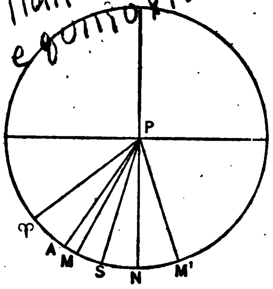
This is the time kept by the ordinary clocks and watches, and by the chronometers carried on shipboard to give navigators Greenwich mean time.

172. Equation of time.—The difference at any instant between apparent and mean solar time is the equation of time. It is also the difference between the right ascensions of the true and mean suns; in other words, it is the difference between the true sun's right ascension and mean longitude, the true sun's mean longitude being the same as the right ascension of the mean sun.



Plane of Horizon

FIG. 98.



Plane of equinoxial

FIG. 99.

In Fig. 98, a projection on the plane of the horizon and in Fig. 99, a projection on the plane of the equinoctial,

P is the pole, Z is the zenith, and PN is the meridian.

W is the West, and E the East point of the heavens.

WNE the equinoctial.

CC' the ecliptic.

$P♈$ the hour circle through the first point of Aries.

PA the hour circle through the true sun.

$♈PA$ the right ascension of the true sun.

PM the hour circle of the mean sun.

$♈PM$ the right ascension of the mean sun.

NPM the hour angle of the mean sun, or L. M. T.

NPA the hour angle of the true sun, or L. A. T.

MPA the equation of time $= ♈PM - ♈PA = NPA - NPM$.

173. Relation of local sidereal time, the hour angle, and right ascension of a given body.—In Fig. 98 and Fig. 99 $P♈$, PA, PM, and PS are hour circles through the first point of Aries, true sun, mean sun, and a star (or moon or planet), respectively;

$♈PN$ equals the right ascension of the meridian equals local sidereal time;

√

ΥPA equals the right ascension of the true sun;
ΥPM equals the right ascension of the mean sun;
ΥPS equals the right ascension of a star (moon or planet);
NPA equals the hour angle of the true sun;
NPM equals the hour angle of the mean sun;
NPS equals the hour angle of a star (moon or planet),

$$\text{but } \Upsilon PN = \Upsilon PA + NPA;$$

therefore, *local sidereal time equals the right ascension plus the hour angle of the true sun at the same instant;*

$$\text{also } \Upsilon PN = \Upsilon PM + NPM, \text{ or}$$

local sidereal time equals the right ascension of the mean sun plus the hour angle of the mean sun at the same instant;

$$\text{also } \Upsilon PN = \Upsilon PS + NPS, \text{ or}$$

local sidereal time equals the right ascension of a star (moon or planet) plus the hour angle of the same star (moon or planet) at the same instant. In case any one of these bodies is East of the meridian so that its hour circle is PM′, the hour angle will be considered negative,

$$\text{and } \Upsilon PN = \Upsilon PM' - NPM'.$$

To state the case generally, **the local sidereal time at any instant equals the right ascension plus the hour angle of the same heavenly body at that instant** (see Art. 141).

In this connection, we may now more clearly define the hour angle of a heavenly body as the angle at the pole between the celestial meridian and the hour circle passing through the body, and which indicates in hours, minutes, and seconds the time elapsed since that body was on the upper branch of the meridian; the length of said hours, minutes, and seconds, and, hence, the duration of time required by the body to describe its hour angle depending on the day established by that body's diurnal motion, and the length of the day depending upon the rate of the body's real motion, *i. e.*, or the rate of change of its right ascension

✓

Thus the moon, the planets, the stars, the true sun, and the mean sun, all have different rates of speed in their apparent diurnal motion, and while the hour angle of any one of them might, for example, be 3 hours, the length of time required for each to pass from the meridian through an hour angle of 3 hours, measured by some independent standard, would be different for each one.

Commence at noon

174. Astronomical time.—The solar day (apparent or mean) is regarded by astronomers as commencing at noon (apparent or mean), when the sun (apparent or mean) is at its upper culmination, and is reckoned from 0 hours at that time to 24 hours at the next upper culmination of the same body. The day so considered is called the astronomical day.

175. Civil time.—The time used in the ordinary phases of life is called civil time. It begins at midnight, 12 hours before the astronomical day of the same date, and is divided into two periods of 12 hours each, marked a. m. and p. m.

176. Rules for conversion of civil into astronomical time.—*If civil time is p. m., drop p. m. and the hours, minutes, and seconds will be those of the astronomical time of the same date. If civil time is a. m., subtract one from the date, add 12 to the hours, and drop the a. m.*

Examples.

✓ February 10, 2 p. m., civil time, is February 10, 2 hours, astronomical time.

✓ March 3, 8 a. m., civil time, is March 2, 20 hours, astronomical time.

To convert astronomical time into civil time.—*If the astronomical time is less than 12 hours, it will be the civil time p. m. of the same date, so simply add p. m.*

If the astronomical time is greater than 12 hours, add one to the date, reject 12 hours, and add a. m.

Examples.

March 21, 3 hours, astronomical time, is March 21, 3 p. m., civil time.

November 10, 15 hours, astronomical time is November 11, 3 a. m., civil time.

177. Standard time.—This is the time of meridians 15° apart known as standard meridians; the time of any standard meridian is used, for the convenience of railways and in the business world, in a belt of territory extending as nearly as possible 7½° each side of that standard meridian.

The standard meridians used in the United States and Canada are the 60th, 75th, 90th, 105th, and 120th meridians West from Greenwich; the times being known as Intercolonial, Eastern, Central, Mountain, and Pacific, respectively. In Alaska, the time of the 135th meridian is kept.

15° = 1hr.

To reduce local mean time to standard time.—*If the local meridian is West of the standard meridian, add the difference of longitude in time to the local time; if the local meridian is East of the standard meridian, subtract the difference of longitude from the local time to obtain the standard time.*

178. Conversion of arc and time.—The elements of the Nautical Almanac are tabulated for Greenwich time, and to obtain them for a given local instant, the longitude in time must be known. If expressed, as usual, in degrees it must be properly converted.

Under the subject of hour angles and hour circles, the relation between arc and time has been shown, and further reflection will show that, as the earth revolves on its axis, 360° of its surface as measured along the equator pass under the sun in 24 hours, or 15° in one hour. Since longitude is measured

1° = 4 min
1' = 4 sec

along the equator it may be expressed in arc or time, the relation being:

15° of arc = 1 hour of time or 1° of arc = 4 minutes of time.

15' of arc = 1 minute of time or 1' of arc = 4 seconds of time.

15" of arc = 1 second of time or 1" of arc = $\frac{1}{15}$ second of time.

If X be a given number of degrees or minutes of arc, then, remembering that $\frac{X}{15}$ (result in hours or minutes) will be $\frac{X}{15} \times 60 = X \times 4$ (result, respectively, in minutes or seconds depending on whether X is in degrees or minutes), any number of degrees or minutes not exactly divisible by 15 may be reduced to the lower denomination in time by multiplying by 4, and the reverse also holds true.

Hence, to convert arc into time:

(1) Divide the degrees of arc by 15; the result will be hours.

(2) Multiply remaining degrees, if any, by 4; the result will be minutes of time.

(3) Divide minutes of arc by 15; the result will be minutes of time, to be added to minutes of time obtained by rule (2).

(4) Multiply remaining minutes of arc by 4; the result will be seconds of time.

(5) Divide seconds of arc by 15; the result will be seconds and decimals of a second of time, to be added to seconds of time obtained by rule (4).

To convert time into arc:

(1) Multiply the hours by 15; the result will be degrees.

(2) Divide the minutes of time by 4; the result will be degrees, to be added to the degrees obtained by rule (1).

(3) Multiply the remaining minutes by 15; the result will be minutes of arc.

(4) Divide seconds of time by 4; the result will be minutes of arc, to be added to the minutes of arc obtained by rule (3).

(5) Multiply remaining seconds and decimals by 15; the result will be seconds of arc.

Ex. 72.—Convert 111° 32′ 40″ into time.

$$\frac{111^\circ}{15} = 7^h \text{ with remainder } 6 = 7^h\ 24^m\ 00^s$$

$$\frac{32'}{15} = 2^m \text{ with remainder } 2 = \quad 2\quad 08$$

$$\frac{40''}{15} = \quad 2.67$$

$$\text{Therefore, } 111^\circ\ 32'\ 40'' = 7^h\ 26^m\ 10^s.67$$

Ex. 73.—Convert $9^h\ 58^m\ 14^s.5$ into arc.

$$9^h \times 15 = 135^\circ\ 00'\ 00''$$

$$\frac{58^m}{4} = 14^\circ \text{ with remainder } 2 = 14\quad 30\quad 00$$

$$\frac{14^s.5}{4} = 3' \text{ with remainder } 2.5 = \quad 3\quad 37.5$$

$$\text{Therefore, } 9^h\ 58^m\ 14^s.5 = 149^\circ\ 33'\ 37''.5$$

Table 7 of Bowditch gives the inter-conversion by inspection.

Examples.—Convert into arc.

(74)	$6^h\ 15^m\ 32^s$	*Ans.*	$93^\circ\ 53'\ 00''$
(75)	10 53 45	*Ans.*	163 26 15
(76)	11 35 13	*Ans.*	173 48 15

Convert into time.

(77)	$29^\circ\ 43'\ 30''$	*Ans.*	$1^h\ 58^m\ 54^s$
(78)	155 13 43	*Ans.*	10 20 54.87
(79)	177 15 30	*Ans.*	11 49 02

179. Relation between the local times at two places.—In comparing corresponding times at different meridians, since the hour angles increase positively to westward, the most easterly meridian is that at which the hour angle of the body, or the time, is the greatest; and, since the longitude of a place is the inclination of the meridian of that place to that of Greenwich, it is the Greenwich hour angle of a body, when on the local meridian. Therefore, if the longitude be added to the hour angle of a heavenly body at a given place, the result will be that body's hour angle from Greenwich.

Let λ_1 and λ_2 be the longitudes of two places; t_1 and t_2, the hour angles at the two places of one and the same body or point, as for instance the vernal equinox, of the true or mean sun, and hence the local sidereal time, local apparent or mean time. Then, using the quantities at the first meridian,

the Greenwich sidereal, apparent, or mean time $= \lambda_1 + t_1$;
and using the quantities at the second meridian,

the Greenwich sidereal, apparent, or mean time $= \lambda_2 + t_2$.

Therefore, $\lambda_1 + t_1 = \lambda_2 + t_2$ and $\lambda_2 - \lambda_1 = t_1 - t_2$, or *the difference between the local times at two places equals the difference of their longitudes, the times being sidereal times, apparent times, or mean times.* This is shown graphically in Fig. 100.

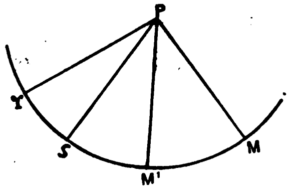


FIG. 100.

Let *PM* and *PM'* be the meridians of two places, and *MPM'* the difference of longitudes of the two meridians; *PS*, the hour circle of the sun (apparent or mean); *P*♈, the equinoctial colure (an hour circle); then *MPS* is the hour angle of the sun (apparent or mean) at all places on the meridian *PM*, *M'PS* is the hour angle of the same body at all places on the meridian *PM'*, *MP*♈ and *M'P*♈ are the corresponding sidereal times at the two meridians, and $MPM' = MPS - M'PS = MP♈ - M'P♈$; or the difference of the times at two places, whether apparent, mean, or sidereal times, equals the difference of longitudes of the two places.

If PM be the meridian of Greenwich, or the origin of longitudes, the difference of longitude of Greenwich and places on the meridian PM' will be the longitude of those places from Greenwich, in other words, MPM' is the longitude of the meridian PM'; MPS becomes the Greenwich time (apparent or mean), and $MP♈$ becomes Greenwich sidereal time, and $MPM' = MPS - M'PS = MP♈ - M'P♈$; or the longitude of any meridian equals the difference between the time at that meridian and the Greenwich time for the same instant.

If G. T. represents Greenwich time, and L. T. represents local time at a place in longitude λ, then, if λ is West longitude, the meridian of Greenwich is to the eastward of the local meridian and G. T. is greater than L. T.; therefore,

$$\text{G. T.} = \text{L. T.} + \lambda,$$

G. T. and L. T. being reckoned astronomically from 0 hours to 24 hours, to the westward. However, if λ is East longitude, the local meridian is to the eastward of the Greenwich meridian, the local time is greater than the corresponding Greenwich time, and

$$\text{G. T.} = \text{L. T.} - \lambda.$$

Hence the following rules in finding the Greenwich time and date:

(1) *Express the local or ship's date and time astronomically in days, hours, minutes, and seconds.*

(2) *To this local astronomical date and time add the longitude in time, if West, and the sum in days, hours, minutes, and seconds will be the Greenwich astronomical date and time.*

(3) *If in East longitude, subtract the longitude in time from the local astronomical time expressed in days, hours, minutes, and seconds, and the result will be the Greenwich astronomical date and time.*

Ex. 80.—In Long. 76° 26′ W., the local time being 8ʰ 09ᵐ 41ˢ p. m., April 3, find the Greenwich time.

	d	h	m	s
Local astronomical time April	**3,**	**8**	**09**	**41**
Longitude in time West	**+**	**5**	**05**	**44**
Greenwich astronomical time April	**3,**	**13**	**15**	**25**

Ex. 81.—In Long. 70° 22′ 00″ W., the local time being 8ʰ 48ᵐ 17ˢ a. m., January 6, find the Greenwich time.

	d	h	m	s
Local astronomical time Jan.	**5,**	**20**	**48**	**17**
Longitude in time West	**+**	**4**	**41**	**28**
Greenwich astronomical time Jan.	**6,**	**1**	**29**	**45**

Ex. 82.—In Long. 103° 58′ E., the local time being February 15, 7ʰ 35ᵐ 40ˢ a. m., find the Greenwich time.

	d	h	m	s
Local astronomical time Feb.	**14,**	**19**	**35**	**40**
Longitude in time East	**(−)**	**6**	**55**	**52**
Greenwich astronomical time Feb.	**14,**	**12**	**39**	**48**

Ex. 83.—In Long. 135° 15′ E., the local time being January 20, 5ʰ 10ᵐ 30ˢ p. m., find the Greenwich time.

	d	h	m	s
Local astronomical time Jan.	**20,**	**5**	**10**	**30**
Longitude in time East	**(−)**	**9**	**01**	**00**
Greenwich astronomical time Jan.	**19,**	**20**	**09**	**30**

By pursuing a course just the reverse of the above, subtracting West longitudes from, and adding East longitudes to given Greenwich times expressed astronomically, the local astronomical times may be found and converted into local civil times.

180. To find the Greenwich date and mean time from the time data of an observation.—Navigators are supplied with chronometers from which to obtain the Greenwich mean time of their observations, and for this time the various elements involved, such as declination, right ascension, etc., are taken from the Nautical Almanac.

Having the error on G. M. T. and the daily rate before leaving port, the error of the chronometer at any given instant can be found.

To the watch time of an observation add the C—W, or the difference between the chronometer and watch, obtained by comparison just before or after the observation, to get the corresponding chronometer time. To this chronometer time, apply its error on Greenwich mean time, adding if the chronometer is slow, subtracting if it is fast. The result, or the result plus 12 hours, will be the G. M. T. This ambiguity in the number of hours arises from the fact that chronometer dial plates are graduated like watches from 0 hours to 12 hours, instead of from 0 hours to 24 hours, and it is necessary to know whether the chronometer is a. m. or p. m. in order to fix the true Greenwich time and date. However, there need be no trouble or ambiguity, if approximate Greenwich time and date are gotten from the approximate local time by the rules just given in Art. 179, and compared with the result obtained as above from the watch time and chronometer comparison.

The following examples will show how to decide whether the G. M. T., in a given case, is the corrected chronometer face or this quantity plus 12 hours.

Ex. 84.—October 31, about 5 a. m., the time data of an observation were, W. 7h 25m 12s, C—W 1h 44m 17s, chronometer fast on G. M. T. 27m 31s, Long. 8h E. Find G. M. T.

		d	h	
Civil time Oct. 31 (a. m.)			5	**approx.**
Local ast. time Oct.		30,	17	**approx.**
Longitude East	(—)		8	
Greenwich ast. time Oct.		30,	9	**approx.**

		h	m	s
W.		7	25	12
C—W		1	44	17
C.		9	09	29
C. C.	(—)		27	31
G. M. T. Oct. 30,		8	41	58

These results are so close as to remove all doubt.

Ex. 85.—November 17, about 10h 53m a. m., in Long. 2h 20m 10s E., the time data of an observation were W. 4h 15m 27s, C—W 4h 07m 20s, chronometer slow on G. M. T. 10m 15s. Find G. M. T.

		d	h	m	s	
Civil time Nov. 17 (a. m.)			10	53		**approx.**
Local ast. time Nov.		16,	22	53		**approx.**
Long. East	(—)		2	20	10	
Greenwich ast. time.		Nov. 16,	20	32	50	**approx.**

		h	m	s
W.		4	15	27
C—W		4	07	20
C.		8	22	47
C. C.	+		10	15
G. M. T. Nov. 16,		20	33	02

It will be noticed that in the above example 12 hours must be added to the chronometer time and the date of the previous day taken to make the two Greenwich times and dates more nearly agree.

It may sometimes be the case that the watch by which observations are taken is not regulated to local time, in fact, may be far out, and that the approximate time of observation is known no closer than by the words a. m. or p. m. (as in the case of set examples). But the result may be reasoned out correctly thus: Apply the C—W and C. C. to the W. T. of observation. The corrected chronometer reading, or this reading plus 12 hours, will be the required G. M. T.

To determine which proceed thus: Express the given approximate local civil time astronomically and find the astronomical date and hour-limits between which the true local time should lie, these limits being determined by the words a. m. or p. m.; thus to illustrate, for Sept. 15, a. m., the limits of astronomical date and time would be Sept. 14, 12^h to 24^h, and for Oct. 10, p. m., they would be Oct. 10, 0^h to 12^h.

Apply the longitude (adding if West, subtracting if East) to the local astronomical date and hour-limits, and the result will be the Greenwich astronomical date and hour-limits between which the true Greenwich time must lie.

Then, if the corrected chronometer reading falls between the Greenwich limits, it is the correct G. M. T.; if not, add 12^h to the corrected chronometer reading and the result, falling between the limits, will be the G. M. T., the date in either case being that of which the hours are a part.

Ex. 86.—November 15, a. m. time, in Long. 10^h W., W. T. of an observation was $7^h\ 50^m\ 30^s$, C—W $6^h\ 10^m\ 20^s$, C. C. + $4^m\ 20^s$. Find the G. M. T.

	h	m	s
W.	7	50	30
C—W	6	10	20
C.	2	00	50
C. C.	+	4	20
Corrected chro. reading.	2	05	10

Approximate local civil time Nov. 15, a. m.

Local astronomical date and hour limits.	**Nov. $14^d\ 12^h$ to $14^d\ 24^h$**
Longitude West	**+ 10 10**
Greenwich astronomical date and hour limits.	**Nov. $14^d\ 22^h$ to $15^d\ 10^h$**

The corrected chronometer reading falls between the limits, the hours are of the 15th, ∴ G. M. T. is Nov. 15, $2^h\ 05^m\ 10^s$.

Ex. 86(a).—July 11, p. m. time, in Long. 150° E., the time data of an observation were as follows: W. = $9^h\ 12^m\ 17^s$, C—W $8^h\ 15^m\ 14^s$, C. C. + $10^m\ 20^s$. Find G. M. T.

	h	m	s
W.	9	12	17
C—W	8	15	14
C.	5	27	31
C. C.	+	10	20
Corrected chro. reading.	5	37	51

Approximate local civil time July 11, p. m.

Local astronomical date and hour limits.	**July $11^d\ 0^h$ to $11^d\ 12^h$**
Longitude East	**−10 10**
Greenwich astronomical date and hour limits.	**July $10^d\ 14^h$ to $11^d\ 2^h$**

The corrected chronometer reading does not fall between the above limits, so adding 12^h to it gives $17^h\ 37^m\ 51^s$, which falls between the limits; the hours are a part of the 10th; and ∴ G. M. T. = July 10, $17^h\ 37^m\ 51^s$.

In case the local time is given as an exact time and the longitude as merely East or West, the same method holds good.

Ex. 87.—Jan. 10, 8 a. m., in West longitude, given the following data: W. T. $9^h\ 40^m\ 30^s$, C—W $5^h\ 20^m\ 20^s$, C. C. + $5^m\ 10^s$. Find G. M. T.

	h	m	s
W.	9	40	30
C—W	5	20	20
C	3	00	50
C. C.	+	5	10
Corrected chro. reading.	3	06	00

Local civil time Jan. 10, 8 a. m.

Local astronomical time	Jan.	$9^d\ 20^h$
Longitude West	+	0 to 12^h
Greenwich astronomical date and hour limits.	Jan.	$9^d\ 20^h$ to $10^d\ 8^h$

The corrected chronometer reading falls between the limits, the hours being of the 10th. ∴ G. M. T. = Jan. 10, $3^h\ 06^m\ 00^s$.

If both time and longitude are given as within a twelve-hour range; *i. e.,* time only as a. m. or p. m. and longitude only as E. or W., the limits of the approximate Greenwich astronomical time will be 24 hours apart and two solutions will result.

Ex. 87(a).—Aug. 10, a. m. time, in West Long., given the following data: W. T. $8^h\ 00^m\ 10^s$, C—W $6^h\ 40^m\ 20^s$, C. C. + $5^m\ 30^s$. Find G. M. T.

	h	m	s
W.	8	00	10
C—W	6	40	20
C	2	40	30
C. C.	+	5	30
Corrected chro. reading.	2	46	00

Approximate local civil time Aug. 10, a. m.

Local astronomical date and hour limits.	Aug.	$9^d\ 12^h$ to	$9^d\ 24^h$
Longitude West	+	0^h—12^h	0^h—12^h
Greenwich astronomical date and hour limits.	Aug.	$9^d\ 12^h$ to	$10^d\ 12^h$

Here both the corrected chronometer reading and that reading + 12 hours fall between the limits, and hence we have a double solution:

$$\text{G. M. T.} = \text{Aug. } 9,\ 14^h\ 46^m\ 00^s,$$
$$= \text{Aug. } 10,\ 2^h\ 46^m\ 00^s.$$

Examples for practice. Find G. M. T., given,

	W. T. of Obs.	C—W	C. C.	Approx. Local date.	Long.	Answers.
	h m s	h m s	m s		°	h m s
8)	3 23 43	6 11 33	(—) 7 18	April 23, a. m.	90 W	April 22, 21 27 58
9)	7 53 26	4 38 56	(—) 9 27.6	Jan. 19, a. m.	30 W	Jan. 19, 0 22 54.4
0)	11 49 33	3 59 30	(—)30 22	Oct. 22, a. m.	120 E	Oct. 21, 15 18 41
1)	5 20 21	3 16 24	(—)25 21	Nov. 5, p. m.	135 E	Nov. 4, 20 11 24
2)	7 35 10	10 20 17	+ 10 04	Nov. 9, 8 a. m.	30 E	Nov. 8, 18 05 31

181. Gain or loss of time with change of position.—Crossing the 180th meridian.—It has been shown that local noon at any meridian is when the sun is on the upper branch of that meridian, that at all places East of that meridian at that instant it is past noon, or time is fast of that of the given meridian; at all places West of that meridian it is not yet noon, or time is slow of that of the given meridian. Hence it is evident that if a navigator travels East, carrying a watch regulated to the time of the meridian departed from, and if he desires to set the watch to the time of a meridian to the eastward, he must set it ahead at the rate of 1 hour for 15° change of longitude, or 24 hours for every 360°; in other words, in going eastward around the world, or through 360° of longitude measured in an easterly direction, he gains a day in his reckoning of time.

In the same way, if sailing westward around the world, or through 360° of longitude measured in a westerly direction, he loses a day in his reckoning of time.

So that if he leaves the given meridian and goes around to the eastward, keeping his time regulated to each successive local meridian, his reckoning of time on return to his point of departure will be one day ahead of the local reckoning; in other words, he would think it, say, Thursday when in reality it was Wednesday.

Had he gone around to the westward, he would have logged his return as Tuesday, if the day in reality was a Wednesday.

To avoid such misconceptions and to keep accurate run of dates, when crossing the meridian of 180°, going eastward, repeat one day; when crossing it, going westward, drop one day from the calendar; at the same time changing the name of the longitude.

Illustration.—Suppose a ship, going eastward, crosses the 180° meridian at local apparent noon, April 10; find the corresponding Greenwich time and date. Then, from this result, considering the longitude as of opposite name to that first used, find the local time and date.

	h	m	s	
Local apparent time	00	00	00	April 10
Longitude 180° East	12	00	00	East
Greenwich apparent time	12	00	00	April 9
Longitude 180° West	12	00	00	West
Local apparent time	00	00	00	April 9

In other words, going eastward, and crossing the 180° meridian, repeat a day.

Suppose the conditions of the illustration to be as above, except the ship is going westward, and the change being from West to East longitude, then,

	h	m	s	
Local apparent time	00	00	00	April 10
Longitude 180° West	12	00	00	West
Greenwich apparent time	12	00	00	April 10
Longitude 180° East	12	00	00	East
Local apparent time	00	00	00	April 11

In other words, going westward and crossing the 180° meridian, omit one day from the calendar.

CHAPTER XIII.

NAUTICAL ALMANAC AND SUBORDINATE COMPUTATIONS.

182. The Ephemeris and Nautical Almanac published by authority of Congress is subdivided into three general parts as follows: Part I, Ephemeris for the meridian of Greenwich, gives the ephemerides of the sun and moon, the geocentric and heliocentric positions of the major planets, the sun's coordinates, and other fundamental astronomical data for equidistant intervals of Greenwich mean time; Part II, Ephemeris for the meridian of Washington gives the ephemerides of the fixed stars, sun, moon, and major planets for transit over the meridian of Washington; and Part III, Phenomena, contains predictions of phenomena to be observed with data for their computation. Tables are also appended for the interconversion of mean and sidereal time and for finding the latitude and azimuth by an altitude of Polaris.

The American Nautical Almanac is a smaller book made up from the "Ephemeris and Almanac" just described, and is designed especially for the use of navigators, being adapted to the meridian of Greenwich throughout. The values and differences of the elements are tabulated to a lower degree of accuracy than in the Ephemeris, but the refinement is sufficient for all computations at sea. It contains the position of the sun and moon, together with the ephemerides of the planets Venus, Mars, Jupiter, and Saturn, and the apparent places of 55 stars for the first of each month and the Greenwich mean time of transit at Greenwich for each of these stars, also the mean places of 110 additional stars; solar and lunar eclipses are described, and the tables for the interconversion of mean and sidereal time and for finding the latitude by Polaris are included.

The elements dependent upon the sun and moon are placed in the first part of the book, arranged according to hours, days, and months of the year. The right ascension of the mean sun for the entire year is given at one opening, also, the mean time of sidereal noon at Greenwich, The declination of the sun, equation of time, the right ascension and declination of the moon and the moon's horizontal parallax and semidiameter are given for every even hour throughout the year. Right ascension and declination of the four brightest planets are given for Greenwich mean noon of each day, also, the Greenwich mean time of transit of the planet over the meridian of Greenwich for each day in the year and, at the bottom of the page, the semidiameter and horizontal parallax of the planet for the first day of each month.

Signs.—In the Nautical Almanac the sign + must be used with the H. D. when the equation of time or declination, if itself positive, is increasing, or if negative, is decreasing numerically; contrariwise, the sign − must be used with the H. D. when the equation of time or sun's declination, if positive, is decreasing, or if negative, is increasing numerically.

In the examples of this book, declinations will be characterized by the letter N. when North, by letter S. when South; and hourly differences by letter N. if the body is moving toward the North, otherwise by the letter S.

183. Greenwich time essential.—The elements must be taken from the Almanac for some definite instant of Greenwich mean time. In computations from observations that depend upon the time of the sun's meridian passage, at which instant the local apparent time is 0^h, and the Greenwich apparent time is equal to the longitude, if West, or to 24^h minus the longitude, if East, it becomes necessary to correct the equation of time for longitude before it is applied to the

Greenwich apparent time to obtain a Greenwich mean time for use in taking out other desired data. This Greenwich mean time is sufficiently correct for all practical purposes, as the equation of time never changes more than $1^s.3$ in an hour.

184. Second differences.—The method of second differences aims at the greatest possible accuracy, and should proceed from the most accurate data obtainable; hence, in cases where a high degree of accuracy is required, as in obtaining the sun's declination and equation of time for use in connection with equal altitudes for chronometer error, the elements should be taken out from the American Ephemeris in which the hourly difference for the equation of time is given to thousandths of a second of time and the hourly difference for the declination to hundredths of a second of arc.

Omit

In the navigator's use of the Nautical Almanac, into which no higher degree of exactness has been admitted in the tabulations than is adequate to meet the needs of the usual course of practice of navigation, the first differences of the elements only need to be regarded, since simple interpolation in which the differences of the quantities are assumed to be proportional to the differences of the times results in sufficient refinement in computations at sea.

As all the examples of this work are meant to be practical, second differences will be used only in the cases of the sun's declination and equation of time as referred to above.

Letting H_1 be the hourly difference for the Greenwich noon preceding the given Greenwich time,
H_2 be that for the following noon,
t be the number of hours of Greenwich time after the first date, for which the value of the quantity is required,

then $H_1 \pm \frac{H_2 - H_1}{24} \times \frac{t}{2}$ will be the mean hourly difference. In case t equals the hours of Greenwich time before the second date, and the value of the quantity is required for that instant, then $H_2 \mp \frac{H_2-H_1}{24} \times \frac{t}{2}$ will be the mean hourly difference.

Ex. 93.—On April 2, 1918, the hourly difference of declination of the sun at Greenwich mean noon was N. 57″.76. At the same time on April 3, it was N. 57″.55. Find the mean hourly rate at local mean noon in Long. 75° W. April 2.

$$\begin{array}{rr} \text{Here } H_1= & 57''.76 \\ H_2= & \underline{57''.55} \\ H_2-H_1=- & 0''.21 \\ \frac{H_2-H_1}{24}=- & 0''.01 \end{array} \left\{ \begin{array}{l} \text{At local mean noon in longitude } 75^\circ \text{ W.} \\ \text{G. M. T.}=t \text{ (in formula)}=+5^h, \text{ and } \frac{t}{2}=+2^h.5 \\ H_1+\frac{H_2-H_1}{24}\times\frac{t}{2}=\text{N. } 57''.76+(-''.01)\,(2^h.5) \\ \qquad\qquad =\text{N. } 57''.76-''.025=\text{N. } 57''.785 \\ \text{Mean hourly difference}=\text{N. } 57''.785 \end{array}\right.$$

Ex. 94.—On April 2, 1918, at Greenwich mean noon the sun's H. D. of declination was N. 57″.76; at the same time April 3, it was N. 57″.55. Find the mean hourly rate and the correction of the declination for local mean noon April 3, at a place in longitude 45° E.

$$\begin{array}{rr} \text{As before, } H_1= & 57''.76 \\ H_2= & \underline{57''.55} \\ \frac{H_2-H_1}{24}=- & 0''.01 \end{array} \left\{ \begin{array}{l} \text{At local mean noon in longitude } 45^\circ \text{ E.} \\ \text{G. M. T.}=t \text{ (in formula)}=-3^h \text{ and } \frac{t}{2}=-1^h.5 \\ H_2+\frac{H_2-H_1}{24}\times\frac{t}{2}=\text{N. } 57''.55+(-''.01)\,(-1^h.5) \\ \qquad\qquad =\text{N. } 57''.55+''.015=\text{N. } 57''.565 \\ \text{Mean hourly rate}=\text{N. } 57''.565 \end{array}\right.$$

$$\text{The correction}=\left(H_2+\frac{H_2-H_1}{24}\times\frac{t}{2}\right)t=\text{N. } 57''.565\times(-3^h)$$
$$=\text{S. } 172''.695=\text{S. } 2'\,52''.695$$

The mean hourly rate, when working forward, being

$H_1 \pm \frac{H_2-H_1}{24} \times \frac{t}{2}$ and the correction being $\left\{H_1 \pm \frac{H_2-H_1}{24} \times \frac{t}{2}\right\} t$, we see that the expression for the correction corresponds to that for uniformly accelerated or retarded motion in mechanics,

$$S=V_0 t+a\frac{t^2}{2}=\left(V_0+a\frac{t}{2}\right)t,$$

V_0 representing the initial velocity or change of the element;

a, the acceleration or retardation, or the increase or decrease of the change per unit of time next smaller than that for which V_0 is given; and S, the correction.

This formula is general in its application, but it must be remembered that, if, as in the case of the sun, V_0 is a difference for one hour, given in the Ephemeris for each day, and taken at the nearest Greenwich date, a will be $\frac{1}{24}$ of the change in V_0 for 24 hours, in other words, the hourly change of V_0; and, if, as in the case of the moon, V_0 is the difference for one minute given in the Ephemeris for each hour, a will be $\frac{1}{60}$ of the change in V_0 for 60 minutes, or, in other words, the change in V_0 for one minute of time.

In case the given time is nearer to a subsequent date than a preceding Ephemeris date, the formula may be used, working backwards; V_0 will be the quantity for the subsequent date in the Ephemeris and t the time before this date.

Taking the first of the two preceding examples to find the correction,

$$S = \text{correction} = \{\text{N. } 57''.76 + (-0''.01 \times \tfrac{5}{2})\}5$$
$$= \text{N. } 288''.675 = \text{N. } 4'\, 48''.675.$$

Taking the second one, we have

$$S = \text{correction} = \{\text{N. } 57''.55 + (-0''.01 \times (-\tfrac{3}{2}))\}(-3)$$
$$= \text{N. } 57''.565 \times (-3) = \text{S. } 2'\, 52''.695.$$

185. To find from the Almanac a certain element for a given mean time at a given place.

(1) *Find the Greenwich mean time corresponding to the local mean time, as previously explained.*

(2) *Take from the Nautical Almanac, for the nearest mean time date preceding the given Greenwich mean time, the required quantity, and the corresponding "difference for 1 hour" noting the name and sign of each. Multiply the "dif-*

ference for 1 hour" by the hours and decimals of an hour of the remaining Greenwich mean time. Apply this quantity algebraically according to sign to the quantity first taken out. Or, take out the quantity for the nearest subsequent date, and the proper difference. Multiply this difference by the fraction of time from the given Greenwich date to the Almanac date, then subtract the product algebraically.

(3) *In the case of the moon and planets the interpolation can be more conveniently performed by the use of Table IV, Nautical Almanac (proportional parts).*

To take out the R. A. M. ⊙ or "sidereal time of mean noon."—Find what it is for the Greenwich mean noon preceding the given Greenwich mean time; the table at the foot of the page contains the correction to be added to the right ascension of the mean sun in order to reduce it to any Greenwich mean time other than noon. The correction is tabulated for every six minutes and can be interpolated to any time required. Table III at the end of the Nautical Almanac for converting a mean solar into a sidereal time interval may also be used for finding this correction.

Ex. 95.—For a L. M. T. January 8, 1918, $8^h\ 16^m\ 54^s$ a. m., in longitude 80° 31′ 30″ W., find the sun's declination; semidiameter; equation of time; and right ascension of mean sun, using first differences only.

First find the Greenwich mean time.

	h	m	s
Local astronomical mean time Jan. 7,	20	16	54
Longitude from Greenwich West	+ 5	22	06
Greenwich mean time Jan. 8,	1	39	00
Jan. 8,	$1^h.65$		

NOTE.—All the data from the Nautical Almanac of 1918 necessary for the solution of examples in this book may be found on pages 745 to 752.

To find the sun's declination.

	Sun's Dec.		H. D
	° ′		
Jan. 8, at Greenwich mean noon	S 22 18.9		N 0′.3
H. D. N 0′.3 × $1^h.65$	N 0.5	G.M.T.	$1^h.65$
Required declination of the sun	S 22 18.4	Corr.	N 0′.495

To find the sun's semi-diameter.

The change of sun's semi-diameter is so small, even in 24 hours, that it is tabulated every 10 days only for Greenwich mean noon. In ordinary practice it would be taken out only to the nearest tenth of a minute of arc.

January 8, sun's S. D. = 16′.3 or 16′ 18″.

To find the equation of time.

	Eq. of time.		H. D.
	m s		
Jan. 8, at Greenwich mean noon	6 36.6		+ $1^s.1$
H. D. $1^s.1$ × $1^h.65$	+ 1.8	G. M. T.	$1^h.65$
Required equation of time	6 38.4	Corr.	+ $1^s.815$

(—) to mean time.

To find the right ascension of the mean sun.

	R. A. M. ⊙
	h m s
Jan. 8, at Greenwich mean noon	19 08 54.3
Corr. for 1^h 39^m (or using Table III)	16.3
Required R. A. M. ⊙	19 09 10.6

Ex. 96.—**Find the right ascension, declination, semi-diameter, and horizontal parallax of the moon for 1918, January**

3, L. M. T. 10^{h} 10^{m} 06^{s} p. m. in longitude 45° East; also the right ascension and declination of the planet Jupiter.

		h	m	s
Local astronomical mean time Jan. 3,		10	10	06
Longitude from Greenwich East,	(—)	3	00	00
Greenwich astronomical mean time Jan. 3,		7	10	06

$= 7^{h}\ 10^{m}.1 = 7^{h}.17$

	Moon's R. A. h m s	H. D.
Jan. 3, at 6 hrs. of G. M. T.	11 28 14	108^{s}
H. D. $108^{s} \times 1^{h}.17 =$	+ 2 06.4	$1^{h}.17$
Jan. 3, at 7^{h} 10^{m} 06^{s} of G. M. T. R. A. =	11 30 20.4	+ $126^{s}.36$

(By proportional parts, Table IV, Nautical Almanac.)

		h m s
R. A. Jan. 3, 8 hrs.		11 31 50
Corr. for 49.9 minutes	—	1 30
Moon's R. A.		11 30 20

	Moon's Dec. ° ′	H. D.
Jan. 3, at 6 hrs. of G. M. T.	S 2 15.4	S 12′.0
H. D. 12′.0 S × $1^{h}.17 =$	S 14.0	$1^{h}.17$
Jan. 3, at 7^{h} 10^{m} 06^{s} of G. M. T. Dec. =	S 2 29.4	S 14′.04

(By proportional parts, Table IV, Nautical Almanac.)

	° ′
Dec. Jan. 3, 8 hrs.	S 2 39.4
Corr. for 49.9 minutes	N 10.0
Moon's Dec.	S 2 29.4

The moon's semi-diameter and horizontal parallax for G. M. T. 7^{h} 10^{m} 06^{s} January 3.—The moon's semi-diameter and horizontal parallax are tabulated for every 2 hours and change so slowly that they may be picked out directly by inspection of the table.

Moon's semi-diameter for G. M. T. 7^h.17 Jan. 3, = 14′.8
Moon's horizontal parallax for G. M. T. 7^h.17 Jan. 3, = 54′.2

To find the right ascension of the planet Jupiter.

	Jupiter's R. A.		H. D.
	h m s		
Jan. 3, at Greenwich mean noon	4 01 49.0		— 0^s.79
H. D. — 0^s.79 × 7^h.17 =	— 5.7	G. M. T.	7^h.17
Required R. A. of Jupiter	4 01 43.3	Corr.	— 5^s.66

(By proportional parts, Table IV, Nautical Almanac.)

	h m s
R. A. Jan. 3, at G. M. noon	4 01 49
Corr. for 7^h 10^m.1	— 05.7
R. A. Jupiter	4 01 43.3

To find the declination of Jupiter.

	Jupiter's Dec.		H. D.
	° ′		
Jan. 3, at Greenwich mean noon	N 19 52.9		S 0′.03
H. D. S 0′.03 × 7^h.17 =	S .2	G. M. T.	7^h.17
Required declination of Jupiter	N 19 52.7	Corr.	S 0′.21

(By proportional parts, Table IV, Nautical Almanac.)

	° ′
Dec. Jan. 3, at G. M. noon	N 19 52.9
Corr. for 7^h 10^m.1	S 00.2
Dec. Jupiter	N 19 52.7

For a given mean time to find the right ascension and declination of a star.

Ex. 97.—Let these elements of the star Polaris (α Ursæ Minoris) be required for the L. M. T., 1918, January 19, 1^h 48^m 15^s a. m., at a place in longitude 40° 15′ W.

In the Nautical Almanac, under "Apparent Places for Stars," the right ascensions and declinations of 55 of the most important stars are tabulated for the first day of each month at the time of upper transit at Greenwich and in the next following table this Greenwich mean time of transit on the first day of each month is given.

From these tables we find the change in the elements occurring in a certain interval and take as a correction to the tabulated values for the nearest date the proportional part of the change indicated by the Greenwich mean time of the given instant of the problem.

		h	m	s
Local civil mean time Jan. 19 (a. m.),		1	48	15
Local astronomical mean time Jan. 18,		13	48	15
Longitude West	+	2	41	00
G. M. T. of given instant =	Jan. 18^d	16	29	15

		h	m		h	m	s
Polaris' transit Jan. 1		6	49	R. A.	1	31	09.0
Polaris' transit Feb. 1		4	46	R. A.	1	30	36.1
Interval	30^d	21	57	Change	—		32.9
	= 30.917 days						

Change of R. A. in 1 day = — $1^s.06$

		d	h	m
Polaris transit	Feb.	1	04	46
G. M. T. of given instant	Jan.	18	16	29
Interval from given instant to nearest tabulated date	= —	13	12	17
	= — 13.512 days			

	h	m	s		
R. A. Feb. 1	1	30	36.1	Daily change	— 1.06
Corr. for — 13.512 days +			14.3	Interval	— 13.512
R. A. Polaris =	1	30	50.4	Corr.	+ 14.323

There is no change in declination from Jan. 1 to Feb. 1.

Declination Polaris = + 88° 52′.5

The above accurate interpolation has been used for finding the right ascension of Polaris on account of the comparatively rapid change in right ascension of that star. For ordinary navigational purposes the elements of the other stars may be picked out to a sufficient degree of accuracy by a simple inspection of the tables.

When work of extreme precision is to be done the elements of stars should be taken from the Ephemeris where they are tabulated to a high degree of accuracy for every tenth upper transit at Washington. In using the star tables of the Ephemeris Washington time is used instead of Greenwich time.

186. To find from the Almanac a certain element of the sun for a given local apparent time.—(1) Find the corresponding Greenwich apparent time; to do which express the local apparent time astronomically, applying to it the longitude, plus if West, minus if East, as already explained for finding G. M. T. Correct the equation of time, tabulated in the Nautical Almanac for the even hours of mean time of the given astronomical day, for the odd hours and decimals of an hour in the Greenwich apparent time, and apply this result to the Greenwich apparent time to get the approximate G. M. T. Entering the Nautical Almanac with the approximate G. M. T. thus obtained, a new value of the equation of time may be obtained, which, being applied to the Greenwich apparent time, will give a more exact value of the G. M. T. with which to take out the required element; but this second approximation to the G. M. T. is an unnecessary refinement for navigators, since the required element may be satisfactorily obtained with the first approximation to the G. M. T. because the equation of time never changes more than 1.3 seconds in an hour.

187. To find a certain element of the sun when it is on the meridian of a given place or at local apparent noon.

Proceed exactly as explained above.

The most common use of this problem is when finding the sun's declination in the case of a meridian altitude of the sun, of an altitude near noon, or in the case of finding declination of sun and equation of time in equal altitudes for chronometer error. In the last case, it is convenient, since the ephemerides of the sun are tabulated for Washington apparent noon in the American Ephemeris, pages 514-521, to reduce the local apparent time to Washington apparent time instead of to Greenwich apparent time.

At the instant of apparent noon, the local apparent time is $0^h\ 0^m\ 0^s$. Therefore, if in longitude 60° W. on January 5, we have

	h m s
Local astronomical apparent time Jan. 5,	0 00 00
Longitude West	+ 4 00 00
Greenwich ast. apparent time Jan. 5,	4 00 00

But if in longitude 60° E. at local apparent noon on January 5, we would have

	h m s
Local astronomical apparent time Jan. 5,	0 00 00
Longitude East	(—) 4 00 00
Greenwich apparent time Jan. 5,	(—) 4 00 00
Or Jan. 4,	20 00 00

From the above it is clear that in longitude West, the G. A. T. of local noon is equal to the longitude, or it is after noon of the same date by the number of hours in the longitude; but that in East longitude at local apparent noon the G. A. T. is before the noon of local date by the number of hours in the longitude, or G. A. T. = (–) longitude.

Ex. 98.—Find the sun's declination and equation of time for local apparent noon at a place in longitude $5^h.1$ W. on January 2, 1918.

	h m s
G. A. T. of local apparent noon	5 06 00
Eq. T.	+ 4 00
G. M. T. of local apparent noon	5 10 00
	$=5^h.17$

Dec. 4^h,	S 22° 57′.3		H. D.
Corr.	N .2		N 0′.2
At L. A. noon		G. M. T.	$1^h.17$
Jan. 2,	S 22 57.1	Corr.	N 0′.234

	m s	H. D.
Eq. of T. 4^h =	3 59.2	$+1^s.2$
Corr.	+ 1.3	$\lambda=1^h.1$
Eq. T.	= 4 00.5	Corr. $+1^s.3$
+to App. T.		

Ex. 99.—January 11, 1918, in longitude 96° 08′ 51″ W., find the sun's declination and equation of time at local apparent noon, using 2d differences.

Here longitude $=6^h\ 24^m\ 35^s.4$ W. and longitude of Washington (Am. Ephemeris, page 684) $=5^h\ 08^m\ 15^s.8$ W. Then $6^h\ 24^m\ 35^s.4$ W. $-\ 5^h\ 08^m\ 15^s.8$ W. $=1^h\ 16^m\ 19^s.6=$ longitude W. of Washington $=1^h.272$.

(Washington apparent time, Ephemeris, pages 514-521.)

Times.	Sun's Dec.	H. D. at Wash. App. Noon and Change.	
	° ′ ″		″
At Wash. app. noon		Jan. 11, H. D.	N 23.18
Jan. 11,	S 21 51 17.9	Jan. 12, H. D.	N 24.23
Corr. N	29.52	Change in 24 hours	(+) 1.05
At L. A. noon Jan. 11,	S 21 50 48.88	Change in 1 hour	(+) 0.0438
		Change in $\frac{\lambda}{2}$ hrs. $=0^h.636$ W	(+) 0.0278
		Jan. 11, H. D.	N 23.18
		Mean H. D.	N 23.208
		Wash. app. T. $=$ (λ place −λ Wash.) $=$	$1^h.272$
			Corr. N 29″.520

Times.	Eq. of T.	H. D. at Wash. App. Noon and Change.	
	m s		s
At Wash. app. noon Jan. 11,	7 56.62	Jan. 11, H. D.	+ 0.998
Corr. +	1.27	Jan. 12, H. D.	+ 0.974
At L. A. noon Jan. 11,	7 57.89	Change in 24 hours	(−) 0.024
+ to app. time		Change in $\frac{\lambda}{2}$ hrs. from Wash.	(−) 0.001
		Jan. 11, H. D.	+ 0.998
		Mean H. D.	+ 0.997
		Wash. A. T. $=$ λ W from Wash.	$1^h.272$
			Corr. + $1^s.268$

When longitude is East of Washington, the Washington apparent time of local noon equals (−) longitude from Washington of the local civil date.

Ex. 100.—April 4, 1918, in longitude $10^h\ 04^m\ 49^s.6$ East, find the sun's declination and equation of time at local apparent noon, using 2d differences. Here the longitude $= 10^h\ 04^m\ 49^s.6$ E. + (Wash. λ) $5^h\ 08^m\ 15^s.8$ W. $= 15^h\ 13^m\ 05^s.4$ E. of Washington $= (-)\,15^h.218$.

Times.	Sun's Dec.	H. D. at Wash. App. Noon and Change.	
	° ′ ″		″
At Wash. app. noon		Apr. 3, H. D.	N 57.49
Apr. 4,	N 5 34 09.5	Apr. 4, H. D.	N 57.26
	Corr. S 14 32.49		
		Change in 24 hours	(−) 0.23
At L. A. noon Apr. 4,	N 5 19 37.01	Change in 1 hour	(−) 0.0096
		Change in $\frac{\lambda}{2}$ hrs. from Wash.	(+) 0.073
		Apr. 4, H. D.	N 57.26
		Mean H. D.	N 57.333
		Wash. app. T. (−) λ from Wash.	(−) $15^h.218$
		Correction	S 872″.493

Times.	Eq. of T.	H. D. at Wash. App. Noon and Change.	
	m s		s
At Wash. app. noon Apr. 4	8 08.62	Apr. 3, H. D.	(−) 0.745
	Corr. + 11.26	Apr. 4, H. D.	(−) 0.738
At local app. noon Apr. 4, + to app. time	8 19.88	Change in 24 hours	(+) 0.007
		Change in $\frac{\lambda}{2}$ hrs. from Wash.	(−) 0.002
		Apr. 4, H. D.	(−) 0.738
		Mean H. D.	(−) 0.740
		Wash. app. T. (−) λ from Wash.	(−) $15^h.218$
		Correction	$11^s.26132$

To find the sun's declination and equation of time at local apparent midnight, proceed as in the above examples, using for Washington A. T. in the first of the two preceding examples (12 hours + λ − long. of Washington) $= 13^h.272$ January 11, and in the second (12 hours $-(\lambda+$long. of Washington$)$ $=12^h-15^h.218=(-)3^h.218$ April 4.

188. To find the local mean time of transit of the moon over a given meridian on a given date, and the moon's right ascension, declination, semi-diameter, and horizontal parallax at that instant.—The Nautical Almanac, pages 76 and 77, contains the Greenwich mean time of each transit of the moon over the meridian of Greenwich. This time is the hour angle of the mean sun when the moon is on the meridian, and, therefore, equals the difference of the right ascensions of the moon and mean sun. Both the moon and mean sun increase their right ascensions daily, but the increase for the moon is greater than that for the mean sun, so that each day the moon gets further and further to the eastward of the mean sun, and in the diurnal revolution comes to the meridian later each day than on the preceding day; the number of minutes varying with the moon's motion, but approximating an average value of 50 minutes.

This retardation, represented by *R*, occurs during a passage of the moon over 24 hours of longitude, and for any longitude λ hours the retardation will be $\frac{R}{24}\times\lambda$, so it is easily seen that if there were no retardation whatever, the local time of the moon's meridian passage in any longitude would be the same as that at Greenwich, but there is retardation, and the local mean time of transit over a meridian is gotten from the Greenwich mean time of Greenwich transit by computing the amount of retardation corresponding to the number of hours and decimals of an hour of longitude, and applying it to the Greenwich mean time of Greenwich transit, adding that amount for West longitude, subtracting for East longitude; West longitude being regarded as $+$, East longitude as $(-)$.

The value of *R*, or the daily retardation, is given in its appropriate column opposite the time of transit. The reduction for longitude is tabulated in Table 11, Bowditch, the arguments being "longitude," and "daily variation of the moon's passing the meridian"; and it may also be found from Table IV, Nautical Almanac.

The times given in the Almanac are for the astronomical date, and care must be exercised in finding the meridian passage on a given civil date; hence the rules:

(1) *Take the time of the moon's meridian passage from the Nautical Almanac for the given civil date when the time tabulated plus the correction for retardation is less than 12 hours, because then the astronomical date is the same as the civil date.*

(2) *When it is seen that the sum of the tabulated time of passage plus the correction for retardation on the given civil date will be greater than 12 hours, take out the time of passage for the day before, since in this case the astronomical date is one day less than the civil date.*

(3) *Take out the retardation from the tables for the given longitude, adding the amount to the G. M. T. of meridian passage at Greenwich when longitude is West, subtracting if longitude is East. The result will be the local mean time of local transit* (see Art. 196 (c)).

Ex. 101.—In longitude 100° 30′ W. find the time of meridian transit of the moon for 1918, January 19 (civil date), then the corresponding G. M. T.

Meridian Transit of the Moon.			Retardation.	
		h m		
Long. 100° 30′ W	G. M. T. of Gr. transit Jan. 19,	5 54.0	For 1^h,	$2^m.25$
or	Corr. for longitude West	+ 15.1	λ=	+ $6^h.7$
Long. 6^h 42^m W				
=$6^h.7$ W	L. M. T. of local transit Jan. 19,	6 09.1	Corr.	+$15^m.07$
	Longitude West	+6 42	Or (Tab. 11, Bowditch) =	+$15^m.1$
	G. M. T. of local transit Jan. 19,	12 51.1		

Ex. 102.—In longitude 100° 30′ E. find the time of meridian transit of the moon for 1918, January 28 (civil date), then the corresponding G. M. T.

Meridian Transit of the Moon.				Retardation.	
			h m		
Long. 100° 30′ E	G. M. T. of Gr. transit Jan. 27,		12 49	For 1^h,	$1^m.88$
or	Corr. for longitude East	(−)	12.6	λ=	− $6^h.7$
Long. 6^h 42^m E					
=$6^h.7$ E	L. M. T. of local transit Jan. 27,		12 36.4	Corr.	$-12^m.60$
	Longitude East	(−)	6 42	Or (Tab. 11, Bowditch) =	$-12^m.55$
	G. M. T. of local transit Jan. 27,		5 54.4		

The times of transits at Greenwich are given only to the nearest minute, and the resulting local time of local transit will be only approximate, though sufficiently exact for navigators. A more exact time may be found by first finding the approximate L. M. T. of local transit and then the approximate G. M. T. of local transit for which the moon's right ascension may be taken out.

This right ascension is the local sidereal time of the moon's local transit and the local mean time corresponding may be found (see Ex. 132).

If then other elements are desired at the time of the moon's local transit, find the G. M. T. corresponding to the L. M. T. just found, and take out for this G. M. T. the required elements.

189. To find the local mean time of transit of a planet over a given meridian on a given date, and the corresponding G. M. T., also the planet's right ascension and declination at that instant.—The mean time of each meridian transit for the meridian of Greenwich is given in the Almanac for Mars, Venus, Jupiter, and Saturn. On certain dates there may be retardation, on others acceleration, in the times of return to the meridian. In the case of a retardation, the time of local transit is found as in the case of the moon; in the case of acceleration, the sign of the reduction for longitude is reversed. Or, considering the hourly retardation +, the hourly acceleration (−), West longitude +, and East longitude (−), the rule of signs will determine the sign of the reduction.

Having found the L. M. T. of local transit, deduce the G. M. T. by applying the longitude, and take out for this

G. M. T. the planet's right ascension and declination (Art. 185, Ex. 96).

Having found the right ascension of a planet when it is on the meridian, take this as local sidereal time and find the corresponding local mean time; the result will be closer than the time tabulated which, however, is sufficiently exact for navigators.

It will be noticed that, while in the case of the moon the retardation was tabulated for each day, in the cases of planets, the retardation or acceleration will be obtained for 24 hours by taking the difference of the times of Greenwich transit for the given day and the day following when in West longitude, the difference of those times for the given day and day preceding when in East longitude.

The change for 1 hour will then be one twenty-fourth of the difference for 24 hours. This hourly change multiplied by the hours and decimals of an hour in the longitude will be the change for longitude.

Ex. 103.—In longitude 75° W. find the L. M. T. of transit of Jupiter, 1918, January 4, civil date; also right ascension and declination of Jupiter for that instant.

Meridian Transit of Jupiter.				Difference.
		h	m	h m
Long. 75° W=5^h W.	At Gr. noon Jan. 4,	9	07	For 24=−5.0
	Correction for longitude W	—	1	For 5=−1.0
	L. M. T. of local transit Jan. 4,	9	06	
	Longitude West	+ 5		
	G. M. T. of local transit Jan. 4,	14	06	

Times.	Jupiter's R. A.	H. D.	Jupiter's Dec.	II. D.
	h m s		° ′	
At Gr. noon Jan. 4,	4 01 30.0	− $0^s.75$	N 19 52.2	S 0′.025
Corr. for G. M. T.	— 10.6	G. M. T. $14^h.1$	Corr. S .4	G. M. T. $14^h.1$
At local transit	4 01 19.4	Corr. −$10^s.57$	N 19 51.8	Corr. S 0′.352

CHAPTER XIV.

RELATION OF MEAN, APPARENT, AND SIDEREAL TIMES.—CONVERSION OF TIME.—RELATION OF TIME, HOUR ANGLES, AND RIGHT ASCENSIONS, AND A CONSIDERATION OF PROBLEMS INVOLVING THEM.—FINDING LOCAL AND WATCH TIMES OF A BODY'S TRANSIT, ETC.

190. To interconvert apparent and mean time.—The equation of time being the difference between the hour angles of the true and mean suns, or, in other words, between apparent and mean times, when one is given, the other is obtained by applying the equation of time with its proper sign of application to the given time. Thus, if for the same instant,

t_m represents local mean time,

t_a represents local apparent time,

E represents equation of time with positive sign of application to apparent time,

then

$$\left.\begin{aligned} t_m &= t_a + E, \\ t_a &= t_m - E. \end{aligned}\right\} \qquad (136)$$

Hence for the given local time (apparent or mean), expressed astronomically, find the Greenwich time (apparent or mean). Take out of the Nautical Almanac for the Greenwich instant the equation of time. The reduction then is made by applying the corrected equation of time to the given time, with the proper sign as shown at the top of the column in which it is found.

The equation of time found in the Nautical Almanac is the apparent time of mean noon at Greenwich, and if corrected for longitude it is the apparent time of local mean noon.

Ex. 104.—January 2, 1918, in longitude 75° 30′ W., find the local apparent time corresponding to a local mean time $8^h\ 10^m\ 10^s$ p. m.

	h m s		m s				h m s
L. M. T.	8 10 10	Jan. 2. Eq. t. at 12^h	= 4 08.6	H. D.	$+1^s.2$	L. M. T.	=8 10 10
Long.	5 2 00 W	Corr.	= + 1.4	G. M. T.	$1^h.2$	Eq. t.	− 4 10
G. M. T.	13 12 10	Jan. 2. Eq. t.	= 4 10.0	Corr.	$+1^s.44$	L. A. T.	8 06 00
	$=13^h.2$		(−) to M. T.				

Ex. 105.—April 3, 1918, in longitude 100° 45′ E., find the local mean time corresponding to $5^h\ 10^m$ a. m., local apparent time.

	h m s		m s				h m s
L. A. T.	17 10 00	Apr. 2. Eq. t. at 10^h	=3 40.8	H. D.	$-0^s.7$	L. A. T.	17 10 00
Long.	6 43 00 E	Corr.	= − 0.3	G. A. T.	$0^h.45$	Eq. t.	3 40.5
G. A. T.	10 27 00	Apr. 2. Eq. t.	=3 40.5	Corr.	$-0^s.3$	L.M.T.	=17 13 40.5
			+ to App. T.			or Apr. 3, a. m.,	5 13 40.5

191. Formulæ for the interconversion of mean and sidereal time intervals.—Since a sidereal year contains 365.25636 mean solar days, or 366.25636 sidereal days, each unit of mean solar time will contain $\frac{366.25636}{365.25636}$ sidereal units of the same denomination, or each unit of sidereal time will contain $\frac{365.25636}{366.25636}$ units of mean time of the same denomination.

Since both are uniform measures of time, any interval of time expressed either in mean solar or sidereal units may be expressed in units of the other denomination.

Thus, if any interval of time be represented by t if expressed in mean solar time, by s if expressed in sidereal time,

then $$\frac{s}{t}=\frac{366.25636}{365.25636}=1.0027379,$$

whence $$s=t+.0027379t, \tag{137}$$

$$t=s-.0027304s, \tag{138}$$

and by these formulæ any interval of the one kind of time can be converted into an interval of the other kind of time.

The reduction is facilitated by the use of Table II of the Nautical Almanac for converting sidereal intervals into mean solar time intervals, which contains for each second of s the value $.0027304s$ expressed in minutes and seconds; also by Table III, for converting a mean solar time interval into a sidereal time interval, which contains for each second of t the value $.0027379t$ expressed in minutes and seconds. Tables 8 and 9 of Bowditch are for the same purpose.

If t and s are in units of hours, the above formulæ become

$$s = t\ (1 + 9^s.8565) = t + 9^s.8565t, \qquad (139)$$

$$t = s\ (1 - 9^s.8296) = s - 9^s.8296s, \qquad (140)$$

so that in the absence of the above mentioned Tables the reduction may still be conveniently calculated.

Acceleration and retardation.—If in (137) $t = 24$ hrs., s will equal $24^h\ 3^m\ 56^s.5553$; or in a mean solar day sidereal time gains on mean time $3^m\ 56^s.5553$, and this is called the acceleration of sidereal on mean time. If in (138) $s = 24$ hrs., $t = 24^h$ minus $3^m\ 55^s.9094$, or in a sidereal day mean time loses on sidereal time $3^m\ 55^s.9094$, and this is the retardation of mean solar on sidereal time.

Examples on the conversion of a mean solar time interval into a sidereal time interval.

Ex. 106.—Express 10 hours of mean solar time in sidereal time.

Taking formula $s^h = t^h(1 + .0027379)$, we have

$$s = 10^h.027379 \qquad = 10^h\ 01^m\ 38^s.564.$$

Taking formula $s^h = t^h(1 + 9^s.8565)$,

we have $\qquad s = 10^h\ 01^m\ 38^s.565.$

Using table III, Nautical Almanac, we have

t = a mean solar time interval,	$10^h\ 00^m\ 00^s$
From table III, reduction to a sidereal interval	$+\quad 1\ \ 38\ .6$
The required sidereal time interval	$10^h\ 01^m\ 38^s.6$

Ex. 107.—Express $15^h\ 33^m\ 29^s$ of mean time in sidereal time.

t = a mean solar time interval	$15^h\ 33^m\ 29^s$
From Table III, reduction to a sidereal interval	$+\ 2\ \ 33\ .3$
The required sidereal time interval	$15^h\ 36^m\ 02^s.3$

Express in sidereal time:

Ex. 108.—$7^h\ 29^m\ 30^s.5$ of mean time.

Ans. $7^h\ 30^m\ 44^s.4$ sidereal time.

Ex. 109.—$1^h\ 14^m\ 03^s$ of mean time.

Ans. $1^h\ 14^m\ 15^s.2$ sidereal time.

Ex. 110.—$23^h\ 15^m\ 10^s$ of mean time.

Ans. $23^h\ 18^m\ 59^s.2$ sidereal time.

Examples on the conversion of a sidereal interval into a mean solar time interval.

Ex. 111.—Express $10^h\ 30^m\ 00^s$ of sidereal time in mean solar time.

Taking formula $t^h = s^h(1-.0027304)$, we have

$$t = 10^h.5 - 0^h.0286692 = 10^h.471331 = 10^h\ 28^m\ 16^s.79.$$

Taking formula $t^h = s^h(1-9^s.8296)$, we have

$$t = 10^h.5(1-9^s.8296) = 10^h\ 30^m - 1^m\ 43^s.2108 = 10^h\ 28^m\ 16^s.789.$$

Using Table II, Nautical Almanac, we have

s = a sidereal time interval =	$10^h\ 30^m\ 00^s$
From Table II, reduction to a mean time interval	$-\ 1\ \ 43\ .2$
The required mean solar time interval	$10^h\ 28^m\ 16^s.8$

Express in mean solar time:

Ex. 112.—$11^h\ 04^m\ 12^s.9$ of sidereal time.

Ans. $11^h\ 02^m\ 24^s.1$ mean time.

Ex. 113.—$15^h\ 08^m\ 33^s.4$ of sidereal time.

Ans. $15^h\ 06^m\ 04^s.5$ mean time.

Ex. 114.—$19^h\ 13^m\ 36^s.6$ of sidereal time.

Ans. $19^h\ 10^m\ 27^s.7$ mean time.

192. Having the mean time at any place, to find the corresponding sidereal time.

Let λ represent the longitude of the place expressed in time, + when West, (−) when East.

t the hour angle of the mean sun expressed positively and, therefore, the local mean time.

$(t+\lambda)$ the G. M. T. or elapsed mean time interval since Greenwich mean noon.

S the hour angle of ♈ and hence the local sidereal time.

$(S+\lambda)$ the Greenwich hour angle of ♈ or Greenwich sidereal time.

a_0 the right ascension of the mean sun (R. A. M. ⊙) at Greenwich mean noon.

If a mean time interval since Greenwich mean noon is $(t+\lambda)^h$, the corresponding sidereal time interval will be $(t+\lambda)^h\,(1+.0027379)$. Having now the sidereal interval since Greenwich mean noon and the sidereal time of Greenwich mean noon, or a_0, the Greenwich sidereal time will be

$$(S+\lambda)^h = a_0 + (t+\lambda)^h(1+.0027379)$$

$$S+\lambda \quad = a_0+\lambda+t+(t+\lambda)(.0027379)$$

$$\left.\begin{array}{r}\text{therefore, } S\\ \text{or the L. S. T.}\end{array}\right\} = a_0+t+(t+\lambda)(.0027379) \qquad 141)$$

The Almanac contains a_0 for each Greenwich mean noon under the heading Right Ascension of the Mean Sun. As $t+\lambda$ is the G. M. T., a_0 should be taken out for Greenwich mean noon of the given Greenwich date, and corrected for the hours, minutes, and seconds of Greenwich mean time, using the table at the foot of pages 2 and 3 of the Almanac or else Table III of the Almanac.

Hence the rule: *Express the local mean time astronomically and find the G. M. T. and date. Then to the local astronomical mean time add the sidereal time or the right ascension of* *the mean sun taken from the Nautical Almanac for noon of the Greenwich date, and also the reduction from Table III for the hours, minutes, and seconds of the Greenwich mean time. The sum, if less than 24 hours, will be the local sidereal time (L. S. T.). If the sum is greater than 24 hours, reject 24 hours and the remainder will be the L. S. T.*

Since the sidereal time (R. A. M. ⊙) at Greenwich mean noon, corrected for the G. M. T. corresponding to the given L. M. T., is the right ascension of the mean sun at the instant of the given L. M. T., the above equation (141) is simply an algebraic expression of what has already been proven, namely: "The sidereal time at a given place is equal to the right ascension of the mean sun plus the local mean time" (Art. 173).

It is usual to keep the solar day; but should it be desired to state the sidereal day, prefix to a_0 the sidereal day at the instant of Greenwich mean noon, which is the same as the astronomical day for six months after the vernal equinox, one day less for six months before the vernal equinox. At the instant of the vernal equinox, the sidereal time and mean solar time coincide. Before that time the mean sun transits before the vernal equinox; after that time, it transits after the vernal equinox.

Examples on the conversion of local mean time into local sidereal time.

Ex. 115.—January 18, 1918, in longitude 55° 15′ W., the local mean time is $8^h\ 06^m\ 29^s.5$ p. m. Find the local sidereal time (see rule in this article).

	h	m	s
The local astronomical mean time Jan. 18,	**8**	**06**	**29.5**
Longitude from Greenwich West	**+ 3**	**41**	**00**
The Greenwich mean time Jan. 18,	**11**	**47**	**29.5**

	h	m	s
R. A. M. ⊙ Jan. 18, at Greenwich mean noon	**19**	**48**	**19.9**
Reduction for G. M. T., Table III		**1**	**56.2**
Add the local astronomical mean time	**8**	**06**	**29.5**
The required local sidereal time (rejecting 24 hrs.)	**3**	**56**	**45.6**

Ex. 116.—January 10, 1918, in Long. 137° 35′ E., the L. M. T. is $5^h 17^m 30^s$ a. m. Find the L. S. T.

	h	m	s
Local astronomical mean time Jan. 9,	17	17	30
Longitude from Greenwich East	9	10	20
Greenwich mean time Jan. 9,	8	07	10
	h	m	s
R. A. M. ⊙ at Greenwich mean noon Jan. 9,	19	12	50.9
Reduction for G. M. T., Table III		1	20.0
Add the local astronomical mean time	17	17	30
Required local sidereal time (rejecting 24 hrs.)	12	31	40.9

Ex. 117.—April 16, 1918, the Greenwich mean time is $9^h 10^m 30^s$ a. m. Find the Greenwich sidereal time.

	h	m	s
Greenwich astronomical mean time April 15,	21	10	30
R. A. M. ⊙ at Greenwich mean noon April 15,	1	31	20.1
Reduction for G. M. T., Table III		3	28.7
Required Greenwich sidereal time	22	45	18.8

Ex. 118.—January 20, 1918, at the U. S. Naval Academy, when the 75th meridian mean noon signal was received, a sidereal clock-read $20^h 23^m 19^s.5$. Shortly after the receipt of this signal a comparison of this clock with a mean time chronometer was: sidereal clock, $20^h 43^m 29^s$; mean time chronometer, $5^h 24^m 16^s$. Find the error of chronometer on G. M. T. (see Ex. 125, Art. 193). Note that here the error of the sidereal clock is not given.

	h	m	s
At 75th meridian mean noon sidereal clock reads	20	23	19.5
At time of comparison sidereal clock reads	20	43	29
Sidereal interval since 75th mer. mean noon	0	20	09.5
Reduction to a mean time interval, Table II	—		3.3
Mean time interval since 75th mer. mean noon	0	20	06.2
Longitude of 75th meridian West	+ 5	00	00
Greenwich mean time of comparison	5	20	06.2
Chronometer time of comparison	5	24	16
Error of mean time chronometer, fast on G. M. T.		4	09.8

Use This Form

Ex. 119.—At Cebu I. Plaza, Lat. 10° 17′ 30″ N., Long. 123° 54′ 18″ E., April 10, 1918, L. M. T. = $5^h\ 45^m\ 30^s$ a. m. Find L. S. T.

	h	m	s
Local astronomical mean time April 9,	17	45	30
Longitude East from Greenwich	— 8	15	37.2
Greenwich mean time April 9,	9	29	52.8
	h	m	s
R. A. M. ⊙ at Greenwich mean noon April 9,	1	07	40.8
Reduction for G. M. T.		1	33.6
Local astronomical mean time	17	45	30
Local sidereal time	18	54	44.4

Ex. 120.—April 1, 1918, in longitude $2^h\ 13^m\ 20^s$ West, the L. M. T. is $9^h\ 48^m\ 06^s$ p. m. Find first the G. S. T., then the L. S. T.

Omit

	h	m	s
April 1, the local astronomical mean time is	9	48	06
Longitude from Greenwich West	+ 2	13	20
April 1, Greenwich mean time	12	01	26
R. A. M. ⊙ April 1 at Greenwich mean noon	0	36	08.4
Reduction for G. M. T., Table III		1	58.5
Greenwich sidereal time	12	39	32.9
Longitude from Greenwich West	(—) 2	13	20
Local sidereal time	10	26	12.9

Ex. 121.—January 25, 1918, at the Naval Academy, Annapolis, Md., in longitude $5^h\ 05^m\ 56^s.5$ W., when the time signal was received from Washington indicating noon of 75th meridian West longitude, mean time, a sidereal clock read

$20^h\ 15^m\ 09^s$. Required the error of the sidereal clock on local sidereal time.

	h	m	s
75th meridian mean time at 75th mer. mean noon	0	00	00
Longitude of 75th meridian West	+ 5	00	00
Jan. 25, G. M. T. of 75th mer. mean noon	5	00	00
Longitude of Naval Academy West	(—) 5	05	56.5
L. M. T. at instant of 75th mer. M. N.	23	54	03.5

	h	m	s
Jan. 25, R. A. M. ⊙ at G. M. noon	20	15	55.8
Reduction for G. M. T. of 75th mer. mean noon (5 hrs.)		0	49.3
Local ast. mean time at Naval Academy	23	54	03.5
Local sidereal time at Naval Academy	20	10	48.6
Reading of sidereal clock at the instant	20	15	09
Error of sidereal clock on L. S. T., fast		4	20.4

193. Having the sidereal time at any place, to find the local mean time.—Since λ, the longitude, is the G. M. T. of local mean noon, or of the instant when the mean sun is on the upper branch of the local meridian, according to the notation of Art. 192,

$a_0 + .0027379\lambda$ will be the local sidereal time at local mean noon.

$S - (a_0 + .0027379\lambda)$ will be the sidereal interval since noon as it is L. S. T. − the sidereal time of local mean noon.

$[S - (a_0 + .0027379\lambda)][1 - .0027304]$ will be the mean time interval since noon, and to find L. M. T. it is only necessary to add the astronomical day to this mean time interval.

Hence the rules:

(1) *Take from the Nautical Almanac for Greenwich mean noon of the given local astronomical day the right ascension of*

the mean sun; apply to this the reduction for longitude (which is the change in the mean sun's right ascension for that number of hours) taken from Table III, Nautical Almanac; adding for West, subtracting for East longitude. The result will be the right ascension of the mean sun at local mean noon, or sidereal time at that instant (local 0 hrs. of mean time).

(2) *Subtract this from the given L. S. T. (adding 24 hrs. to the L. S. T. if necessary for subtraction) and the result will be the sidereal interval from local mean noon.*

(3) *Apply to this the reduction of a sidereal to a mean time interval taken from Table II, Nautical Almanac, which is always subtractive. The result, after prefixing the given astronomical day, is the required local mean time.*

In the absence of Tables, the reduction may be made by using the formulæ (139) and (140) of Art. 191.

Caution.—It is much better to convert a given L. S. T. and afterwards, if desired, find the G. M. T., than to first find G. S. T. and then convert it into G. M. T., for the reason that the right ascension of the mean sun must be taken out for the given astronomical day. To convert G. S. T. the Greenwich astronomical date must be known, and as this may or may not be the same as the local astronomical date, an error might result.

As a little thought can easily determine the Greenwich date, this caution may seem unnecessary to those thoroughly familiar with the subject; to others, however, it is most important.

In cases where the G. M. T. is known in addition to the L. S. T., the method of reduction is very simple.

From formula (141),

$$t = S - [a_0 + (t + \lambda)\ (.0027379)] \qquad (142)$$

Rule: *For the G. M. T. $(t+\lambda)$ take out the right ascension of the mean sun, subtract it from the given L. S. T., and the result will be L. M. T. of the given astronomical date.*

So also from Art. 173, it is plain that local apparent time equals L. S. T. minus the apparent right ascension after correction for G. M. T.

Examples on the conversion of time; L. S. T. into L. M. T.

Ex. 122.—January 8, a. m., 1918, at Royal Observatory, Lisbon (Long. $0^h\ 36^m\ 44^s.68$ W.), local sidereal time is $10^h\ 44^m\ 30^s$. Find the local mean time.

First find the astronomical day, which is January 7.

		h	m	s
R. A. M. ⊙ or sidereal time at Greenwich mean noon Jan 7,		19	04	57.8
Reduction for longitude West	+			6.0
The sidereal time of local mean noon		19	05	03.8
The given local sidereal time (+ 24 hrs. for the subtraction)		10	44	30
The sidereal interval from noon		15	39	26.2
Reduction of a sid. to a M. T. interval, Table II	—		2	33.9
The required astronomical mean time Jan. 7,		15	36	52.3
Or civil time Jar. 8 (a. m.)		3	36	52.3

Ex. 123.—April 15 (civil date), 1918, in Long. 129° 30′ 45″ E., the local sidereal time is $23^h\ 56^m\ 30^s$. Find the local mean time.

The above example does not say whether it is a. m. or p. m., but the astronomical date must be known before taking out the R. A. M. ⊙. To determine this look up the approximate R. A. M. ⊙, which is found to be about 1½ hours. Subtracting this from the L. S. T. leaves an approximate astronomical mean time of over 22 hours; the civil time is, therefore, a. m., and hence the local astronomical date is April 14.

		h	m	s
R. A. M. ⊙ or sidereal time at Greenwich mean noon April 14,		1	27	23.5
Reduction for Long. (8^{h} 38^{m} 03^{s} East), Table III	—		1	25.1
The sidereal time of local mean noon		1	25	58.4
The given local sidereal time		23	56	30
The sidereal interval from noon		22	30	31.6
Reduction of a sid. to a M. T. interval, Table II	—		3	41.3
The required astronomical L. M. T. April 14,		22	26	50.3
Or civil date April 15 (a. m.)		10	26	50.3

Ex. 124.—On January 11, astronomical time, 1918, the sidereal clock time of transit of α Leonis (Regulus) over the middle wire of a transit instrument at the U. S. Naval Academy was 10^{h} 05^{m} $22^{s}.5$. Later a comparison of the sidereal clock and a mean time chronometer was: Sid. clock, 10^{h} 30^{m} $20^{s}.5$; M. T. chro., 8^{h} 05^{m} 10^{s}. Find the error of chronometer on G. M. T. Longitude of Naval Academy 5^{h} 05^{m} $56^{s}.5$ West.

		h	m	s
R. A. of ✳ α Leonis at transit equals the L. S. T.		10	04	02.6
Reading of sidereal clock at star's transit		10	05	22.5
Error of sidereal clock on L. S. T. fast			1	19.9
Reading of sidereal clock at comparison		10	30	20.5
L. S. T. at instant of comparison		10	29	00.6

		h	m	s
R. A. M. ⊙ or sidereal time at G. M. noon Jan. 11,		19	20	44.0
Reduction for longitude West (5^{h} 05^{m} $56^{s}.5$)	+			50.3
Sidereal time of local 0 hrs.		19	21	34.3
The given L. S. T. at comparison		10	29	00.6
Sidereal interval from noon		15	07	26.3
Reduction of a sid. to a M. T. interval, Table II	(—)		2	28.7
The local mean time at instant of comparison		15	04	57.6
Longitude of Naval Academy West	+	5	05	56.5
G. M. T. at instant of comparison		20	10	54.1
Reading of M. T. chronometer at comparison		8	05	10
Error of chronometer (dropping 12 hrs.), slow on G. M. T.			5	44.1

The following example worked under Art. 192 without first finding the error of the sidereal clock, by considering only the sidereal interval from noon to time of comparison, and finding the corresponding mean time interval from noon and then the G. M. T., will now be worked by finding the clock error on L. S. T. as indicated in the solution.

Ex. 125.—January 20, 1918, at U. S. Naval Academy, when the 75th meridian mean noon signal was received, a sidereal clock read $20^h\ 23^m\ 19^s.5$. Shortly after the receipt of this signal a comparison of this clock with a mean time chronometer was: Sid. clock, $20^h\ 43^m\ 29^s$; M. T. chro., $5^h\ 24^m\ 16^s$. Find the error of the clock on L. S. T. and the error of the chronometer on G. M. T. (see Ex. 118, Art. 192).

	h	m	s
The 75th mer. mean time of 75th mer. mean noon	**0**	**00**	**00**
Longitude of 75th meridian West	**+ 5**	**00**	**00**
G. M. T. of 75th mer. mean noon Jan. 20,	**5**	**00**	**00**
R. A. M. ⊙ or sidereal time G. M. noon Jan. 20,	**19**	**56**	**13.0**
Correction for G. M. T.			**49.3**
G. S. T. of 75th meridian mean noon	**0**	**57**	**02.3**
Longitude of Naval Academy West	**5**	**05**	**56.5**
L. S. T. of 75th meridian mean noon	**19**	**51**	**05.8**
Sidereal clock time of 75th meridian noon	**20**	**23**	**19.5**
Error of sidereal clock on L. S. T. fast	**0**	**32**	**13.7**
Sidereal clock time of comparison	**20**	**43**	**29**
L. S. T. at instant of comparison	**20**	**11**	**15.3**

	h	m	s
R. A. M. ⊙ or sidereal time at G. M. noon Jan. 20,	**19**	**56**	**13.0**
Reduction for longitude ($5^h\ 05^m\ 56^s.5$ W.)	**+**		**50.3**
Sidereal time of local 0 hrs.	**19**	**57**	**03.3**
Given local sidereal time	**20**	**11**	**15.3**
Sidereal interval from noon	**0**	**14**	**12.0**
Reduction of a sid. to a M. T. interval, Table II	**—**	**0**	**02.3**
Required L. M. T. at instant of comparison	**0**	**14**	**09.7**
Longitude of Naval Academy West	**+ 5**	**05**	**56.5**
The G. M. T. at instant of comparison	**5**	**20**	**06.2**
M. T. chronometer reading at comparison	**5**	**24**	**16**
Error of M. T. chronometer on G. M. T., fast		**4**	**09.8**

194. Relation between apparent time and sidereal time.—From Art. 173 it is seen that local sidereal time is equal to the true sun's right ascension plus the local apparent time. The right ascension of the time sun is not tabulated in the Nautical Almanac however, so that, having a given local apparent time, to find the local sidereal time:

(1) *Find the G. A. T. and date.*

(2) *For this G. A. T. take out the equation of time from the N. A. Apply the equation of time with its proper sign to the G. A. T. obtaining the corresponding G. M. T.*

(3) *Apply the longitude to the G. M. T. obtaining the corresponding L. M. T. which can be converted into L. S. T. as before explained.*

Ex. 126.—On January 8, 1918, in Long. 135° 15′ E., the local apparent time is $5^h\ 10^m\ 30^s$ a. m. Find the local sidereal time.

		h	m	s
Jan. 7, local astronomical apparent time		17	10	30
Longitude from Greenwich East	(—)	9	01	00
Greenwich apparent time Jan. 7,		8	09	30
Equation of time	+		6	19.6
Greenwich mean time Jan. 7,		8	15	49.6
Longitude East		9	01	00
Local astronomical mean time		17	16	49.6
		h	m	s
R. A. M. ⊙ at G. M. noon Jan. 7,		19	04	57.8
Corr. for G. M. T. Table III			1	21.5
Required local sidereal time		12	23	08.9

195. Relation of time, hour angles, right ascensions, and a consideration of problems involving them.

A great many problems arise in everyday practical navigation which involve the consideration of hour angle and right ascension. Such problems are readily solved if the definitions of these terms and of local sidereal time are well understood. Some will be illustrated in the following articles.

196. To find the local mean time of transit of a particular heavenly body across the meridian of a given place, the longitude of the place, or G. M. T., being known.

(*a*) **In case the L. M. T. of transit of the sun is desired,** it is only necessary to remember that the instant of transit of the true sun is apparent noon, and at this instant the equation of time corrected for longitude (which is the G. A. T. of the instant) is the hour angle of the mean sun. If the equation of time is additive to apparent time, the L. M. T. of the sun's transit is the equation of time, and the local date is the given astronomical date; if the equation of time is subtractive from apparent time, the L. M. T. of the sun's transit is 24 hours—the equation of time, and the local date is that of the day preceding the given astronomical day.

Ex. 127.—January 27, 1918, in Long. 52° 30′ W., find the local mean time of upper transit of the true sun, or of local apparent noon.

λ=G. A. T. of sun's transit=$3^h\ 30^m$
L. M. T. of local apparent noon
equals the equation of time,
or Jan. 27, $0^h\ 12^m\ 50^s.00$ (p. m.)

Equation of Time.
m s
At 2^h 12 49.2
Corr. + 0.8
Eq. of T. =12 50.0
+to Apparent time.

H. D.
+ $0^s.5$
G. A. T. $1^h.5$
Corr. + $0^s.75$

Ex. 128.—April 27, 1918, in Long. 52° 30′ W., find the local mean time of the upper transit of the true sun, or of local apparent noon.

λ=G. A. T. of sun's transit=$3^h\ 30^m$
L. M. T. of local apparent noon
equals the equation of time,

	h m s
or April 27,	(−) 0 02 21.2
or April 26,	23 57 38.8
or April 27,	11 57 38.8(a. m.)

Equation of Time.
m s
At 2^h 2 20.6
Corr. + 0.6
Eq. of T. 2 21.2
(−) to Apparent time.

H. D.
+ $0^s.4$
G. A. T. $1^h.5$
Corr. + $0^s.6$

When any heavenly body is on the upper branch of the meridian of a place, its right ascension is the right ascension of the meridian, or the local sidereal time (Art. 173), and to find the L. M. T. of transit it is only necessary to obtain from the Nautical Almanac the right ascension of the body at that instant, and, remembering that this is the local sidereal time, reduce it to L. M. T. (Art. 193).

The time of transit of the sun across the meridian may be found in this way, which, however, is a longer way than the method used on page 394.

(*b*) **To find the L. M. T. of transit of a given star across the upper branch of a given meridian.**

In the table on pages 94 and 95, the American Nautical Almanac contains the apparent right ascensions and declinations of 55 of the principal stars for the upper culmination at Greenwich on the first day of each month. These right ascensions may be taken as the right ascensions for the upper culmination at any other meridian, except in the case of Polaris, whose right ascension may be reduced by interpolation for differences of longitude, if desired.

In the table on pages 98 and 99, the mean places of 110 additional stars, with their annual variation, are given for the beginning of the Besselian fictitious year at Greenwich.

In both tables the stars are arranged in the order of their right ascensions.

Having the longitude of the place and the right ascension of the star, which is the L. S. T. at the instant of the star's upper transit of that meridian, we find the L. M. T. of transit by the method explained in Art. 193. A close approximation to the L. M. T. of local transit may be obtained by taking out the G. M. T. of Greenwich transit from the table on page 96, N. A., and correcting for day and difference of longitude.

Ex. 129.—April 12 (civil date), 1918, in longitude $5^h\ 05^m\ 56^s.5$ W., find the local mean time of the upper transit of the star *α* Scorpii (Antares), whose R. A. is $16^h\ 24^m\ 25^s.8$.

An examination of the approximate R. A. M. ⊙ and R. A. of the star shows the astronomical date to be the 11th.

		h	m	s
April 11, R. A. M. ⊙ at G. M. noon		**1**	**15**	**33.9**
Reduction for longitude West, Table III	+			**50.3**
Sidereal time of local 0 hrs.		**1**	**16**	**24.2**
The given L. S. T. = ✱'s R. A.		**16**	**24**	**25.8**
The sidereal interval from noon		**15**	**08**	**01.6**
Reduction of the sid. to a M. T. interval, Table II	(—)		**2**	**28.8**
The required L. M. T. of ✱'s transit April 11,		**15**	**05**	**32.8**
Or civil date April 12 (a. m.)		**3**	**05**	**32.8**

Ex. 130.—January 5, 1918, at the Naval Academy, Annapolis, Md. (Long. $5^h\ 05^m\ 56^s.5$ W.), find the 75th meridian West longitude mean time of the upper transit of the star *α* Canis Majoris (Sirius) across the local meridian.

		h	m	s
Jan. 5, R. A. M. ⊙ at G. M. noon		**18**	**57**	**04.6**
Reduction for longitude West, Table III	+			**50.3**
The sidereal time of local 0 hrs.		**18**	**57**	**54.9**
The L. S. T. at time of transit = star's R. A.		**6**	**41**	**34.3**
The sidereal interval from mean noon		**11**	**43**	**39.4**
Reduction of sid. to a mean time interval, Table II	(—)		**1**	**55.3**
L. M. T. of transit of Sirius Jan. 5,		**11**	**41**	**44.1**
Diff. of longitude of local and 75th meridians	+		**5**	**56.5**
The 75th meridian time of local transit		**11**	**47**	**40.6**

(*c*) **To find the L. M. T. of transit of the moon across the upper branch of a given meridian.**

In the case of the moon the L. M. T. of transit may be found from the Nautical Almanac, where the mean time of the upper transit at Greenwich is given to the nearest minute accompanied by a column of differences for 24 hours.

These differences express, for each day, the mean hourly increase of the moon's right ascension. The number of minutes in this column divided by 24 hours, when multiplied by the hours of longitude, will give a correction, + for West longitude, (−) for East longitude, to be applied to the time of Greenwich transit to give the L. M. T. of local transit. (See Table IV, N. A.)

In taking out the time of the meridian passage of the moon, it must not be forgotten that the result will be astronomical time and not civil time. When the Almanac time of passage, after correction for longitude, gives a time greater than 12 hours of a given astronomical day, it is plain that this is not the time of passage on the civil day of the same date. *Hence, if the time of passage over the meridian is desired for a given civil date, and it is seen by inspection that the tabulated time after correction for the longitude will be greater than 12 hours, then it will be necessary to take out the Greenwich time of Greenwich transit for the day before* (see Art. 188).

Ex. 131.—Find the time of the meridian passage of the moon over the upper branch of the meridian in longitude 60° 30′ East for January 2 (civil date), 1918.

Since an inspection of the Nautical Almanac shows the astronomical time of transit will be greater than 12 hours, the meridian transit of the preceding astronomical date, January 1, must be used in order to make the civil date January 2.

	h m		
Approx. M. T. of transit of moon over the mer. of Greenwich Jan. 1,	15 35.0	H. D.	+ $1^{m}.71$
Corr. or retardation for Long. East	− 6.9	λ	− $4^{h}.03$
L. M. T. of local transit Jan. 1,	15 28.1	Corr.	− $6^{m}.89$
Or civil date Jan. 2 (a. m.)	3 28.1	Or (Tab. 11, Bowditch) −7^{m}	

The above method is sufficiently accurate for all purposes of everyday practical navigation, but if a more accurate time of transit of the moon is desired, find the above L. M. T. of local transit, apply the longitude to obtain the G. M. T. of local transit, and for this G. M. T. take out the moon's right ascension which is given in the Nautical Almanac for each even hour of G. M. T. with corresponding differences.

The moon being on the upper branch of the meridian, its right ascension is the L. S. T., which can be reduced to L. M. T. (Art. 193).

In the solution on page 399 will be shown the method of re-correcting the L. M. T. of transit of the moon for the difference between itself and the approximate L. M. T. as found above.

(*d*) **To find the L. M. T. of transit of a planet across the upper branch of a given meridian.**

In the case of a planet, the Nautical Almanac gives the G. M. T. of the transit over the Greenwich meridian to the nearest minute for each day of the year. The difference of the times for two successive days will give the daily retardation or acceleration. This divided by 24 and the result multiplied by the number of hours of longitude, + for West longitude, (–) for East longitude, will give the retardation or acceleration to be applied to the Greenwich time of Greenwich transit to give the L. M. T. of local transit.

As in the case of the moon, if the sum of the approximate time of transit of the Greenwich meridian and the retardation (or acceleration) is less than 12 hours, the time of the transit of the planet should be taken out of the Nautical Almanac for the given civil date; if that sum is greater than 12 hours, the time of transit must be taken for the day before the given civil day.

Ex. 132.—Find the correct time of the moon's meridian passage over the upper branch of the meridian in longitude 60° 30′ East for Jan. 2, civil date, 1918, by using the right ascension of the moon.

	h m s
Approx. M. T. of transit of moon over the meridian of Greenwich Jan. 1,	15 35 00
Retardation for Long. E.	— 6 53
L. M. T. of local transit	15 28 07
Long. East	(—) 4 02 00
G. M. T. of local transit Jan. 1,	11 26 07

H. D.	+ $1^m.71$
λ	— $4^h.03$
Corr.	— $6^m.89$
	$= -6^m\ 53^s$

R. A. of Moon, using 2^d Differences.

	h m s
Jan. 1 R. A. of moon at 11^h	10 10 28.17
Correction for G. M. T.	47.958
R. A. of moon at transit = L. S. T. =	10 11 16.128
R. A. M. ⊙ corrected for G. M. T.	18 43 11.121
L. M. T. of transit =	15 28 05.007
Approx. L. M. T.	15 28 07
Diff. from approx. L. M. T.	— 1.993

M. D.

	s
At 11^h	1.8368
At 12^h	1.8346
Ch. in 60^m	— .0022
Ch. in 13^m	— .0005
	+1.8368
Mean M. D.	1.8363
G. M. T.	$26^m.117$
Corr.	$+47^s.958$

R. A. M. ⊙.

	h m s
At G. M. N.	18 41 18.41
Corr. G. M. T. +	1 52.711
	18 43 11.121

		s		h m s
In (—) $1^s.993$	Ch. of R. A. ☾ =	—0.061	L. M. T. of transit	15 28 05.007
	—Ch. of R. A. M. ⊙	+0.005	Corr.	— .056
Correction		—0.056	Correct L. M. T.	15 28 04.951

or Jan. 2, 3h 28m 04s.951 a. m.

NOTE.—Data for approximate L. M. T. of transit from Nautical Almanac.
Data for correct L. M. T. of transit from Ephemeris.

If a more accurate time of transit of a planet is desired, proceed as explained for the case of the moon.

The following example illustrates the finding of the approximate as well as of the more exact time of transit:

Ex. 133.—Find the local mean time of the meridian passage of planet Mars over the upper branch of the meridian in longitude 52° 30′ W., for February 14, 1918, civil date.

An inspection of page 751, extracts from Ephemeris, 1918, shows that the astronomical time of transit is >12 hours; therefore, the astronomical date corresponding to the time of transit February 14, civil date, is February 13.

		Acceleration.		R. A. M. ☉.
	h m			h m s
Approx. M. T. of Gr. Tr. Feb. 13	14 41.5	For 24^h, $4^m.4$	At G. M. N.	21 30 50.35
Acceleration for $\lambda = 3^h.5$ W	— 0.6	For 1^h, 0.18	Corr. G. M. T.	2 59.207
Approx. L. M. T. of local transit	14 40.9	For λ W, 0.630		21 33 49.557
Longitude West	+ 3 30			
G. M. T. of local transit Feb. 13 (approx.) } =	18 10.9			
		h m s		s
Feb. 13, R. A. of Mars at G. M. noon		12 14 58.72	Feb. 13, H. D.	−1.056
Correction for G. M. T. (2^d diff.)		— 20.04	Feb. 14, H. D.	−1.179
R. A. of Mars on meridian = L. S. T.		12 14 38.68	Change in 24^h	0.123
R. A. M. ☉ corrected for G. M. T.		21 33 49.557	Change in 1^h	0.0051
			Change in $9^h.1$	0.046
L. M. T. of transit of planet Mars, Feb. 13,		14 40 49.123	Feb. 13, H. D.	−1.056
or civil date Feb. 14 (a. m.)		2 40 49.123		
			Mean H. D.	−1.102
			G. M. T.	$18^h.182$
			Correction	$-20^s.036$

197. To find the time of transit of the moon, a planet, or of a given star across the lower branch of a given meridian.

To find the time of a body's lower culmination, the L. S. T. is taken as 12 hours plus the right ascension, or, what amounts to the same thing, 12 hours may be added to the longitude of the place. The latter method is preferable when finding the approximate times in case of the moon and planets.

Ex. 134.—Find the L. M. T. of the lower culmination of the star α Argus (Canopus) in longitude 60° East on April 4, a. m., 1918.

In this case (12 hours + *'s R. A.) $= 18^h\ 22^m\ 08^s.4 =$ L. S. T. at the instant of lower culmination.

	h	m	s
April 3, R. A. M. ⊙ at G. M. noon	0	44	01.5
Reduction for λ 60° E, Table III	—		39.4
The sidereal time of local 0 hrs.	0	43	22.1
The L. S. T. of lower culmination	18	22	08.4
The sidereal interval from mean noon	17	38	46.3
Reduction of sidereal to a M. T. interval, Table II	—	2	53.5
The L. M. T. of lower culmination April 3,	17	35	52.8
Or civil time April 4 (a. m.)	5	35	52.8

198. To find the watch time of transit of a given heavenly body across the upper branch of a given meridian.

The simplest and most practical way of observing the meridian altitude of a heavenly body is to calculate beforehand its watch time of transit, and then to observe the altitude when the watch indicates that time.

(*a*) **Watch time of sun's transit.**—In the case of the sun, the a. m. longitude brought up to noon by means of the run in longitude from the time of a. m. sight to noon, expressed in time, is the G. A. T. of noon of the given astronomical date, if in West longitude; or, if in East longitude, it is a negative, or (−), G. A. T. of the given astronomical date.

For this G. A. T. take out the equation of time, and find the G. M. T. of noon; apply the chronometer correction with the sign of application reversed, and get the C. T. of noon from which, by subtracting the C—W, find the watch time of local apparent noon. Every navigator should do this before going on deck to observe his meridian altitude. Another way of arriving at the same result is to obtain from his forenoon sight the watch error on L. A. T., and apply to this error the difference in longitude for the run from sight to noon.

Ex. 135.—April 4, 1918, in Long. 85° 30′ W., given the C—W = 5^h 52^m 05^s, chronometer fast on G. M. T. 5^m 03^s.7, find the W. T. of local apparent noon.

		h m s
Long. = G. A. T. of local apparent noon April 4		5 42 00
Equation of time	+	3 08.3
G. M. T. of local apparent noon		5 45 08.3
Chronometer fast on G. M. T.	+	5 03.7
C. T. of local apparent noon		5 50 12
C—W		5 52 05
W. T. of local apparent noon		11 58 07

Equation of Time.

	m s		
At 4^h April 4	3 09.5	H. D.	−.7
Corr.	− 1.2	G. A. T.	1.7
Eq. of T. +to App. T.	3 08.3	Corr.	−1.19

Ex. 136.—January 20, 1918, in Long. 132° 15′ E., if the C—W is 3^h 17^m 30^s, and the chronometer is slow on G. M. T. 6^m 19^s.2, what is the watch time of local apparent noon?

		h m s
Long. = G. A. T. of noon Jan. 20,	(−)	8 49 00
or G. A. T. is Jan. 19,		15 11 00
Equation of time	+	10 55.2
G. M. T. of apparent noon		15 21 55.2
Chronometer slow	(−)	6 19.2
C. T. of local apparent noon		3 15 36
C—W		3 17 30
W. T. of apparent noon		11 58 06

Equation of Time.

	m s		
At 14^h Jan. 19	10 54.3	H. D.	+ .8
Corr.	+ .9	G.A.T.	+1.18
Eq. of T. +to App. T.	10 55.2	Corr.	+ .944

(*b*) **Watch time of a star's transit.**—In the case of stars, the right ascension at the instant of upper transit is the L. S. T. Knowing the longitude, find the corresponding G. M. T. of local transit; apply the chronometer correction and C—W as in Exs. 135 and 136 and get the watch time of transit.

Remember that at the instant of lower transit the L. S. T. equals the right ascension plus 12 hours.

Ex. 137.—January 10, 1918, in longitude $5^{h}\ 32^{m}\ 15^{s}$ West, find the watch time of upper transit of the star α Aurigæ (Capella) if the C—W is $5^{h}\ 35^{m}\ 10^{s}$ and the chronometer slow on G. M. T. $2^{m}\ 04^{s}.9$. The star's R. A. $= 5^{h}\ 10^{m}\ 41^{s} =$ L. S. T. at transit.

		h	m	s
Jan. 10, R. A. M. ⊙ at G. M. noon		**19**	**16**	**47.4**
Reduction for Long. ($5^{h}\ 32^{m}\ 15^{s}$ W), Table III	**+**		**0**	**54.6**
The sidereal time of local 0 hrs.		**19**	**17**	**42.0**
The given L. S. T. = star's R. A.		**5**	**10**	**41.0**
The sidereal interval		**9**	**52**	**59.0**
Reduction, Table II	**—**		**1**	**37.1**
L. M. T. of star's local transit		**9**	**51**	**21.9**
Longitude West	**+**	**5**	**32**	**15**
G. M. T. of local transit		**15**	**23**	**36.9**
Chronometer slow on G. M. T.	**(—)**		**2**	**04.9**
C. T. of star's local transit		**3**	**21**	**32**
C—W		**5**	**35**	**10**
W. T. of transit of star Capella		**9**	**46**	**22**

(*c*) **Watch time of the transit of the moon or a planet.**—For the moon or planets, find from the Nautical Almanac the G. M. T. of local transit to the nearest minute (Art. 188 and Art. 189), apply the chronometer correction and C—W as above and find the W. T. of transit.

199. To find the hour angle of any heavenly body at a given time and place.

(*a*) In the case of the sun, the hour angle reckoned positively from the upper meridian towards the West is the L. A. T. If the sun is East of the meridian, the hour angle is negative and is equal to 24 hours—the apparent time.

Having then a given mean time or sidereal time, the longitude or G. M. T. being known, the L. A. T. may be found by Art. 190, Art. 193, or Art. 194.

Ex. 138.—April 10, a. m., 1918, Long. 129° 30′ 45″ E., L. M. T. $10^h\ 25^m\ 19^s$, find the true sun's hour angle.

	h m s
Local ast. mean time April 9,	22 25 19
Longitude East	8 38 03
G. M. T. April 9,	13 47 16

Equation of Time.	m s	
At 12^h April 9	1 38.0	$-0^s.7$
Corr.	− 1.3	$1^h.79$
Eq. of T. (−) to M. T.	1 36.7	$-1^s.253$

	h m s
L. M. T. (astronomical) April 9,	22 25 19
Equation of time	− 1 36.7
L. A. T. = sun's H. A. April 9,	+22 23 42.3
or April 10,	− 1 36 17.7

Ex. 139.—April 6, 1918, a. m., Long. 162° 49′ 15″ W., L. S. T. = $18^h\ 42^m\ 10^s$, find the H. A.'s of mean and true suns.

	h m s
R. A. M. ☉ April 5 at G. M. noon	0 51 54.6
Reduction for longitude	+ 1 47.0
Sidereal time of local 0^h	0 53 41.6
The given L. S. T.	18 42 10
The sidereal interval	17 48 28.4
Reduction to a M. T. interval	− 2 55.0
L. M. T. April 5, (H. A. mean sun)	17 45 33.4
Longitude W.	+10 51 17
G. M. T. April 6,	4 36 50.4

Equation of Time (−) to M. T.

m s		
2 34.4	H. D.	$-0^s.7$
− 0.4	G. M. T.	$+0^h.6$
2 34.0	Corr.	$-0^s.42$

	h m s
L. M. T. April 5,	17 45 33.4
Eq. of time	− 2 34.0
L. A. T. = H. A. ☉ April 5,	17 42 59.4
or April 6,	(−) 6 17 00.6

Ex. 140.—January 3, 1918, a. m., in Long. 150° 09′ 54″ W., the W. T. of obs. of the sun was $8^h\ 04^m\ 35^s$, C—W $10^h\ 07^m\ 15^s$,

chronometer fast on G. M. T. 7^m 11^s.5. Find the true sun's H. A.

	h m s	Equation of Time.			H. D.		h m s
W=	8 04 35		m s			G. M. T.	6 04 38.5
C—W	10 07 15	At 6^h	4 29.5		+1^s.2	Long. W	10 00 39.6
		Corr. G.M.T.	+ .1	G. T.	0^h.08		
C.	6 11 50					L. M. T.	20 08 58.9
C. C.	(−) 7 11.5	Eq. of T.	4 29.6	Corr.	+0^s.096	Eq. of T.	− 4 29.6
		(−) to M. T.					
G. M. T. Jan. 3	6 04 38.5					L. A. T.=	
						H. A. ⊙	+19 59 29.3
						or H.A.⊙=	−4 00 30.7

(*b*) In the case of the moon, a planet, or a fixed star, it is only necessary to find from the given time, knowing the longitude or Greenwich mean time, the local sidereal time. Subtracting from this L. S. T. the right ascension of the moon, planet, or fixed star, the algebraic difference, if plus, will be the hour angle, West of the meridian, of the moon, planet, or fixed star; if the algebraic difference is minus it will be the hour angle, East of the meridian.

If the body is the moon or a planet or a star, its right ascension is corrected for the Greenwich mean time of the instant.

If the given time is local mean time, the right ascension of the mean sun for the Greenwich instant must be added to it to give the L. S. T.

Ex. 141.—About 5 a. m. April 22, 1918, took an observation of a star α Aquilæ (Altair), W. T. obs. 4^h 55^m 20^s, C—W 1^h 12^m 00^s, chronometer fast on G. M. T. 57^m 07.^{s}6, Long. by D. R. 3° 10′ West. Find the star's hour angle.

	h m s	R. A. M. ⊙.			h m s
W	4 55 20		h m s	L. ast. M. T.	16 57 32.4
C—W	1 12 00	At G. M. N.	1 54 59.4	Corr. R. A. M. ⊙	1 57 48.6
		Corr. G. M. T.	2 49.2		
C.	6 07 20			L. S. T.	18 55 21.0
C. C.	− 57 07.6	R. A. M. ⊙	1 57 48.6	Altair's R. A.	19 46 49.0
G.M.T. Apr. 21,	17 10 12.4			Altair's H. A. (−)	0 51 28.0
Long. W	− 12 40				
L.M.T. Apr. 21,	16 57 32.4				

Ex. 142.—April 10, 1918, in longitude 45° 15′ W., wh were the hour angles of the moon and the planet Mars 8ʰ 06ᵐ 14ˢ p. m., L. M. T.?

	h m s
L. ast. M. T. April 10,	**8 06 14**
Long. West	**+3 01 00**
G. M. T. April 10,	**11 07 14**
	or 11ʰ.12

	h m s
L. ast. M. T. April 10,	**8 06 14**
R.A.M.⊙ at G. M. N.	**1 11 37.3**
Red. for G. M. T.	**1 49.6**
Local sidereal time	**9 19 40.9**

R. A. and H. A. of the Moon.

	h m s		
☽'s R. A. at 10ʰ	**0 52 48**	**H. D.**	**+144ˢ**
Corr. for 1ʰ.12	**2 41.8**		**1ʰ**
☽'s R. A.	**0 55 29.8**		**+161ˢ.**
L. S. T.	**9 19 40.9**		**=2ᵐ 41**
☽'s H. A.	**+8 24 11.1**		
	or West 8 24 11.1		

R. A. and H. A. of Mars.

	h m s		
R. A. of Mars Apr. 10,	**11 10 36**	**H. D.**	**— 1ˢ.9**
Corr. for G. M. T.	**— 21.8**		**11ʰ.1**
R. A. of Mars	**11 10 14.2**		**—21ˢ.**
L. S. T.	**9 19 40.9**		
H. A. of Mars	**—1 50 33.3**		
	or East 1 50 33.3		

Ex. 143.—On April 5, 1918, in longitude 34° 52′ 30″ W the H. A. of the true sun is +3ʰ 10ᵐ 30ˢ, find the H. A. o the vernal equinox and stars Sirius (α Canis Majoris) and Achernar (α Eridani).

	h m s
Local astronomical apparent time April 5,	**3 10 30**
Longitude from Greenwich West	**+ 2 19 30**
Greenwich apparent time April 5,	**5 30 00**
Equation of time	**+ 2 50.8**
Greenwich mean time April 5	**5 32 50.8**

	h m s
R. A. mean ⊙ at G. M. N. April 5,	**0 51 54.6**
Correction for G. M. T. Table III	**0 54.7**
Local astronomical mean time	**3 13 20.8**
L. S. T. equals the H. A. of the vernal equinox	**4 06 10.1**

	h m s		
L. S. T.	**4 06 10.1**	**L. S. T.**	**4 06 10.1**
R. A. ✳ Sirius	**6 41 33.4**	**R. A. ✳ Achernar**	**1 34 38.7**
H. A. ✳ Sirius	**(—) 2 35 23.3**	**H. A. ✳ Achernar**	**+2 31 31.4**

***Ex. 144.*—At sea, in longitude $3^h\ 40^m\ 30^s$ W. April 9, 1918, during evening twilight, took an observation of Polaris (α Ursæ Minoris) W. T. of obs. $7^h\ 05^m\ 20^s$, C—W, $3^h\ 25^m\ 30^s$, chro. fast on G. M. T. $21^m\ 25^s$. Find the local sidereal time and hour angle of Polaris.**

	h m s		h m s	To Find R. A. of Polaris by Inspection.	
W.	7 05 20	R. A. M. ⊙ at G. M. noon	1 07 40.8	Change in R. A. from Apr. 1 to May 1	+5ˢ.4
C—W	3 25 30	Reduction for G. M. T.	1 40.1		
		G. M. T.	10 09 25	Change in 1 day	+0 .18
C.	10 30 50			Change in 9 days	+1 .62
C. C.	− 21 25	G. S. T.	11 18 45.9		h m s
		Longitude W.	3 40 30	R. A. Polaris Apr. 1	1 29 56.8
G. M. T. April 9	10 09 25	L. S. T.	7 38 15.9	Change in 9 days	+ 1.6
		R. A. ✳ Polaris	1 29 58.4	R. A. Polaris Apr. 9	1 29 58.4
		H. A. ✳ Polaris	+ 6 08 17.5		

The above method is sufficiently accurate for all purposes of everyday practical navigation, but if a more accurate time of transit of the moon is desired, find the above L. M. T. of local transit, apply the longitude to obtain the G. M. T. of local transit, and for this G. M. T. take out the moon's right ascension which is given in the Nautical Almanac for each even hour of G. M. T. with corresponding differences.

The moon being on the upper branch of the meridian, its right ascension is the L. S. T., which can be reduced to L. M. T. (Art. 193).

In the solution on page 399 will be shown the method of re-correcting the L. M. T. of transit of the moon for the difference between itself and the approximate L. M. T. as found above.

(*d*) **To find the L. M. T. of transit of a planet across the upper branch of a given meridian.**

In the case of a planet, the Nautical Almanac gives the G. M. T. of the transit over the Greenwich meridian to the nearest minute for each day of the year. The difference of the times for two successive days will give the daily retardation or acceleration. This divided by 24 and the result multiplied by the number of hours of longitude, + for West longitude, (−) for East longitude, will give the retardation or acceleration to be applied to the Greenwich time of Greenwich transit to give the L. M. T. of local transit.

As in the case of the moon, if the sum of the approximate time of transit of the Greenwich meridian and the retardation (or acceleration) is less than 12 hours, the time of the transit of the planet should be taken out of the Nautical Almanac for the given civil date; if that sum is greater than 12 hours, the time of transit must be taken for the day before the given civil day.

Ex. 132.—**Find the correct time of the moon's meridian passage over the upper branch of the meridian in longitude 60° 30′ East for Jan. 2, civil date, 1918, by using the right ascension of the moon.**

---

	h m s
Approx. M. T. of transit of moon over the meridian of Greenwich Jan. 1,	15 35 00
Retardation for Long. E.	− 6 53
L. M. T. of local transit	15 28 07
Long. East	(−) 4 02 00
G. M. T. of local transit Jan. 1,	11 26 07

H. D.	+ $1^m.71$
λ	− $4^h.03$
Corr.	− $6^m.89$
	$= -6^m\ 53^s$

R. A. of Moon, using 2ᵈ Differences.

	h m s
Jan. 1 R. A. of moon at 11^h	10 10 28.17
Correction for G. M. T.	47.958
R. A. of moon at transit=L. S. T.=	10 11 16.128
R. A. M. ⊙ corrected for G. M. T.	18 43 11.121
L. M. T. of transit=	15 28 05.007
Approx. L. M. T.	15 28 07
Diff. from approx. L. M. T.	− 1.993

M. D.

	s
At 11^h	1.8368
At 12^h	1.8346
Ch. in 60^m	− .0022
Ch. in 13^m	− .0005
	+1.8368
Mean M. D.	1.8363
G. M. T.	$26^m.117$
Corr.	$+47^s.958$

R. A. M. ⊙.

	h m s
At G. M. N.	18 41 18.41
Corr. G. M. T. +	1 52.711
	18 43 11.121

$$\text{In } (-)\ 1^s.993 \left\{ \begin{array}{lr} \text{Ch. of R. A. } ☾ = & -0.061 \\ -\text{Ch. of R. A. M. } \odot & +0.005 \\ \hline \text{Correction} & -0.056 \end{array} \right\} \begin{array}{lr} \text{L. M. T. of transit} & 15\ 28\ 05.007 \\ \text{Corr.} & -\ .056 \\ \hline \text{Correct L. M. T.} & 15\ 28\ 04.951 \end{array} \text{ or Jan. 2, } 3\ 28\ 04.951 \text{ a. m.}$$

NOTE.—Data for approximate L. M. T. of transit from Nautical Almanac.
Data for correct L. M. T. of transit from Ephemeris.

of observing the body's altitude when on the meridian. It must not be forgotten that the ship's clock was regulated to apparent time at noon, and that the navigator must learn his watch error on the local time of the meridian over which he is to observe a transit. If his watch was correct at noon, it will be too fast on the local time of a meridian to the westward, too slow on the local time of a meridian to the eastward of the noon meridian by four minutes of time for each degree difference of longitude. It would be well, however, for the navigator to carry a watch regulated to G. M. T., and, having found the Greenwich mean time corresponding to the required transit, to observe by that watch.

Ex. 147.—What stars of a magnitude greater than the second magnitude crossed the upper branch of the meridian of Annapolis, Md., above the visible horizon, between the hours of 8 p. m. and 12 midnight of 75th meridian West longitude, mean time, January 18, 1918?

	h m s	h m s
75th meridian mean time	**8 00 00**	**12 00 00**
Long. of 75th meridian W	**5 00 00**	**5 00 00**
G. M. T. Jan. 18,	**13 00 00**	**17 00 00**
Longitude of Annapolis West	**— 5 05 56.5**	**— 5 05 56.5**
Local astronomical mean times	**7 54 03.5**	**11 54 03.5**
R. A. M. ⊙ Jan. 18 at G. M. noon	**19 48 19.9**	**19 48 19.9**
Correction for G. M. T.	**+ 2 08.1**	**+ 2 47.6**
L. S. T.'s = limits of R. A.'s	**3 44 31.5**	**7 45 11.0**

All stars of a greater magnitude than the second whose right ascensions fall between the above limits and whose South declination is $<51° 01' 07''$ S. (Considering only those stars which appear in the apparent place table of the Nautical Almanac.)

α Tauri.
α Aurigæ.
β Orionis.
ε Orionis.
α Orionis.
α Canis Majoris.
ε Canis Majoris.
α Canis Minoris.
β Geminorum.

CHAPTER XV.

CORRECTIONS TO AN OBSERVED ALTITUDE.

202. The observed altitude of a heavenly body above the sea horizon, at a given place, is the altitude of the body as indicated by the reading of the sextant with which the observation was made, after correction for the index error (I. C.) previously explained.

The true altitude of the body, at the given place, is the altitude of its center observed above the celestial horizon, the eye of the observer supposed to be at the center of the earth. This point is selected as the common point to which to refer observations made at the surface, when combining them with the tabulated elements from the Nautical Almanac, in the solution of the astronomical triangle.

To reduce an observed altitude of a heavenly body to a true altitude it is necessary to apply the following corrections: Dip, refraction, parallax, semi-diameter. Theoretically they should be applied in the above order; following that order would give:

after applying dip,
(1) the apparent altitude of the limb;
after applying refraction and parallax,
(2) the true altitude of the limb;
after applying semi-diameter,
(3) the true altitude of the center.

When an artificial horizon is used, the observed and apparent altitudes are the same; in other words, there is no correction for the dip. As already explained under the head of artificial horizon, when a body is observed, the artificial horizon being used, the reading of the sextant is first corrected for I. C., and the corrected reading divided by 2 to get what is known as the observed altitude.

In case of fixed stars, owing to their great distances, the semi-diameter and parallax are inappreciable, so that the only corrections to be applied are I. C., dip, and refraction.

In case of planets, for sea observations, parallax and semi-diameter may be disregarded; however, if the observation is made with a telescope so powerful that the limb can be distinguished, the semi-diameter should be applied.

For refined observations ashore, both these corrections should be applied.

For the ordinary sea observations of a planet, it will be sufficient to correct the altitude for I. C., dip, and refraction.

In ordinary nautical practice, it is unnecessary to follow the theoretical order, except that in the case of the moon, it is essential to find, first, the apparent altitude of the moon's center, and for this to take out the correction for parallax and refraction combined. (Bowditch, Table 24.)

The combined corrections to be applied to the observed altitude of the sun's lower limb may be taken directly from Table 46, Bowditch, or II in the back of this book; to the observed altitude of a star, from Table 46, Bowditch, or III in the back of this book; to the observed altitude of the moon's lower or upper limb, from Table 49, Bowditch. In the case of the sun and star corrections, the arguments for entering the tables are "height of eye" and "observed altitude" with a correction for "semi-diameter" for the sun; in the case of the moon, "horizontal parallax" and "observed altitude" with a correction for "height of eye." These tables are sufficiently accurate for ordinary navigation.

203. Refraction.—It is a fundamental law of optics that a ray of light, when passing obliquely from one medium into another of different density, is bent towards, or from, a normal to the separating surface at the point of entrance, according as it passes from a lighter into a denser medium, or the reverse.

The ray before entering the second medium is called the incident ray, after entering it the refracted ray. The incident ray makes with the normal what is called the angle of incidence, the refracted ray makes with the normal the angle of refraction, and the difference between these two angles is called the refraction.

Astronomical refraction.—A ray of light from a heavenly body must pass through the atmosphere before reaching the observer.

The earth's atmosphere may be considered as formed of concentric spherical strata, that nearest the surface of the earth being of greatest density, and each succeeding stratum decreasing in density as its distance from the surface increases till the upper limit of the atmosphere is reached at a height of perhaps 50 miles from the surface.

If the space between the upper limit of the atmosphere and a star be regarded as a vacuum, or filled with a medium which exerts no sensible effect on the direction of a ray of light, its path will be a straight line till it meets the upper limit of the atmosphere.

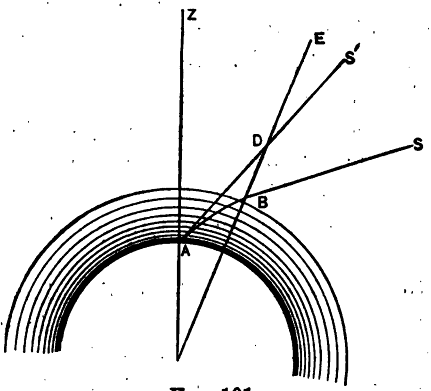


FIG. 101.

At this upper limit, the effect of refraction is very small, but, as the ray continuously passes through the atmosphere whose density increases by insensible degrees from stratum to stratum of infinitely small thickness, its path is a curve concave to the surface of the earth; the plane of its path being in the plane of the normals which meet at the center of the earth.

The last direction of a ray, or that at which it enters the eye of the observer, is in a tangent to the curve at this point and indicates the direction in which a body appears to the observer; so it is apparent that the effect of the bending of the rays is to apparently increase the altitude of the body without altering its azimuth. Astronomical refraction, then, which is the difference of direction between the ray that enters the eye of an observer and of the same ray before entering the atmosphere, making as it does an altitude appear greater than it really is, must be subtracted from an observed altitude of a body.

The ray from a star *S*, entering the atmosphere at *B* (Fig. 101) is bent into the curve *BA*. The observer at *A* apparently sees *S* in direction *AS'*. The angle *EBS* is the angle of incidence; *ZAS'*, the angle of refraction; and the ratio of their sines is a constant at a given place for a given atmospheric condition. Refraction equals *EBS* — *EDS'*, because the angle between *AS'* and *BS* is equal to the difference of the angles that these lines make with any straight line cutting both.

The refraction for a mean state of the atmosphere, that of a height of barometer of 30 inches and temperature of 50° F., can be found in Table 20A, Bowditch.

A rise of temperature, or a fall of barometer, indicates a decrease of density of the atmosphere and hence a diminution of refraction. A fall of temperature, or a rise of barometer, would indicate the reverse (see Tables 21 and 22, Bowditch). In Table 20B, Bowditch, will be found, in the case of the sun only, the value of combined parallax and refraction.

Refraction is zero when the body is in the zenith, about 36′ when it is in the horizon, and for intermediate altitudes may be said to vary as the tangent of the zenith distance of the body, provided the zenith distance does not exceed 80°.

Owing to the irregularity of refraction at low altitudes it is advisable not to observe, at sea, altitudes of less than 10°.

The oval form of the sun and moon after rising and before setting is due to the difference of refraction for the altitudes of the lower and upper limbs.

Refraction affects the dip, decreasing it by about $\frac{1}{18}$th of the whole.

204. Parallax.—In general, parallax may be defined as a change in direction of an object due to a change of the point of view. In astronomical observations, the observer is on the surface of the earth, and it is desired to reduce observations to what they would be if the observer were at the center of the earth. It is by the application of parallax that observations are so reduced.

Geocentric parallax is the angle at the body subtended by that radius of the earth which passes through the observer's position at the surface. When the heavenly body is in the horizon at *H* (Fig. 102), this angle has its greatest value and *CAH* is a right angle. Letting this angle, called the **horizontal parallax,** be represented by *P*, the earth's equatorial radius by *R*, the distance of the body by *d*, we have

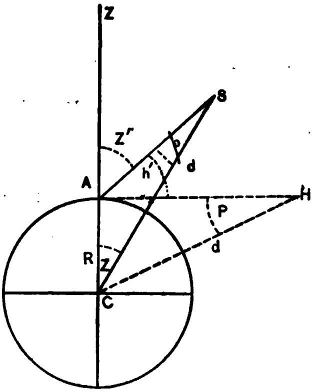


FIG. 102.

$$\sin P = \frac{R}{d}\,.$$

The value of *P* from this formula is given in the Nautical Almanac for the sun, moon, and planets.

Parallax in altitude.—When a heavenly body is observed in any position other than in the horizon, the parallax to be applied is known as parallax in altitude.

In triangle CAS, p is the parallax in altitude, ZAS the apparent zenith distance $= z' = 90° - h'$,

ZCS the true zenith distance of body $= z$,
d the distance of the body, and we have

$$\frac{\sin p}{\sin z'} = \frac{R}{d} = \sin P,$$

$$\sin p = \sin P \cos h'.$$

Since p and P are small angles, they are proportional to their sines; therefore,

$$p = P \cos h'. \tag{143}$$

Parallax is additive to the observed altitude.

205. Dip of the horizon.—The visible sea horizon is the small circle where tangents from the observer's eye meet the sea. The sensible and celestial horizons have already been defined (Art. 138).

The dip of the horizon is the angular depression of the visible below the celestial horizon, and is due to the elevation of the observer's eye above the surface.

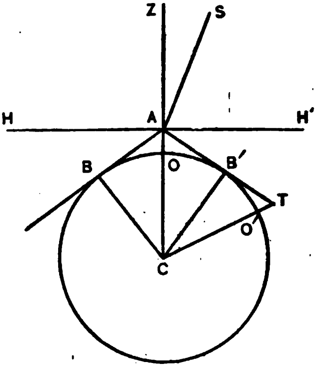


FIG. 103.

In Fig. 103, let BB' be a portion of the earth's surface, C the center, CO a radius prolonged to A, the eye of an observer; CB and CB', radii making angles of 90° respectively with AB and AB', tangents to the surface. If this figure be revolved about AC, HH' will generate a plane parallel to the celestial horizon and either AB or AB' will generate a cone tangent to the earth at the visible horizon.

Letting R be the earth's radius in feet, h the height of observer's eye in feet, and D the dip, since $D = HAB = ACB$, we have

$$\cos D = \frac{R}{R+h}; \text{ but } \cos D = 1 - 2\sin^2 \tfrac{1}{2} D,$$

therefore, $\sin^2 \frac{1}{2} D = \frac{h}{2(R+h)};$ and $\sin \frac{1}{2} D = \sqrt{\frac{h}{2(R+h)}}.$

As D is small, $\sin^2 \frac{1}{2} D'' = \left(\frac{D''}{2}\right)^2 \sin^2 1''$, and as h is very small in comparison with R, justifying the assumption that $R + h$ is sensibly equal to R, we have

$$D'' = \frac{1}{\sin 1''}\sqrt{\frac{2h}{R}}.$$

The value of the mean radius in feet being 20,902,433 feet,

$$D'' = 63''.803\sqrt{h}, \qquad (144)$$

$$D' = 1'.063\sqrt{h}. \qquad (145)$$

However, the value of the dip, as found above, is affected by refraction which raises the visible horizon, increases the distance at which an object in the horizon can be seen, and lessens the dip, so that when the effect of refraction is to be considered a change must be made in the formula.

For a mean state of the atmosphere, barometer 30 inches, thermometer 50° F., it has been computed that the value of the dip, considering refraction, is given by the formula,

$$D_r'' = \frac{.9216}{\sin 1''}\sqrt{\frac{2h}{R}} = 58''.801\sqrt{h} \qquad (146)$$

$$D_r' = 0'.98\sqrt{h} \qquad (147)$$

Application of dip.—Table 14, Bowditch, gives the dip for various heights of the eye, computed so as to allow for the effect of the refraction of the atmosphere under normal conditions. Dip is one of the corrections to be applied to an observed altitude of a heavenly body to obtain the true altitude, and is subtractive from the observed altitude, as the visible horizon is below the celestial horizon.

Error of dip.—The position of the visible horizon, and hence the amount of dip, depends on the relative temperature of sea and air. The horizon is depressed below its mean position, and the dip is increased over the tabulated amount, when the sea is warmer than the air; the reverse is true, when the air is warmer than the sea.

Hence it is easily understood that tabulated dip for given conditions may be in error, and that this error will affect all altitudes observed under those conditions. The error of position thus caused may be considerable, especially in the Red Sea and in regions of the Gulf stream. For this reason, the navigator must be cautious and, as experience shows that the error decreases with the height of the observer's eye, it would be well for him to observe from elevated positions.

Chauvenet gives the following formula from which to find a correction, always subtractive to D_r'':

$$\text{Corr.} = \frac{24021'' \, (t - t_0)}{D_r''} \text{ when}$$

t is the temperature of air, and t_0 that of water, using a Fahrenheit thermometer.

206. To find the distance of the visible horizon for a given height h of observer's eye.—It has been seen that refraction reduces the angle of dip and increases the distance of the visible horizon, so that the distance of the visible horizon from an altitude h, when the dip is affected by refraction, may be considered to be the same that it would be from an altitude $h + x$, provided there was no effect of refraction. Letting d be the distance of the visible horizon from the height of eye of $h + x$ feet and as before R the radius of the earth in feet, refraction not being considered,

$$\begin{aligned} d &= \sqrt{(R + h + x)^2 - R^2} \\ &= \sqrt{R^2 + h^2 + x^2 + 2Rh + 2Rx + 2hx - R^2} \\ &= \sqrt{(h + x)^2 + 2R\,(h + x)}\,. \end{aligned}$$

Since $(h + x)^2$ is very small in comparison with $2R\ (h + x)$, let $d = \sqrt{2R\ (h + x)}$; x is a side of a triangle which, without appreciable error, may be considered as right angled, and the angle opposite x may also, without appreciable error, be taken as $D - D_r$;

$$\text{therefore, } x = d \sin\ (D - D_r),$$

$$x = \sin\ (D - D_r)\ \sqrt{2R\ (h + x)};$$

$$\text{but } D - D_r = 5''.002\ \sqrt{h},$$

$$\text{hence } x = 5.002\ \sqrt{h}\ \sqrt{2R\ (h + x)}\ \sin 1''$$

$$x^2 = 50.04R\ (h^2 + hx)\ \sin^2\ 1''$$

$$x^2 - 50.04Rhx\ \sin^2\ 1'' + (25.02)^2R^2h^2\ \sin^4\ 1''$$
$$= 50.04Rh^2\ \sin^2 1'' + (25.02)^2R^2h^2\ \sin^4 1''$$

$$x\ - 25.02Rh\ \sin^2\ 1''$$
$$= \pm\ h\ \sqrt{50.04R \sin^2 1'' + (25.02)^2R^2 \sin^4 1''}$$

$$x\ = 25.02Rh\ \sin^2\ 1''$$
$$\pm\ h\ \sqrt{50.04R \sin^2 1'' + (25.02)^2R^2 \sin^4 1''}$$

Whence, since $R = 20902433$ feet,

$$25.02R\ \sin^2\ 1'' = .01229.$$

$$\left.\begin{array}{r} 50.04R\ \sin^2\ 1'' = .02458 \\ (25.02)^2R^2\ \sin^4\ 1'' = .00015 \end{array}\right\} \text{ and } \sqrt{.02473} = .15726.$$

$x = .16955h$, and $d = \sqrt{2R(h + x)} = \sqrt{2.3391Rh}$ in feet.

$$d \text{ (in nautical miles)} = \sqrt{\frac{2.3391 \times 20902433\,h}{(6080.27)^2}}$$
$$= \sqrt{1.3225h} = 1.15\sqrt{h}.$$

$$\left.\begin{array}{ll} d = 1.15\sqrt{h}, & d \text{ in nautical miles} \\ d = 1.324\sqrt{h}, & d \text{ in statute miles} \end{array}\right\} h \text{ in feet.} \qquad \begin{array}{r} (148) \\ (149) \end{array}$$

207. Range of visibility at sea.—If an observer whose eye is at A, height h feet, sees in his horizon at T (Fig. 103) a light or object of known height h', then since

$$AB' = 1.15\sqrt{h} \text{ and}$$
$$B'T = 1.15\sqrt{h'},$$

the distance of the light in nautical miles will be

$$d = AB' + B'T = 1.15 (\sqrt{h} + \sqrt{h'}),$$

and in statute miles,

$$d = 1.324 (\sqrt{h} + \sqrt{h'}).$$

Table 6, Bowditch, gives the distance of visibility of objects at sea in both nautical and statute miles for a given height of eye. Entering this table with heights of observer and object, respectively, the sum of the corresponding distances will be the distance of the object from the observer.

Ex. 148.—A light 121 feet above the level of the sea is just visible from a bridge of a steamer 49 feet above the water. Required the distance of the light in nautical miles.

$$d = 1.15 (\sqrt{121} + \sqrt{49}) = 1.15 (11 + 7)$$
$$= 1.15 \times 18 = 20.7 \text{ miles.}$$

208. To find the dip or depression of a point nearer than the horizon, as of a land horizon.—In Fig. 104, x represents the height of an observer, having in sight a shore horizon B, which corresponds to the visible sea horizon of a height y on the perpendicular through the eye of the observer, and d equals the known distance of B. But by a previous article $d = 1.15\sqrt{y}$ (in nautical miles), therefore $\sqrt{y} = \frac{d}{1.15}$, but the dip at height y after correction for refraction is D_r'' $= 58''.801\sqrt{y} = \frac{58''.801d}{1.15} = 51''.13d.$

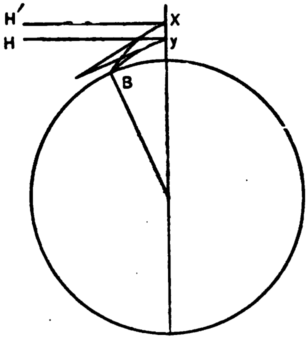


FIG. 104.

The refracted rays Bx and By make with each other a small angle which represents, without appreciable error, the difference of dip, or depression of *B*, as seen from the heights x and y; letting ϕ'' represent this angle in seconds of arc, since Bxy is nearly right angled at y, we have

$$\text{Tan } \phi'' = \frac{x - y}{6080.27d} \text{ and, as } \phi'' \text{ is a small angle,}$$

$$\psi'' = \frac{x - y}{6080.27d \tan 1''} = \frac{x - \left(\frac{d}{1.15}\right)^2}{6080.27d \tan 1''}$$

$$= \frac{x}{6080.27d \tan 1''} - \frac{d}{(1.15)^2 \, 6080.27 \tan 1''}$$

$$\psi'' = 33''.924 \frac{x}{d} - 25''.651d.$$

Remembering what ϕ'' is, and knowing that the dip at height y equals $D_r'' = 51''.13d$, to find the dip of B at height x, represented by D_{rx}'', we have only to add ϕ'' to D_r'' at height y.

$$\text{Therefore } D_{rx}'' = 25''.479d + 33''.924 \frac{x}{d}.$$

For practical purposes it is only necessary to use the formula to the nearest tenth,

or

$$D_{rx}'' = 25''.5d + 33''.9 \frac{x}{d} \left\{ \begin{array}{l} x \text{ in feet,} \\ d \text{ in nautical miles,} \\ D_{rx}'' \text{ dip in seconds of arc,} \\ \quad \text{corrected for refraction.} \end{array} \right\} \quad (150)$$

Shore horizon.—When sailing near shore, or when in a harbor at anchor, an observer may be forced to use an altitude from a shore horizon. The dip may be calculated by the above formula, or taken out of Table 15, Bowditch.

209. Apparent semi-diameter.—The apparent semi-diameter of a body is the angle subtended by its radius at the place of the observer, and for the same body varies with the distance of that body from the observer. The value given in the Nautical Almanac is the angle at the center of the earth subtended by the radius of the body.

At sea, in sextant observations of the sun and moon, the upper or lower limb is brought into contact with the sea horizon; in observations on shore, when using an artificial horizon, the opposite limbs of direct and reflected images are made tangent to each other.

Since the altitude of the center of the body is required, the angular semi-diameter of the heavenly body observed must be applied, plus or minus, according as the altitude observed was that of the lower or upper limb; and as the observation of the limb is reduced, by the application of parallax, to what it would be if taken at the center of the earth, it is necessary to find the apparent semi-diameter of a heavenly body as it would be seen at that point.

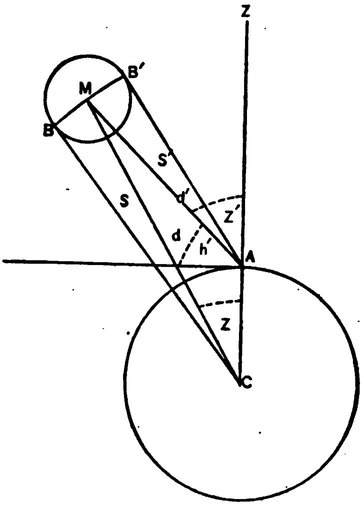


FIG. 105.

In Fig. 105, let M be the body,

h' its apparent altitude,

h its true altitude, or $90° - ZCM$.

z' its apparent zenith distance,

z its true zenith distance,

d' its distance from A,

d its distance from the center of the earth,

$S = MCB$, apparent S. D., as viewed from C, the center of the earth,

$S' = MAB'$, apparent S. D., as viewed from A, the observer's position on the surface,

$R = CA =$ earth's radius,
$r = MB = MB' =$ linear radius of body.

From the right-angled triangle MCB, $\sin S = \frac{r}{d}$.

When the body M is in the horizon of A, AM and CM are sensibly equal and, hence, the angle S is called the horizontal semi-diameter.

It has been shown that $\sin P = \frac{R}{d}$ or $d = \frac{R}{\sin P}$.

Therefore, $\sin S = \frac{r}{R} \sin P$.

Since S and P are small, they are proportional to their sines, hence $S = \frac{r}{R} P$.

$\frac{r}{R}$ is a ratio, constant for any particular body, and, representing it by C, we have

$$\log S = \log C + \log P. \tag{151}$$

For the moon, $\frac{r}{R} = .272$, so that having the moon's horizontal parallax, its semi-diameter may be gotten by multiplying it by .272; however, it is just as easy to take it out from the Almanac for the given Greenwich mean time.

The Nautical Almanac gives the semi-diameter, also the horizontal parallax, of the sun, moon, and planets.

To find the apparent semi-diameter as viewed from the observer's position on the surface:

From the right-angled triangle $AB'M$ (Fig. 105) $\sin S' = \frac{r}{d'}$, also, from AMC, $\frac{d'}{d} = \frac{\sin z}{\sin z'} = \frac{\cos h}{\cos h'}$, and $d' = \frac{d \cos h}{\cos h'}$.

Substituting value of d' in expression for $\sin S'$,

we have, $\sin S' = \frac{r}{d} \frac{\cos h'}{\cos h}$; but $\frac{r}{d} = \sin S$,

therefore, $\sin S' = \sin S \frac{\cos h'}{\cos h}$.

Now S' and S are small angles and proportional to their sines; therefore

$$S' = S\,\frac{\cos h'}{\cos h} \tag{152}$$

From this formula S' may be found when S, h', and h are known. As h is greater than h', $\cos h$ is less than $\cos h'$; therefore S' is greater than S, or the semi-diameter increases with the altitude of the body. This excess is called the augmentation, but is of appreciable value only in the case of the moon, for which body it is tabulated in Table 18, Bowditch.

210. To find the augmentation of the moon's semi-diameter.

Let ΔS be the augmentation,

$$\Delta S = S' - S = S\,\frac{\cos h'}{\cos h} - S = S\,\frac{(\cos h' - \cos h)}{\cos h},$$

$$\begin{aligned}\cos h' - \cos h &= -2 \sin \tfrac{1}{2}\,(h' - h)\,\sin \tfrac{1}{2}\,(h' + h)\\ &= 2 \sin \tfrac{1}{2}\,(h - h')\,\sin \tfrac{1}{2}\,(h' + h)\end{aligned}$$

$$\Delta S = \frac{S[2 \sin \tfrac{1}{2}\,(h - h')\sin \tfrac{1}{2}\,(h' + h)]}{\cos h}$$

Now, since $h - h' = p =$ parallax in altitude and is very small, $2 \sin \frac{1}{2}(h - h') = 2 \sin \frac{p}{2} = p \sin 1'' = P \cos h' \sin 1''$.

As ΔS is small, $\frac{1}{2}\,(h + h')$ may be taken as h' and $\cos h'$ may be substituted for $\cos h$;

therefore $\Delta S = S\,\dfrac{(P \cos h' \sin 1'')\,\sin h'}{\cos h'}$

$$\Delta S = SP \sin h' \sin 1'' = \frac{R}{r}\,S^2 \sin h' \sin 1'';$$

but $\frac{R}{r} \sin 1''$ is a constant for any one body, and may be represented by K,

$$\text{therefore, } \Delta S = KS^2 \sin h'.$$

In the case of the moon, $\frac{R}{r} = 3.6646$ and $\log K = 5.2496$.

A more rigorous formula may be found in Chauvenet's Astronomy, but the above will not involve an error greater than $\frac{8}{10}''$.

211. The following symbols may be used:

Inst. $\underline{\odot}$ (or $\underline{\odot}$) Instrumental altitude of sun's lower limb.
Obs. $\underline{\odot}$ The above corrected for I. C.
Inst. $\overline{\odot}$ Instrumental altitude of sun's upper limb.
Obs. $\overline{\odot}$ The above corrected for I. C.
$-\!\ominus\!-$ True altitude of sun's center.
2 $\underline{\odot}$ Twice the altitude of sun's lower limb.
☾̅ Altitude of moon's upper limb.
☾̲ Altitude of moon's lower limb.
-☾- Altitude of moon's center.
Alt. ✱ Altitude of star.

212. Theoretical and practical methods.—The various corrections to an instrumental altitude are applied as indicated below in the examples given. Sometimes the theoretical and practical methods may give slightly differing results, which is a matter of no importance at sea. Bowditch's Tables are used. Tables 46 and 49, Bowditch, or Tables II and III in the back of this book can be used in the examples given as illustrations.

Ex. 149.—January 3, a. m., 1918, the instrumental altitude of the sun's lower limb was 23° 42′ 00″ I. C. + 1′ 20″. Height of eye 45 feet. Find the true altitude of the center of the sun.

THEORETICALLY.

		° ′ ″
Instrumental $\underline{\odot}$		23 42 00
I. C.	+	1 20
Observed $\underline{\odot}$		23 43 20
Dip (Tab. XIV)	−	6 36
Apparent $\underline{\odot}$		23 36 44
p. & R. (Tab. XXB)	−	2 04
True $\underline{\odot}$		23 34 40
S. D.	+	16 18
True $-\!\ominus\!-$		23 50 58

PRACTICALLY, AT SEA.

	° ′ ″		′ ″
$\underline{\odot}$	23 42 00	S. D.	+ 16 18
Corr.	+ 8 58	I. C.	+ 1 20
		D.	− 6 36
$-\!\ominus\!-$	23 50 58	p. & R.	− 2 04
		Corr.	+ 8 58
Or (Table 46, Bowditch)			+ 7 38
I. C.			+ 1 20
Corr.			+ 8 58

Ex. 150.—January 19, 1918, the instrumental altitude of star Arcturus was 29° 22′ 10″. I. C. + 2′. Height of eye 45 feet. Find the true altitude.

THEORETICALLY.

	° ′ ″
Inst. alt. ✳	29 22 10
I. C.	+ 2 00
Observed alt. ✳	29 24 10
Dip (Tab. XIV)	− 6 36
Apparent alt. ✳	29 17 34
Ref.	− 1 44
True alt. ✳	29 15 50

PRACTICALLY.

	° ′ ″		′ ″
Inst. alt. ✳	29 22 10	I. C.	+ 2 00
Corr.	− 6 19	D.	− 6 36
		R.	− 1 43
True alt. ✳	29 15 51		
		Corr.	− 6 19
Or (Table 46, Bowditch)			− 8 19
I. C.			+ 2 00
Corr.			− 6 19

Ex. 151.—January 8, 1918, the altitude of the lower limb of the sun, as observed with an artificial horizon, was 34° 36′. I. C. − 2′. Required the true altitude.

THEORETICALLY.

	° ′ ″
Instrumental 2 $\underline{\odot}$	34 36 00
I. C.	− 2 00
Observed 2 $\underline{\odot}$	34 34 00
Observed $\underline{\odot}$	17 17 00
p. & R. (Tab. XXB)	− 2 57
True $\underline{\odot}$	17 14 03
S. D.	+ 16 18
True ⊖	17 30 21

PRACTICALLY.

	° ′ ″		′ ″
2 $\underline{\odot}$	34 36 00	S. D.	+ 16 18
I. C.	− 2 00	p. & R.	− 2 57
	2)34 34 00	Corr.	+ 13 21
Obs. $\underline{\odot}$	17 17 00		
Corr.	+ 13 21		
⊖	17 30 21		

To correct an instrumental altitude of the moon.—Owing to the rapid change of the moon's semi-diameter and horizontal parallax, they must be reduced for the Greenwich mean time of observation; also, as the moon is nearer the observer at all altitudes than when in the horizon, the semi-diameter must be corrected for augmentation (Table 18, Bowditch). The combined correction for parallax and refraction will be found in Table 24, Bowditch, in which the arguments are the horizontal parallax at the top and apparent altitude of the moon's center at the side, the parallax being at intervals of one minute, the apparent altitude at intervals of ten minutes of arc. For seconds of parallax, enter the table abreast the approximate correction where the arguments are tens of seconds at the side and units at the top; opposite the former and under the latter is the correction to be added. The additional correction for minutes of altitude will be found in a table on extreme right of page, to be applied as there directed. Hence the rules:

(1) For the given instant find the corresponding G. M. T. for which correct the moon's semi-diameter and horizontal parallax; also find from Table 18 the augmentation of S. D.

(2) To the instrumental altitude apply the first correction, consisting of the algebraic sum of the I. C., dip, and augmented semi-diameter. The result will be the apparent altitude of the moon's center.

(3) With the horizontal parallax and the apparent altitude find from Table 24, Bowditch, the second correction (for parallax and refraction combined) which, added to the apparent altitude of the center, will give the true altitude of the center.

Ex. 152.—January 25, 1918, in longitude 100° 30′ W., at $11^h\ 29^m\ 00^s$ local mean time, the instrumental altitude of the moon's lower limb was 60° 18′ 22″ bearing N. I. C. + 0′. Height of eye 30 feet above the sea level. Required the true altitude.

	h m		° ′ ″		′ ″	H.P.
L. M. T. Jan. 25	11 29	Instrumental alt.	☾ 60 18 22	S. D.	+15 06	
Longitude W	+ 6 42	1st correction	+ 9 57	Aug.	+ 13	
G. M. T. Jan. 25	18 11	Apparent alt.	☾ 60 28 19	I. C.	+ 0 00	′ ″
	$18^h\ 11^m$	Par. & Ref. (Tab. 24)	+ 26 43	Dip.	− 5 22	55 18
		True alt.	☾ 60 55 02	1st corr.	+ 9 57	
		Total corr. (Table 49, Bowditch)	+ 36′ 40″			

To be strictly accurate the H. P. found from the Nautical Almanac should be further corrected for the latitude of the place of observation (Table 19, Bowditch). However, it is usually disregarded, as in the above example.

Correction of a planet's altitude.—Theoretically a planet's altitude should be corrected for I. C., dip, refraction, parallax, and S. D.; practically, at sea, only the first three are applied.

The parallax is gotten from Table 17, Bowditch, the arguments being altitude and horizontal parallax.

Ex. 153.—April 3, 1918, observed the lower limb of planet Mars 32° 15′. I. C. + 1′. Height of eye 20 feet. Required the true altitude.

THEORETICALLY.
(Using Data from Ephemeris.)

		° ′ ″
Instrumental alt. of Mars		32 15 00
Index correction	+	1 00
Obs. alt. of lower limb		32 16 00
Dip	−	4 23
App. alt. of lower limb		32 11 37
H. P. 12″.8:		
Par. + 11″	−	1 21
Ref. − 1′ 32″		
True alt. of lower limb		32 10 16
Semi-diameter	+	7
True alt. of center		32 10 23

PRACTICALLY, AS AT SEA.

		° ′ ″		′ ″
♂		32 15 00	I. C.	+ 1 00
Corr.	−	4 55	Dip	− 4 23
			Ref.	− 1 32
♂		32 10 05		
			Corr.	− 4 55
Or (Table 46, Bowditch)				− 5 55
I. C.				+ 1 00
Corr.				− 4 55

The only difference between the use of planets and fixed stars for navigational purposes at sea lies in the fact that the R. A. and declination of a planet must be corrected for G. M. T., the corrections of the altitudes being practically the same, I. C., dip, and refraction.

213. Correction of altitude for run.—A ship at sea seldom remains stationary between observations; as the ship moves, the zenith of the observer describes an arc on the celestial sphere, and the number of minutes in the arc is equal to the number of nautical miles run by the ship. A heavenly body, if observed simultaneously from the two ends of the ship's run, would have the two zenith distances and hence the two altitudes differing by an amount called "the correction for the run." This must be found whenever it is desired to reduce an observed altitude for a change of the observer's position.

Now suppose an observer whose zenith is Z, observes a heavenly body M; after running a certain distance ZZ' (Z' being in one of several different positions $Z'_1, Z'_2 \ldots Z'_5$), he again observes the same body. To compare the two observations, one must be reduced to what it would have been if observed at the other place.

Let M be the position of the heavenly body, at the first observation, supposed fixed during the run, or as the observer's zenith shifts from Z to Z'; ZZ', the distance sailed in sea miles; $C = NZZ'$, the course sailed estimated from the elevated pole; and $Z = NZM$, the body's azimuth.

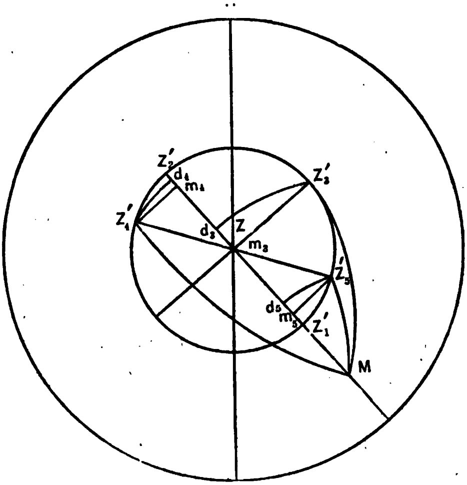


FIG. 106.

If the course is directly towards the body, Z shifts to Z'_1, and the body's zenith distance is lessened, or altitude increased by a number of minutes of arc equal to the run ZZ'_1, in sea miles. If the course is directly away from the body, Z shifts to Z'_2 and the altitude is diminished.

If the zenith shifts to any other position, by regarding the triangle $ZZ'M$ as a spherical triangle (which it is), the reduction may be obtained by a rigorous formula from Chauvenet:

$$\Delta h = d \cos (C-Z) - \tfrac{1}{2} d^2 \sin 1' \tan h \sin^2 (C-Z). \quad (153)$$

Now, with M as a center, and radius MZ', describe an arc cutting the first bearing line in d, that is from Z'_3 in d_3, from Z'_4 in d_4, from Z'_5 in d_5. Also drop perpendiculars from Z'_3, Z'_4, and Z'_5 on the first bearing line at m_3, m_4, m_5.

When the course is away from the body, the zenith going to Z'_4, the correction is subtractive to the first altitude and equals $Zm_4 + m_4d_4$ which are respectively the first and second terms in (153).

When the course is generally towards the body, Z going to Z'_5, the correction is additive to the first altitude and equals $Zm_5 - m_5d_5$, the first and second terms in (153).

When the course is at right angles to the first bearing of the body, Z going to Z'_3, the correction is $-Zd_3$, the second term of (153); the first term of (153) reducing to zero.

Since the distance ZZ' is small, the second term of (153) may be neglected, the triangles $m_4ZZ'_4$ and $m_5ZZ'_5$ may be regarded as plane triangles, and Δh as Zm_4, Zm_5, etc.

$$\left.\begin{aligned} \text{Now } Zm_4 &= ZZ'_4 \cos (C - (180^\circ - Z)), \\ \text{or, } \Delta h &= d \cos (C - Z). \\ Zm_5 &= ZZ'_5 \cos (Z - C), \\ \text{or, } \Delta h &= d \cos (C - Z). \end{aligned}\right\} \quad (154)$$

$$\begin{aligned} &\text{And } \Delta h \text{ is } + \text{ if } (C \sim Z) \text{ is } < 90^\circ, \\ &\Delta h \text{ is } - \text{ if } (C \sim Z) \text{ is } > 90, \\ &\Delta h \text{ is } 0 \text{ if } (C \sim Z) \text{ is } \ \ 90. \end{aligned}$$

Since Δh is zero when $(C \sim Z)$ equals 90°, or is smaller as $(C \sim Z)$ approaches 90°, it is better to reduce that altitude for which the difference of course and azimuth is nearer 90°.

If the second altitude is to be reduced, then C in the formula is the course reversed.

If a single course and distance are run, $(C \sim Z)$ may be by compass, but if a traverse is made from Z to Z' then the magnetic (or true) course and bearing should be used.

The traverse table may be used by taking $(C \sim Z)$, or $180 - (C \sim Z)$ if $(C \sim Z)$ is $>90°$, for the course, the run as a distance, and looking for the correction in the difference of latitude column. This, in minutes and tenths of a minute of arc, is to be added to the first altitude if $(C \sim Z)$ is $<90°$, subtracted if $(C \sim Z)$ is $>90°$.

Rules in finding correction for run.—

(1) Take a bearing of the heavenly body at each observation.

(2) For the elapsed time between observations find the course made good and distance run.

(3) For the distance in sea miles find the altitude correction Δh, in minutes and decimals of a minute of arc, to be applied as already explained.

Ex. 154.—On January 18, 1918, in N. Lat. and W. Long., a.m. time, the sextant altitude of sun's lower limb, bearing N. 137° E. (true), was 22° 18′ 20″. I. C. + 2′ 30″. Height of eye 21 feet. After running NE. (true) 33 miles, observed a p.m. altitude of sun's lower limb 21° 28′ 50″. I. C. + 2′ 10″. Height of eye 21 feet. Reduce first altitude to what it would have been had it been observed at the same time at the second place.

Course	N 45° E			
Azimuth	N 137° E			
				° ″ ′
$(C \sim Z)$	$= 92°$ ....cos	-8.54282	At 1st place	$\underline{\odot}$ 22 18 20
d	$= 33'$ .... log	1.51851	Δh	— 1 09
$\Delta h = -1'\ 09'' = -1'.15$	log	-0.06133	At 2d place	$\underline{\odot}$ 22 17 11

CHAPTER XVI.

SOLUTION OF THE ASTRONOMICAL TRIANGLE.

214. In a consideration of the astronomical triangle, or any of the parts thereof, in this and succeeding chapters, the following notation will be used:

$L =$ latitude.
$h_s =$ the sextant altitude of the heavenly body observed.
$h =$ the true altitude of the heavenly body.
$h' =$ the apparent altitude of the body.
$h_0 =$ the meridian altitude of the body.
$z =$ the true zenith distance of the body.
$z' =$ the apparent zenith distance of the body.
$z_0 =$ the meridian zenith distance of the body.
$d =$ its declination.
$p =$ its polar distance $= 90° - d$ (algebraically).
$Z =$ its azimuth measured from the elevated pole towards the East or West, from 0° to 180°.
$Z_N =$ its azimuth measured from North around to the right, from 0° to 360°.
$t =$ its hour angle.
$A =$ its amplitude.
$M =$ its position angle.

215. Many of the various problems that confront a navigator are solved by a solution of the astronomical triangle whose sides are $90° - h$, $90° - L$, and $90° - d$ or p, and whose angles are t, Z, and M.

Fig. 107 represents this triangle projected on the plane of the horizon. By spherical trigonometry, when any three of the parts are known, the others can be found. The position angle M is of no importance to the navigator, and it will not be further considered. So that for practical purposes there are given three to find the remaining two of the quantities *h, Z, t, L,* and *d.* As the latter quantity is tabulated in the Nautical Almanac, the methods of finding the other quantities will be considered in the order given above.

216. The parts of the triangle, when used in the solutions as given data, are thus found.

The latitudes and longitudes of places on shore, at which observations may be taken, are found from charts, tables of maritime positions as Appendix IV, Bowditch, or from sailing directions. At sea, they are found from previous determinations brought forward, or from subsequent determinations carried back by dead reckoning; or from practically simultaneous observations where one body is so favorably located that an error in time will not affect the resulting latitude to be used in a sight for time taken simultaneously, or where one body is near the prime vertical and an error in the latitude will not affect the resulting longitude to be used in a sight taken simultaneously for latitude.

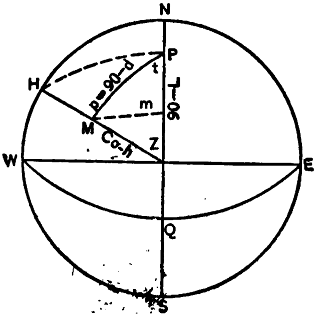


FIG. 107.

Having on board a chronometer regulated to Greenwich mean time, and having a comparison of the deck watch with the chronometer, and the watch time of observation of a given heavenly body, the G. M. T. of this observation is found and for it the body's declination is taken from the Nautical Almanac; the polar distance, which equals 90° — *d* algebraically, will be less than 90°, when the name of the declination is the same as that of the latitude; greater than 90°, or 90° + *d,* when its name is different from that of the latitude.

The instrumental altitude of the heavenly body is reduced to a true altitude (Art. 212), and if the hour angle is to be one of the given parts, it can be found by the methods of Art. 199 when the Greenwich mean time and longitude are both known.

The True Altitude.

217. To find the true altitude of a heavenly body at a given time and place, when its azimuth is not required.

Here the latitude and longitude are given; the Greenwich time is found from the local time and the longitude, and for this Greenwich time the body's declination is taken from the Nautical Almanac.

The hour angle t of the body is next found. In the case of the sun West of the meridian, the H. A. will be the local apparent time; when East of the meridian, the H. A. will be 24 hours—the local apparent time considered astronomically.

When the body is the moon, a planet, or a star, it will be necessary to find in order the L. S. T., the body's right ascension, and then its hour angle (Arts. 192 and 199).

Applying to the triangle of Fig. 107, the fundamental formula from spherical trigonometry

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

we have

$$\sin h = \cos(L \sim d) - 2 \cos L \cos d \sin^2 \tfrac{1}{2}t, \qquad (155)$$

or

$$\cos z = \cos(L \sim d) - 2 \cos L \cos d \sin^2 \tfrac{1}{2}t. \qquad (156)$$

As haversine $x = \dfrac{\text{versine } x}{2} = \dfrac{1 - \cos x}{2} = \sin^2 \tfrac{1}{2}x$ (156) will become by substitution, etc.,

$$\text{haver } z = \text{haver}(L \sim d) + \cos L \cos d \text{ haver } t.$$

$$\left.\begin{aligned} &\text{Defining } \theta \text{ by haver } \theta = \cos L \cos d \text{ haver } t, \\ &\quad \text{We have haver } z = \text{haver}(L \sim d) + \text{haver } \theta. \end{aligned}\right\} \qquad (157)$$

NOTE.—When L and d are of different names $(L \sim d)$ becomes numerically $(L+d)$ since the symbol $\sim$ means "the difference between."

Ex. 155.—Jan. 3, 1918, local mean time $8^h\ 03^m\ 59^s$ a. m., in latitude 7° 28′ 06″ N., longitude 150° 09′ 54″ W., find the sun's true altitude (using formula 155 which does not require a table of haversines).

Times and H. A.		Declination.		H. D.		Equation of Time.		H. D.	
	h m s		° ′		′		m s		
Jan. 2, L. M. T.	20 03 59	Jan. 3, at 6^h	S 22 51.3		N 0.2	At 6^h	4 29.5		$+1^s.2$
Long. West	+ 10 00 39.6	Corr. G. M. T.	0 00	G. M. T.	0.08	Corr. G. M. T.	+ .1	G. M. T. $0^h.08$	
Jan. 3, G. M. T.	6 04 38.6	Dec.	= S 22 51.3	Corr.	N 0.016	Eq. of T.	=4 29.6	Corr.	$+0^s.096$
			h m s						
L. M. T.	20 03 59	t	= 4 00 30.6	log sin $\frac{t}{2}$	9.69981			2 log sin $\frac{t}{2}$	9.39962
Eq. of T.	(−) 4 29.6	2						log	0.30103
			° ′ ″						
L. A. T.	19 59 29.4	L	7 28 6 N					log cos	9.99630
t	= (−) 4 00 30.6	d	22 51 18 S					log cos	9.96449
		$(L \sim d)$	30 19 24	Nat. cos	0.86319				
					0.45861			log	9.66144
		h	23 51 54	Nat. sin	0.40458				

Ex. 156.—Jan. 19, 1918, a. m., Lat. 43° 55′ 30″ N. and Long. 54° 56′ 21″ W., W. T. $12^h\ 43^m\ 24^s$. C—W $3^h\ 48^m\ 16^s$. Chro. fast of G. M. T. $2^m\ 25^s$. Find the true altitude of *α* Canis Minoris (Procyon) (haversine formula).

Times.				R. A. M. ⊙.				Star's R. A. and Dec.			
	h	m	s		h	m	s		h	m	s
W.	12	43	24	At G. M. N.	19	48	19.9	*'s R. A.	7	35	03.2
C—W	3	48	16	Corr. G. M. T.	+	2	42.5		°	′	″
C.	4	31	40		19	51	02.4	*'s Dec.	N 5	26	00
C. C.	−	2	25								
G. M. T. Jan. 18,	16	29	15								
R. A. M. ⊙	19	51	02.4								
G. S. T.	12	20	17.4								
λ W	3	39	45.4								
L. S. T.	8	40	32.0								
*'s R. A.	7	35	03.2								
t	+1	05	28.8								

$t = 1^h\ 05^m\ 28^s.8$ L. haver 8.30684

$L = 43°\ 55′\ 30″$ N. L. cos 9.85748

$d = 5\ 26\ 00$ N. L. cos 9.99804

θ..............L. haver 8.16236.....N. haver .01454

$L \sim d = 38\ 29\ 30$........................N. haver .10865

$z = 41\ 05\ 45$........................N. haver .12319

$h = 48\ 54\ 15$

Table 44, Bowditch, has opposite t in the p. m. column the log sin $\frac{1}{2}$ t in the *sine* column; so, if using formula (155) or (156), look for t, expressed in time, in the p. m. column, and from the *sine* column, directly abreast, take out the log sin $\frac{1}{2}$ t which, when multiplied by 2, will give log $\sin^2 \frac{1}{2} t$.

Formulæ (156) and (157) are useful in connection with the methods of "The New Navigation" (Art. 308).

Method of Time Azimuths.

218. To find both the altitude and azimuth of a certain heavenly body at a given time and place, or given t, L, and d, to find h and *Z*.

For the given time find the body's declination (Art. 185), and its hour angle (Art. 199). Then in the spherical triangle PZM (Fig. 107), the following are given:

$$PZ = 90^\circ - L,$$
$$PM = 90^\circ - d,$$
$$ZPM = t,$$

and it is required to find

$$ZM = 90^\circ - h \text{ and } PZM = Z.$$

Let fall Mm perpendicular to PZ, call Pm, ϕ; then $Zm = 90^\circ - (L + \phi)$, and by Napier's rules,

$$\left.\begin{aligned} \tan \phi &= \cot d \cos t & (1)\\ \sin h &= \sin (L + \phi) \sin d \sec \phi & (2)\\ \cot Z &= \cot t \cos (L + \phi) \operatorname{cosec} \phi & (3) \end{aligned}\right\} \qquad (158)$$

Following Chauvenet's methods, the above can be put into a more convenient form. If $\phi = 90^\circ - \phi'$, the above become

$$\left.\begin{aligned} \tan \phi' &= \tan d \sec t & (1)\\ \sin h &= \frac{\cos (\phi' - L) \sin d}{\sin \phi'} & (2)\\ \cot Z &= \frac{\cot t \sin (\phi' - L)}{\cos \phi'} & (3) \end{aligned}\right\} \qquad (159)$$

In (159), ϕ' is taken out in the same quadrant as t and is given the same sign as the declination; that is, if the declination is of the same name as the latitude, it is $+$, and ϕ' is marked +; if the declination is of a different name from the latitude, it is (—), and ϕ' is marked (—). The mere fact of t being E. or W. has no influence on the signs of the functions sec t and cot t. If t is E. or (—), the body is East of the meridian and the azimuth is marked East; if t is W. or +, the body is West of the meridian, and the azimuth is marked West; in other words, the azimuth, being restricted to 180°, is reckoned from the elevated pole (or the North point of horizon in North latitude, the South point in South latitude) towards the East or West according as the body is East or West of the meridian as indicated by the hour angle. Again, for emphasis, let it be repeated that a t's mark E. or W. does not affect the sign of its function in the above formulæ. However, the signs of functions of other quantities must be followed, and care must be exercised to do so.

When $t = 6$ hours, $\phi' = 90°$, and the formula for Z (159) becomes of an indeterminate form. However,

$$\cos t = \frac{\tan d}{\tan \phi'} \text{ and } \cot t = \frac{\tan d}{\tan \phi' \sin t}$$

and by substitution in (159)

$$\cot Z = \frac{\sin (\phi' - L) \tan d}{\sin \phi' \sin t} \tag{160}$$

which can be used when t is near 6 hours.

Formulæ (158) may be simplified in the case of Polaris (α Ursæ Minoris) because of its small polar distance.

Since $\tan \phi = \tan p \cos t$ (ϕ and p being very small),

$$\phi = p \cos t.$$

$$\sin h = \frac{\sin (L + \phi) \cos p}{\cos \phi}, \text{ or approximately } h = L + \phi.$$

$$\cot Z = \frac{\cot t \cos (L + \phi)}{\sin \phi}, \text{ or } \tan Z = \frac{\tan t \sin \phi}{\cos (L + \phi)} = \frac{\tan p \sin t \cos \phi}{\cos (L + \phi)},$$

but ϕ is so small, $\cos \phi$ is near unity; therefore,

$$Z = p \sin t \sec (L + \phi) \text{ approximately.}$$

Example 157, and Ex. 158 worked first for North latitude, then for South latitude, will illustrate the method (formulæ 159).

Ex. 157.—Jan. 3, 1918, a. m., Lat. 7° 28′ 06″ N., Long. 150° 09′ 54″ W., W. T. 8ʰ 08ᵐ 45ˢ, C—W 10ʰ 08ᵐ 15ˢ, chro. fast on G. M. T. 7ᵐ 21ˢ.4. Find the sun's altitude and azimuth.

Time.		Declination.		H. D.		Equation of Time.		H. D.	
	h m s		° ′			−to M. T.			+1ˢ.2
W.	8 08 45	At 6ʰ	S 22 51.3		N 0′.2		m s	G. M. T.	0ʰ.08
C—W	10 08 15	Corr. G. M. T.	00.0	G. M. T.	0ʰ.08	At 6ʰ	4 29.5		
						Corr. G. M. T.	+ 0.1	Corr.	0ˢ.096
C.	6 12 00		S 22 51.3	Corr.	0′.016				
							4 29.6		
C. C.	7 21.4		° ′ ″						
		d	− 22 51 18	tan	− 9.62479	sin	− 9.58928		
G. M. T. Jan. 3	6 04 38.6 =6ʰ.08	t	E 60 07 39	sec	10.30272			cot	9.75920
G. M. T.	6 04 38.6	ϕ'	− 40 14 25	tan	− 9.92751	cosec	− 10.18977	sec	+ 10.11728
Eq. of T.	4 29.6	L	+ 7 28 06						
G. A. T.	6 00 09.0	$\phi'-L(-)$	47 42 31			cos	+ 9.82795	sin	− 9.86908
λ West	10 00 39.6								
		$h=$	23 51 51			sin	+ 9.60700		
L. A. T.	19 59 29.4	$Z=$	N 119 06 04 E (Z_N=119° 06′ 04″)					cot	− 9.74556
t	h m s = − 4 00 30.6 =E 60° 07′ 39″								

Here it will be noted that N. is +, S. is (−).

Ex. 158.—April 1, 1918, p. m., in Lat. 28° 30′ N., Long. 33° 20′ W. Watch time $9^h\ 47^m\ 47^s$. C—W $2^h\ 17^m\ 09^s$. Chro. fast of G. M. T. $3^m\ 30^s$. It is required to find tne true altitude and azimuth of star (α Bootis) Arcturus.

Times.		R. A. M. ⊙.		Star's Elements.					
	h m s		h m s		h m s				
W.	9 47 47	At G. M. N.	0 36 08.4	R. A.	14 11 58.1				
C—W	2 17 09	Corr. G. M. T. (Tab. III)	1 58.5		° ′				
C.	12 04 56			Dec.	N 19 36.2				
C. C.	— 3 30		0 38 06.9						
G. M. T. April 1	12 01 26		° ′ ″						
		d	+ 19 36 12	tan	+ 9.55163	sin	+ 9.52570		
λ W	2 13 20	t	E 56 26 18	sec	10.25741			cot	9.82180
L. M. T.	9 48 06	ϕ'	+ 32 47 27	tan	9.80904	cosec	10.26634	sec	10.07539
R. A. M. ⊙	0 38 06.9	L	N 28 30						
L. S. T.	10 26 12.9	$\phi'-L+$	4 17 27			cos	9.99878	sin	8.87401
✳'s R. A.	14 11 58.1								
		h =	38 09 11			sin	9.79082		
t =	(−) 3 45 45.2								
t =	(−) 56° 26′ 18″	Z =	N 86 37 15 E (Z_N=86° 37′ 15″)					cot	8.77120

NOTE.—In the above it will be noticed that the latitude being N. and declination being N., declination is +, ϕ' is +, and that the azimuth is reckoned from the elevated pole (N. pole), and to the East as t is E. or (−).

Now work the above example for Lat. 28° 30′ S.

		° ′ ″							
d	=	− 19 36 12		tan	− 9.55163	sin	− 9.52570		
t	=	E 56 26 18		sec	10.25741			cot	9.82180
ϕ'	= (−)	32 47 27		tan	− 9.80904	cosec	− 10.26634	sec	10.07539
L	=	28 30	S						
$\phi' - L$	= (−)	61 17 27				cos	+ 9.68157	sin	− 9.94303
h	=	17 18 46				sin	+ 9.47361		
Z	=	S 124 41 25	E ($Z_N = 55° 18′ 35″$)					cot	− 9.84022

Here latitude is S. and +, declination is N. and (−); signs should be followed as indicated. The azimuth is reckoned from the elevated or S. pole, and to eastward.

The above formula for azimuth is of great importance in finding the deviation of the compass: Suppose that the navigator about $8^h\ 08^m\ 45^s$ a. m., January 3, 1918, in Lat. 7° 28′ 06″ N., Long. 150° 09′ 54″ W., had observed the bearing of the sun per compass to be $Z_N = 108\frac{1}{2}°$, ship's head 45° (p. c.), and it is required to find the deviation (see Ex. 157). Working the time azimuth we have

Sun's bearing (true)		119° 06′
Sun's bearing (p. c.)		108 30
Compass error	= +	10° 36′
Variation	= +	8
For 45° (p. c.), deviation	= +	2° 36′

Time azimuths may be obtained as above by the solution of the astronomical triangle, from azimuth tables, or by graphic methods from azimuth diagrams.

Instead of deducing the above formula for Z by Napier's rules, the third and fourth of Napier's analogies may be used. These, applied to the astronomical triangle, give

$$\left.\begin{aligned}\tan\tfrac{1}{2}(Z-M) &= \cot\tfrac{1}{2}t\sin\tfrac{1}{2}(L-d)\sec\tfrac{1}{2}(L+d)\\ \tan\tfrac{1}{2}(Z+M) &= \cot\tfrac{1}{2}t\cos\tfrac{1}{2}(L-d)\operatorname{cosec}\tfrac{1}{2}(L+d)\end{aligned}\right\}\quad(161)$$

The azimuth Z, however found, should be expressed for practical purposes in the form of Z_N which is measured from North, around to the right, from 0° to 360°.

The Altitude-Azimuth Method.

219. To find the azimuth of a heavenly body from its observed altitude at a given place.

Noting the time of observation by a watch compared with a chronometer regulated to Greenwich mean time, the G. M. T. of observation is found, and for this the declination of the body is taken from the Nautical Almanac. Knowing the latitude, and reducing the sextant altitude to a true altitude, the three sides of the astronomical triangle are known.

By spherical trigonometry,

$$\cos^2 \tfrac{1}{2} A = \frac{\sin S \sin (S-a)}{\sin b \sin c}$$

in which a, b, and c represent the three sides of the triangle, and $S = \frac{a+b+c}{2}$.

Applying this formula to the astronomical triangle PZM, (Fig. 107),

$A = Z =$ the azimuth of the heavenly body;
$a = p =$ the polar distance of body;
$b = 90° - L =$ the co. latitude;
$c = 90° - h =$ the co. altitude of the body.

$$S = \frac{a+b+c}{2} = 90° - \frac{L+h-p}{2},$$

$$S - a = 90° - \frac{L+h-p}{2} - \frac{2p}{2} = 90° - \frac{L+h+p}{2}.$$

Therefore, $$\cos^2 \tfrac{1}{2} Z = \frac{\cos\left(\frac{L+h-p}{2}\right) \cos\left(\frac{L+h+p}{2}\right)}{\cos L \cos h}.$$

Now letting $s = \frac{1}{2}(L+h+p)$,

then $\frac{1}{2}(L+h-p) = s - p$,

and $$\left.\begin{aligned} \cos \tfrac{1}{2} Z &= \sqrt{\frac{\cos s \cos (s-p)}{\cos L \cos h}} \\ &= \sqrt{\cos s \cos (s-p) \sec L \sec h}. \end{aligned}\right\} \quad (162)$$

We also have from spherical trigonometry

$$\sin^2 \tfrac{1}{2} A = \frac{\sin (S - b) \sin (S - c)}{\sin b \sin c}, \text{ where } S = \frac{a + b + c}{2}.$$

In triangle PZM (Fig. 107),

$$a = 90^\circ - d,$$
$$b = \text{co. } L,$$
$$c = 90^\circ - h.$$

$$S = \frac{a + b + c}{2} = 90^\circ - \frac{h + d - co\ I}{2},$$

$$S - b = 90^\circ - \frac{h + d - co\ L}{2} - \frac{2\, co\ L}{2} = 90^\circ - \frac{h + d + co\ L}{2},$$

$$S - c = 90^\circ - \frac{h + d - co\ L}{2} - 90^\circ + h = \frac{h + co\ L - d}{2}.$$

$$\text{Therefore, } \sin^2 \tfrac{1}{2} Z = \frac{\cos\left(\frac{h + d + co\ L}{2}\right) \sin\left(\frac{h + co\ L - d}{2}\right)}{\cos L \cos h}.$$

Now letting $s' = \frac{1}{2}(h + d + \text{co. } L)$,

$s' - d = \frac{1}{2}(h + \text{co. } L - d)$,

$$\text{and } \sin \tfrac{1}{2} Z = \sqrt{\frac{\cos s' \sin (s' - d)}{\cos L \cos h}}. \Big\} \qquad (163)$$

(162) is preferred when Z is $>90^\circ$, (163) when Z is $<90^\circ$. This problem is known as the altitude-azimuth.

Formula (162) is more convenient for use in connection with the problem of finding the hour angle of a heavenly body and is more generally used.

If the bearing of the heavenly body is observed by compass at the time of getting its altitude, or if the bearing at this time is interpolated for from previous and subsequent bearings per compass, the error of the compass can be found.

It has already been shown that compass error is the difference between the true and compass bearings of a heavenly body at the same instant, and is marked E. when the true bearing is to the right of the compass bearing, W. when the true bearing is to the left of the compass bearing.

In addition to the tables requisite for use in computing by the above formulas, navigators are also supplied with log haversines, Table 45, Bowditch (the term haversine ϕ, meaning $\frac{\text{versin}\ \phi}{2}$ or $\sin^2 \frac{1}{2}\phi$), and hence the formula for azimuth, whose deduction follows, may be conveniently used.

From trigonometry $\sin \frac{1}{2} A = \sqrt{\frac{\sin (S-b) \sin (S-c)}{\sin b \sin c}}$,

in which

$$\left.\begin{array}{l} a = p, \\ b = 90^\circ - L, \\ c = 90^\circ - h, \\ A = Z, \end{array}\right\} \begin{array}{l} S = \frac{a+b+c}{2} = 90^\circ - \frac{h+L-p}{2}, \\ S-b = 90^\circ - \frac{h+L-p}{2} - (90^\circ - L) = \frac{p-(h-L)}{2}, \\ S-c = 90^\circ - \frac{h+L-p}{2} - (90^\circ - h) = \frac{p+(h-L)}{2}. \end{array}$$

$$\text{Therefore, } \sin^2 \tfrac{1}{2} Z = \frac{\sin\left(\frac{p-(h-L)}{2}\right) \sin\left(\frac{p+(h-L)}{2}\right)}{\cos L \cos h}$$

$$\text{haversine } Z = \frac{\sqrt{\text{haver}\,(p-(h-L))}\ \sqrt{\text{haver}\,(p+(h-L))}}{\cos L \cos h}$$

$$\log \text{haver } Z = \tfrac{1}{2} \log \text{haver } (p-(h-L)) + \tfrac{1}{2} \log \text{haver } (p+(h-L)) + \log \sec L + \log \sec h. \qquad (164)$$

The solution by formula 162 is illustrated in Ex. 159 on page 445. The points to be noted in that example are the following: The declination being of a different name from the latitude, the polar distance is $>90^\circ$. The true azimuth is marked from the South point of horizon because latitude is South; and, East as the body is East of the meridian.

Amplitudes.

220. The amplitude of a heavenly body is its angular distance from the East or West point when in the true horizon, and is marked N. or S. according as it is N. or S. of that altitude of sun's lower limb 22° 38′; at the same time its center bore (p. s. c.) 89°, ship's head NE. (p. s. c.) W. T. obs. $8^h\ 10^m\ 30^s$. C—W $7^h\ 51^m\ 10^s$, chro. slow on G. M. T. $6^m\ 55^s.5$. I. C. +1′. Height of eye 24 feet. It is required to find the sun's true azimuth and then the deviation of the compass. Variation from chart 17° W.

Times.		Altitude.		Corr. for Altitude.		Declination.		H. D.	
	h m s		° ′ ″		′ ″		° ′		
W	= 8 10 30	⊙	22 38 00	S. D.	+ 16 01	At 16^h	N 5 21.6		N 1′.0
C—W	= 7 51 10	Corr.	+ 10 02	I. C.	+ 1 00	Corr.	N 0.1	G. M. T.	$0^h.14$
C. C.	= + 6 55.5			Dip	(−) 4 48				
		⦵	22 48 02	p. & R.	(−) 2 11	Dec.	N 5 21.7	Corr.	N 0′.14
G. M. T. April 3,	16 08 35.5					p	=95 21.7		
	$=16^h.14$			Corr.	+ 10 02				
				Or (Tab. 46, Bowditch)	=+ 9 01				
				I. C.	+ 1 00				
				Corr.	+10 01				

		° ′ ″		
h		22 48 02	sec	10.03533
L		29 42 30	sec	10.06120
p		95 21 42		
$2s$		147 52 14		
s		73 56 07	cos	9.44205
$s-p$	−	21 25 35	cos	9.96890
				2) 19.50748
$\frac{1}{2}Z$		55 26 40	cos	9.75374

True Bearing or $Z=$ S 110° 53′ 20″ E and $Z_N=$ 69° 06′ 40″
Bearing (p. s. c.) = 89 00 00
Compass error = (−) 19 53 20
Variation = (−) 17
Deviation = (−) 2 53 20

point. In other words, it is the complement of the azimuth when the body is in the true horizon. In Fig. 108 and Fig. 109, let PZM be a projection of the astronomical triangle on the plane of the horizon, the body M being in the horizon, and in both cases just rising. Let NZS be the celestial meridian, WQE the celestial equator, WZE the prime vertical, and $EM = A =$ amplitude $= 90° - NM = 90° - Z$. In Fig. 108, the latitude and declination are of the same name,

$$PM = 90° - d, PN = \text{Lat.}, NM = 90° - A = Z,$$

and in the triangle PNM, the angle PNM is a right angle.

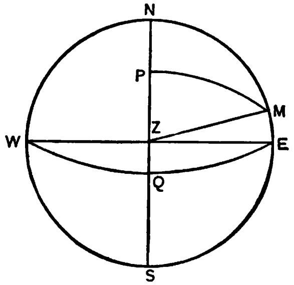


FIG. 108.

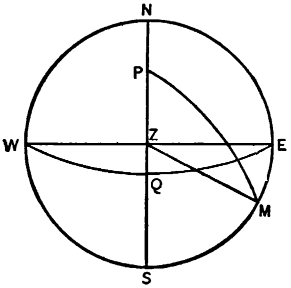


FIG. 109.

In Fig. 109 the latitude and declination are of a different name; therefore,

$$PM = 90° + d, PN = \text{Lat.}, NM = 90° + A = Z,$$

and, as before, PNM is a right angle.

Applying Napier's rules to the triangle PNM,

$$\cos PM = \cos PN \cos NM,$$
$$\cos (90° \pm d) = \cos L \cos (90° \pm A),$$
$$\sin d = \cos L \sin A,$$
$$\sin A = \sin d \sec L. \qquad (165)$$

It is evident from the two figures that a body will rise and set to the northward or southward of the prime vertical according as its declination is N. or S.

Amplitudes, computed by formula (165), are tabulated in Table 39, Bowditch, for which the arguments are declination at the top and latitude in the side column; the true amplitude is found under the former and opposite the latter.

The azimuth tables give not only the azimuth, which is the complement of the amplitude when the body is in the horizon, but also the times of rising and setting.

Time for observing an amplitude.—This problem supposes the body to be in the true horizon, that is, the true altitude of the center to be 0°. If h is a true altitude, h' an observed altitude of the center, then $h = h' - D - R + p$, or $h' = h + D + R - p$, but when the sun's center is in the true horizon $h = 0°$, $D =$ dip depending on height of eye, $R = 36' \, 29''$, $p = 9''$. Therefore, as observed, the altitude of the center will be $36' \, 20'' +$ dip above the visible horizon; hence the rule, in taking an amplitude of the sun, is to observe the bearing per compass of its center when its center is about one sun's diameter, or the lower limb a semi-diameter, above the visible horizon. Note at the same instant the ship's head (p. s. c.), angle and direction of heel, and the time by a watch compared with a chronometer regulated to G. M. T. Or, the bearing of the center in the visible horizon may be obtained by taking the mean of the bearings of the upper and lower limb of the sun when rising or setting, and by applying a correction for the vertical displacement from Table 40, Bowditch, the observed amplitude may be reduced to what it would have been, if taken when the sun's center was in the true horizon.

Stars are not often available for amplitudes, except in the cases of very bright stars or planets before setting, and then the altitude should be $36' \, 29'' +$ dip above the horizon. If observed in the visible horizon, the correction from Table 40, Bowditch, must be applied. In the cases of the sun, a star, or a planet, this correction is applied to the right at rising in North, or setting in South latitude; otherwise to the left.

The moon should not be observed for an amplitude, because when it has its center in the true horizon, the center is not visible, due to the excess of the parallax over the refraction. When the moon's center is seen in the visible horizon, or $h'=0$, the true altitude of the center is (+H. P.−refraction−dip); now, as the H. P. averages about 58′, the refraction about 36′, the dip being dependent on the height of the eye, when the moon's center is just seen, rising or setting, on the visible horizon, it is in reality 22′−dip, or about one of its semi-diameters above the true horizon; for this reason the moon should not be considered available for amplitudes.

Amplitudes of the Sun.

Ex. 160.—At sea, in Lat. 40° 20′ N., Long. 60° 15′ W., about 6^h 20^m p. m. local apparent time on April 5, 1918, the bearing per standard compass of the sun's center at the time when the center was estimated to be one diameter above the visible horizon was 280° 30′. Required the compass error.

It is first necessary to find the Greenwich time and then the declination. If the approximate time had not been given, it could have been found from sunset or azimuth tables; the latter, however, would also give the azimuth=(90°−the amplitude).

	h m s		° ′	H. D.		m s	H. D.
L. A. T. =	6 20 00	Dec. 10^h	N 6 01.6	N 0′.9	Eq. t. 10^h,	2 47.5	$-0^s.7$
λ West	4 01	Corr.	N .4	$0^h.39$	Corr.	− .2	$0^h.35$
G. A. T. =	10 21 00	Dec.	N 6 02.0	N 0′.35	Eq. t.	2 47.3	$-0^s.24$
Eq. t. +	2 47						
G. M. T.	10 23 47						
	$=10^h.39$						

By Computation.

		° ′			
L	=	40 20	N	sec	10.11788
Dec.	=	6 02	N	sin	9.02163
A	=W	7 55 31	N	sin	9.13951

		° ′ ″	
True Amplitude	=W	7 55 31	N
Amplitude (p.s.c.)	W	10 30	N
Compass error		2 34 29	W

By Inspection (Table 39, Bow.)

$\left.\begin{matrix} L=40\frac{1}{3}^\circ \text{ N} \\ d=6^\circ \quad\ \text{ N} \end{matrix}\right\}$ True Amp. = W 7°.8 N

Amp. (p. s. c.) = W 10.5 N

Compass error = 2.7 W

= 2° 42′ W

Ex. 161.—At sea, in Lat. 38° S., Long. 85° E., at sunrise (L. A. T. about 4^h 46^m a. m.), January 9, 1918, the bearing (p. s. c.) of the sun's center at the time when the center was estimated to be one diameter above the visible horizon was 110°. Required the compass error.

L. A. T. of	h m s		° ′	H. D.		m s	H. D.
sunrise	=16 46 00 Jan. 8.	Dec. 10^h	S 22 15.6	N 0′.3	Eq. t. 10^h	=6 47.3	$+1^s.1$
λ East	= 5 40	Corr.	N .4	$1^h.2$	Corr.	+ 1.2	$1^h.1$
G. A. T.	=11 06 Jan. 8.	Dec.	S 22 15.2	N 0′.36	Eq. t.	6 48.5	$+1^s.21$
Eq. t.	+ 6 48						
G. M. T.	11 12 48						
	$=11^h.21$						

By Computation.

L = 38° S....sec 10.10347
d =S 22° 15′ 12″....sin 9.57830
A =E 28 43 26 S...sin 9.68177

	° ′ ″
True Amplitude	E 28 43 26 S
Ampl. (p. s. c.)	E 20 S
Compass error	8 43 26 E

By Inspection (Tab: 39, Bow.)

		°
L 38° S d 22°.3 S	True Amp.	E 28.8 S
Amp. (p. s. c.)		E 20 S
C. E.=8° 48′ 00″ E=		8.8 E

Amplitude of a Star.

Ex. 162.—At sea, October 1, 1918, in Lat. 40° N., Long. 30° W., the bearing (p. s. c.) of star Sirius when in the visible horizon, at setting, was 268°. The star's declination was S. 16° 36′ 06″. Find the compass error.

By Computation.

L=40° N .... sec 10.11575
d=S 16° 36′ 06″............. sin 9.45593

True Amplitude W 21 53 58 S sin 9.57168
Observed Amp. W 2° S
Tab. 40 Corr. left 0.6 S

		° ′ ″
Comp. Amp.	=W 2.6 S	=W 2 36 00 S
True Amplitude		=W 21 53 58 S
Compass error		19 17 58 W

By Inspection.

		°
L 40° N d 16.6 S	True Amp.	W 21.9 S
Amp. (p. c.)		W 2.6 S
C. E.=19° 18′ 00″		W=19.3 W

Amplitude of a Planet.

Ex. 163.—On January 15, 1918, about 7^h 53^m p. m. L. M. T., in Lat. 35° N., Long. 150° 15′ E., Venus' bearing (p. s. c.)

when in the visible horizon, at setting, was 274°. Required the compass error.

	h m		° ′		
L.M.T. of bearing Jan. 15	7 53	Dec.	S 9 35.1	H. D.	N 0′.7
λ East	(−) 10 01	Corr.	S 1.5	G. M. T.	− 2^h.1
G. M. T. Jan. 14,	21 52	Dec.	S 9 36.6	Corr.	S 1′.47
or Jan. 15,	$= -2^h$.1				

By Computation.

$L = 35°$ N	sec	10.08664
$d = 9°\ 36'\ 36''$ S..............	sin	9.22256
True Amplitude W 11° 45′ 32″ S	sin	9.30920

Obs. Amp.	W 4° 00′ N	
Tab. 40 Corr. left	30	° ′ ″
Compass Amp.	= W 3° 30′ N	= W 3 30 00 N
True Amplitude		= W 11 45 32 S
Compass error		15 15 32 W

By Inspection.

$L = 35°$ N $d = 9°.6$ S	True Amp.	W 11°.8 S
Amp. (p. s. c.)		W 3°.5 N
C. E. =		15°.3 W

Azimuth Tables.

221. Besides the special tables for finding the azimuth that are contained in Hydrographic Office publication No. 200, entitled "Altitude, Azimuth, and Line of Position," the azimuth tables issued by the Navy Department are embodied in two separate publications, H. O. No. 71 and H. O. No. 120.

In both, the azimuth is given at intervals of ten minutes of hour angle, the arguments being latitude, declination, and hour angle. In No. 71, the latitude runs from 0° to 61° at intervals of 1°, the declination from 0° to 23° at intervals of 1°. No. 71 is especially adapted to the case of the sun, though applicable to the cases of all bodies of a declination less than 23° North or South. The hour angle is given in the p. m. column, 12 hours − H. A. in the a. m. column, the H. A. in case of the sun being local apparent time. When the body is in the true horizon, the hour angle is the time of sunset and 12 hours − H. A. the time of sunrise. The azimuth is also given when the body is in the true horizon.

No. 120 is intended for use with the stars, planets, and the moon. It is tabulated for latitudes from 0° to 70° and declinations from 24° to 70° (see Appendix C).

To Use the Azimuth Tables.

Take each argument to its next lower tabulated amount and find the azimuth corresponding from the tables, placing it on the first line in each column of the tabulated form following.

Now consider two of the above arguments unchanged and the third to be of the next higher denomination till each argument has been successively changed; find from the azimuth tables the azimuth corresponding to each set of arguments, placing the result in the column whose name at the top indicates the changing argument, and just below the azimuth first taken out.

Then, having the change in azimuth for an interval of one argument, find the change for the given fraction of that interval. The algebraic sum of the changes for fractions of all intervals is a correction to be applied by sign to the azimuth first taken out. Having found Z, express it in the form of Z_N.

Though this may be done mentally, the example and form below will indicate the method of solution.

Ex. 164.—In latitude 30° 30′ N. find the azimuth of a heavenly body whose declination is 21° 10′ N. and whose H. A. is $+ 4^h\ 13^m$.

Arguments.	Differences for		
	10 Min. of Hour Angle.	1° of Declination.	1° of Latitude.
Lat. 30° N Dec. 21° N H. A. + 4^h 10^m	N 83° 28′ W	N 83° 28′ W	N 83° 28′ W
	82° 23′	82° 21′	84° 08′

Change for 10^m of H. A	−65′		
" " 1° of Dec.		−67′	
" " 1° of Lat.			+40′

	′
Change, therefore, for 3^m of H. A.	− 19.5
" " " 10′ of Dec.	− 11.2
" " " 30′ of Lat.	+ 20
Corr.	— 10.7
	N 83° 28′ W

Required azimuth = N 83° 17′.3 W and Z_N = 276° 42′.7

222. In both an altitude azimuth and a time azimuth the declination may be regarded as accurately known; in the former the altitude and latitude, in the latter the time and latitude are liable to error. Therefore, it is necessary to consider the effect on the resulting azimuth of small errors in data, and the determination of the most favorable position of a heavenly body for observations for Z.

(1) **In an altitude azimuth to find the variation in Z due to a variation in h.**

Taking the fundamental trigonometric formulæ

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$
$$\sin A \sin b = \sin B \sin a,$$

and substituting $A = Z$, $B = M$, $a = 90° - d$, $b = 90° - L$, $c = 90 - h$, we have

$$\left.\begin{aligned} \sin d &= \sin L \sin h + \cos L \cos h \cos Z \\ \sin Z \cos L &= \sin M \cos d \\ \sin Z \cos h &= \sin t \cos d \end{aligned}\right\} \qquad (166)$$

By differentiation, h and Z variable,

$$\begin{aligned} 0 = \sin L \cos h dh &- \cos L \cos Z \sin h dh \\ &- \cos L \cos h \sin Z dZ \end{aligned}$$

$$\frac{dZ}{dh} = \frac{\sin L \cos h - \cos L \cos Z \sin h}{\cos L \cos h \sin Z} \qquad (167)$$

From trigonometry, $\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$. By substituting the above quantities in this formula, we have

$$\cos d \cos M = \sin L \cos h - \cos L \sin h \cos Z.$$

Substituting $\cos d \cos M$ and the value of $\cos L$ from (166) in (167), we have

$$\frac{dZ}{dh} = \frac{\cos d \cos M}{\sin M \cos d \operatorname{cosec} Z \cos h \sin Z} = \cot M \sec h. \qquad (168)$$

Also, by substituting cos d cos M and the value of cos h from (166) in (167), we have

$$\frac{dZ}{dh}=\frac{\cos d \cos M}{\cos L \sin t \cos d \operatorname{cosec} Z \sin Z}=\cos M \sec L \operatorname{cosec} t. \quad (169)$$

(168) shows $\frac{dZ}{dh}$ to be least when M is 90° and $h=0°$.

(169) shows $\frac{dZ}{dh}$ to be least when M is 90°, L is 0° and t is 6 hours.

(2) **To find the variation in Z due to a variation in L.**—Differentiating, $\sin d = \sin L \sin h + \cos L \cos h \cos Z$; regarding L and Z as variables, we have

$$0=\sin h \cos L dL - \cos h \cos Z \sin L dL$$
$$- \cos h \cos L \sin Z dZ$$

$$\frac{dZ}{dL}=\frac{\sin h \cos L - \cos h \sin L \cos Z}{\cos h \cos L \sin Z}. \quad (170)$$

By trigonometric substitution as in the previous case,

$$\cos d \cos t = \sin h \cos L - \cos h \sin L \cos Z.$$

Substituting cos d cos t and the value of sin Z in terms of M from (166) in (170), we have

$$\left.\begin{aligned}\frac{dZ}{dL}&=\frac{\cos d \cos t}{\cos h \cos L \sin M \cos d \sec L}\\ &=\frac{\cos t}{\cos h \sin M}=\cos t \sec h \operatorname{cosec} M.\end{aligned}\right\} \quad (171)$$

Substituting cos d cos t and the value of sin Z in terms of t from (166) in (171), we have

$$\frac{dZ}{dL}=\frac{\cos d \cos t}{\cos h \cos L \sin t \cos d \sec h}=\cot t \sec L. \quad (172)$$

(171) shows $\frac{dZ}{dL}$ to be least when M is 90°, h is 0°, and t is 6 hrs.

(172) shows $\frac{dZ}{dL}$ to be least when t is 6 hrs. and L is 0°.

In a Time Azimuth.

(3) To find the variation in Z due to a variation in t.— Taking the trigonometric formula $\cot A \sin C = \sin b \cot a - \cos b \cos C$, and as in the first and second cases above, substituting $A = Z$, $C = t$, $a = 90° - d$, $b = 90° - L$, we have an expression involving only those quantities used in the solution of a time azimuth, namely,

$$\cot Z \sin t = \cos L \tan d - \sin L \cos t.$$

By differentiation, Z and t regarded as variables,

$$-\sin t \operatorname{cosec}^2 Z dZ + \cot Z \cos t dt = \sin L \sin t dt,$$

$$\frac{dZ}{dt} = \frac{\cos Z \cos t - \sin L \sin t \sin Z}{\sin t \operatorname{cosec} Z}.$$

From trigonometry, $\cos B = -\cos A \cos C + \sin A \sin C \cos b$. By making the same substitutions as before, we have

$$\cos M = -\cos Z \cos t + \sin Z \sin t \sin L,$$

$$\text{or, } \cos M = -(\cos Z \cos t - \sin L \sin t \sin Z).$$

Therefore, $\frac{dZ}{dt} = \frac{-\cos M}{\operatorname{cosec} Z \sin t} = -\cos M \sin Z \operatorname{cosec} t.$ (173)

From (166), $\sin Z \operatorname{cosec} t = \cos d \sec h$;

therefore, $\frac{dZ}{dt} = -\cos M \cos d \sec h.$ (174)

(173) shows $\frac{dZ}{dt}$ to be least when M is 90° and $t = 6$ hrs.

(174) shows $\frac{dZ}{dt}$ to be least when M is 90° and $h = 0$.

Conclusions.

It is thus seen that the ideal circumstances for observations in the determination of the azimuth of a heavenly body, and hence of deviation of the compass, would be when the observer is on or near the equator, and the heavenly body is on the prime vertical in the true horizon, rising or setting, its position angle being 90°. However, in the determination of the deviation, azimuths can be taken at any time, provided the change of azimuth is not too rapid, or the altitude so great as to make important the errors arising from the want of verticality of the sight vanes of the azimuth circle or pelorus.

True Bearing of a Terrestrial Object.

223. In the survey of a harbor, it is necessary to know the azimuth of at least one of the triangulation lines, that is the true bearing of some one station from another, that other lines may be laid off in their proper directions, and a meridian line drawn upon the chart.

It may often be desirable to determine the true bearing of a distant peak or point in finding compass error.

If a terrestrial object, whose true bearing has been determined from a shore position, be observed from the same position by compass, the difference between the two bearings will give the variation of the locality.

The azimuth of a terrestrial object may be found by combining in the proper way the angle between the terrestrial object and a heavenly body with the azimuth of the same heavenly body determined at the same instant.

224. First method.—The angle between the two objects may be determined by using the azimuth circle of the standard compass or pelorus on board; but ashore, where more refined observations would be needed, it may be measured by a theodolite. This instrument is brought to bear on the heavenly body, and the time is noted or its altitude measured by a second observer simultaneously with the reading of the circle. In the absence of a second observer, the altitude of the heavenly body may be observed before and after the circle is read; and from the times noted and their corresponding altitudes, by interpolation, the altitude at the instant of reading the circle may be obtained. Turn the telescope in azimuth, bringing it to bear on the terrestrial object and read the circle again. The difference of the two readings of the circle will be the difference of azimuths of the two objects, which being applied to the true azimuth of the heavenly body found by (1) the time-azimuth method or (2) the altitude-azimuth method will give the true azimuth of the terrestrial object.

When the heavenly body has an appreciable diameter, as in the case of the sun, both limbs must be observed thus: Bring the vertical wire of the telescope tangent to one limb of the sun, the neutral glass being used on the eye piece. Note the time and reading of the circle, then quickly bring the same wire tangent to the other limb of the sun. Note the time and reading of the circle. The mean of the times and readings will be those corresponding to an observation of the center. Then turn the instrument in azimuth and read the circle when the line of sight is on the terrestrial object.

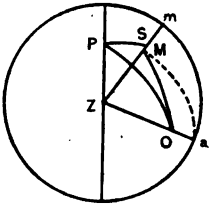


FIG. 110.

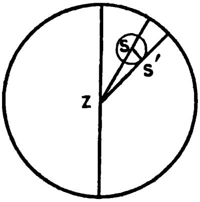


FIG. 111.

In case only one limb is observed, a correction must be applied to the reading of the circle to reduce the bearing to that of the center.

In Fig. 111, this correction is the angle SZS' where $SS' = x$, the sun's semi-diameter.

The triangle SZS' being right angled at S' and $90° - ZS$ being equal to h, the true altitude,

$$\sin SZS' = \frac{\sin SS'}{\sin ZS} = \frac{\sin SS'}{\cos h} = \frac{\sin x}{\cos h}.$$

Since the correction and semi-diameter are small,

$$\text{corr.} = x \sec h, \tag{175}$$

the sign of the correction depending on the limb observed.

225. Second method.—In this case the true azimuth of the heavenly body is found as before, the astronomical triangle being solved for Z by either the time-azimuth or the altitude-azimuth method.

Let Fig. 110 represent the bodies projected on the plane of the horizon and the triangles involved; the heavenly body's true place is S, its apparent place M. The apparent place of the terrestrial object is O, and MO is the observed angular distance of the object O from the heavenly body's center (that is the sextant reading corrected for instrumental errors, and in the case of the sun for semi-diameter).

PZS is the astronomical triangle, PZ the co-latitude, PS polar distance, ZS co-altitude, and PZS the azimuth.

If Z is not gotten from a time azimuth, it is gotten from an altitude azimuth, the altitude being the true altitude of M found by observation when arc MO is measured. Measure with a sextant the angular distance MO between the bodies. At the time of measuring the arc MO, note the time or measure the altitude of M; also measure the altitude of O. Correct the altitudes of both M and O for instrumental errors and dip, thus getting the apparent altitudes of M and O. The correction for dip is taken from Table 14, Bowditch, in case of a free horizon; from Table 15, in case of an obstructed horizon. In the latter case this correction may be computed by formula (150), Art. 208.

When the true altitude is found from a time azimuth, it is reduced to an apparent altitude by adding the refraction and subtracting the parallax.

Letting h' be the apparent altitude of M,
Q " " O.

there are given in the triangle MZO the three sides:

$$ZM = 90^\circ - h',$$
$$ZO = 90^\circ - Q,$$
$MO = D$, the corrected distance.

To find $MZO = \zeta$, the difference of azimuths.

From trigonometry,

$$\sin \tfrac{1}{2} MZO = \sqrt{\frac{\sin \frac{1}{2}(D + Q - h') \sin \frac{1}{2}(D + h' - Q)}{\cos Q \cos h'}};\quad (176)$$

and letting $\dfrac{D + Q + h'}{2} = s$, we have

$$\sin \tfrac{1}{2} \zeta = \sqrt{\left[\frac{\sin (s - h') \sin (s - Q)}{\cos Q \cos h'}\right]}; \quad (177)$$

also from trigonometry,

$$\cos \tfrac{1}{2} MZO = \sqrt{\frac{\cos \frac{1}{2}(h' + Q + D) \cos \frac{1}{2}(h' + Q - D)}{\cos h' \cos Q}} \quad (178)$$

and letting $\dfrac{D + Q + h'}{2} = s$, we have

$$\cos \tfrac{1}{2} \zeta = \sqrt{\left[\frac{\cos s \cos (s - D)}{\cos h' \cos Q}\right]} \quad (179)$$

$$\text{or } \tan \tfrac{1}{2} \zeta = \sqrt{\left[\frac{\sin (s - h') \sin (s - Q)}{\cos s \cos (s - D)}\right]} \quad (180)$$

Formula (177) is preferable when ζ is $< 90^\circ$, (179) when ζ greatly exceeds 90°.

When the body O is in the true horizon, $Q = 0^\circ$, that is the observed altitude is equal to the dip; in Fig. 110, aM is the corrected distance, the triangle a$M$$m$ is right angled at m and by Napier's rules,

$$\cos D = \cos h' \cos \mathrm{a}m,$$
$$\cos \mathrm{a}m = \cos \zeta = \cos D \sec h'. \qquad (181)$$

or in (180), if $Q = 0$,

$$\tan \tfrac{1}{2} \zeta = \sqrt{[\tan \tfrac{1}{2} (D + h') \tan \tfrac{1}{2} (D - h')]} \qquad (182)$$

If the observed object O is exactly in the water line, the apparent altitude is equal to the dip and is negative, or $Q = (-)$ dip.

The most favorable conditions for observation are when the heavenly body is on the prime vertical at a low altitude and the distance MO approximates 90°; the ideal condition being when both bodies are in the true horizon, or $\zeta = D$.

When the terrestrial object presents a vertical line to which the sun may be brought tangent, the sun's diameter through the point of contact O will not be in the direction of the distance OM, but perpendicular to the vertical circle through the terrestrial object, ZO, and a correction must be applied to the measured distance to obtain D. It is obtained from the formula

corr. $= S \sin MOZ$ where $S =$ sun's semi-diameter.

The altitude of O is very small anyhow, and by considering its altitude as zero, MOZ equals $M\mathrm{a}Z$, so that MOZ is found with sufficient accuracy from the right triangle $Mm\mathrm{a}$, taking Ma equal to the uncorrected distance, that is, the sextant arc corrected only for I. C. Letting this uncorrected distance be D', we have by Napier's rules,

$$\sin h' = \sin D' \sin M\mathrm{a}m = \sin D' \cos M\mathrm{a}Z,$$
$$\text{or, } \sin h' = \sin D' \cos MOZ,$$
$$\cos MOZ = \sin h' \operatorname{cosec} D'. \qquad (183)$$

Ex. 165.—At Annapolis, Md., April 19, 1918, at 75th meridian mean time 8ʰ 35ᵐ 46ˢ a. m., the angular distance between the sun's center and a chimney top across the Severn, chimney to left of sun, was 94° 07′ 30″ by sextant, height of chimney by sextant above a shore horizon distant one sea mile, 1° 15′. I. C.—2′. Height of eye 20 feet. Required the true azimuth of the chimney. (Use data from Ephemeris.)

Times.		Sun's Dec.	H. D.	Eq. of T.	H. D.	Altitude of Chimney.		Altitude of Sun.	
	h m s	° ′ ″		m s			° ′ ″		° ′ ″
75th mer. M. T. April 18,	20 35 46	N 10 59 10.6	N 52″.11	0 45.51	+ 0″.562	Obs. Q	1 15 00	h	35 53 25
		N 1 23.4	1ʰ.6	.90	1ʰ.6	I. C.	− 2 00	p. & R. +	1 13
Long. 75th mer.	W 5 00 00	N 11 00 34.0	N 83″.38	0 46.41	+ 0″.899	Dip	− 12 00	h′	35 54 38
(19th) G. M. T.	1 35 46					Appt. Q	1 01 00		
Long. W.	5 05 56.5								

			° ′ ″								
L. M. T.	20 29 49.5	d	N 11 00 34	tan	+ 9.28903	sin	9.28097			Ang. Dist.	° ′ ″
Eq. of T.	+ 0 46.41	t	E 52 21 01	sec	0.21408			cot	9.88733	Obs. D.	94 07 30
L. A. T.	20 30 35.91	φ′	+ 17 40 00	tan	+ 9.50311	cosec	0.51787	sec	0.02098	I. C.	− 2 00
or t=	− 3 29 24.09										
or t=	E 52° 21′ 01″.4	L	+ 38 58 53							D.	94 05 30
		φ′ − L (−)	21 18 53			cos	9.96923	sin	− 9.56049		
		h	35 53 25			sin	9.76807				
		Z	N 106 23 59 E					cot	− 9.46880		

	° ′ ″		
D	94 05 30		
Q	1 01 00	sec	10.00007
h′	35 54 38	sec	10.09155
	131 01 08		
s	65 30 34		
s − Q	64 29 34	sin	9.95546
s − h	29 35 56	sin	9.69366
		2	19.74074
½ζ	47 53 49	sin	9.87037
ζ	95 47 38		

	° ′ ″
True azimuth of sun Z_N	=106 23 59
ζ { horizontal angle sun to chimney }	− 95 47 38
Azimuth of chimney	10 36 21

Hour Angle, Local Time, and Longitude.

226. To find the hour angle of a heavenly body at a given place, and thence the local time, the altitude of the body and the Greenwich time being known.

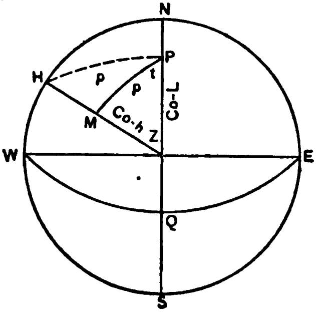


FIG. 112.

Noting the time of observing the body's altitude by a watch compared with a chronometer regulated to Greenwich mean time, the G. M. T. of observation is found, and for this the declination of the body is taken from the Nautical Almanac. Knowing the latitude and reducing the observed to a true altitude, the three sides of the astronomical triangle are known. By spherical trigonometry,

$$\sin \tfrac{1}{2} A = \sqrt{\left[\frac{\sin (S-b) \sin (S-c)}{\sin b \sin c}\right]}$$

in which a, b, and c represent the three sides of the triangle and $S = \frac{a+b+c}{2}$.

Applying this formula to the astronomical triangle PZM (Fig. 112), and

letting $A = t$, the hour angle of the heavenly body;
$a = 90° - h$, the complement of its true altitude;
$b = 90° - d = p$, its polar distance;
$c = 90° - L$, the complement of the latitude;

then

$$S = \frac{a+b+c}{2} = \frac{90° - h + p + 90° - L}{2} = 90° - \frac{h+L-p}{2},$$

$$S - b = 90° - \frac{h+L-p}{2} - p = 90° - \frac{L+h+p}{2},$$

$$S - c = 90° - \frac{h+L-p}{2} - (90° - L) = \frac{L+p-h}{2}$$

Therefore,

$$\sin \tfrac{1}{2} t = \sqrt{\left[\frac{\cos \frac{1}{2}(L+h+p) \sin \frac{1}{2}(L+p-h)}{\cos L \sin p}\right]}.$$

Now letting $s = \frac{1}{2}(L+h+p)$, then $\frac{1}{2}(L+p-h) = s-h$,

and $$\sin \tfrac{1}{2} t = \sqrt{\left[\frac{\cos s \sin (s-h)}{\cos L \sin p}\right]} \qquad (184)$$

In like manner may be deduced from an application to the triangle of the formula $\cos^2 \frac{1}{2} A = \dfrac{\sin S \sin (S-a)}{\sin b \sin c}$ the following:

$$\cos \tfrac{1}{2} t = \sqrt{\left[\frac{\sin (s-L) \cos (s-p)}{\cos L \sin p}\right]} \qquad (185)$$

where $s = \frac{1}{2}(L+h+p)$.

Since, in addition to the tables requisite for use in computing by the above formulæ, navigators are also supplied with log haversines, Table 45, Bowditch, the advantages of another formula, whose deduction follows, may be brought to notice. From trigonometry,

$$\sin^2 \tfrac{1}{2} A = \frac{\sin (S-b) \sin (S-c)}{\sin b \sin c} \text{ in which } \begin{cases} A = t, \\ a = z, \\ b = 90° - d, \\ c = 90° - L, \end{cases}$$

$$S = \frac{a+b+c}{2} = 90° - \frac{L+d-z}{2},$$

$$S - b = 90° - \frac{L+d-z}{2} - (90° - d) = \frac{d+z-L}{2} = \frac{z-(L-d)}{2},$$

$$S - c = 90° - \frac{L+d-z}{2} - (90° - L) = \frac{L+z-d}{2} = \frac{z+(L-d)}{2}.$$

Therefore, $$\sin^2 \tfrac{1}{2} t = \frac{\sin \frac{z-(L-d)}{2} \sin \frac{z+(L-d)}{2}}{\cos d \cos L}$$

Now
$\sin^2 \frac{1}{2} t =$ haversine t, $\sin \frac{z-(L-d)}{2} = \sqrt{\text{haver}\,(z-(L-d))}$,
and $\sin \frac{z+(L-d)}{2} = \sqrt{\text{haver}\,(z+(L-d))}$.

Therefore, using logs:

$$\left.\begin{aligned}\log \text{haver}\, t = {} & \tfrac{1}{2} \log \text{haver}\, (z - (L-d)) \\ & + \tfrac{1}{2} \log \text{haver}\, (z + (L-d)) \\ & + \log \sec d + \log \sec L.\end{aligned}\right\} \quad (186)$$

When t greatly exceeds 6 hours, as is often the case in high latitudes, it should be found from formula (185).

When $L = 90°$, the zenith is at the pole;

in (184) $p + h = 90°$ and $\frac{\cos \frac{1}{2}(L+p+h)}{\cos L} = \frac{\cos 90°}{\cos 90°} = \frac{0}{0}$,

in (185) $p + h - L = 0$ and $\frac{\sin (s-L)}{\cos L} = \frac{\sin 0°}{\cos 90°} = \frac{0}{0}$.

Therefore, in very high latitudes it is impracticable to find with exactness the local time as the formulæ for hour angles then approach the indeterminate form.

The formulæ also reduce to the indeterminate form when $d = 90°$, at which time the star would be at the pole and, therefore, its altitude would be the latitude; for this reason when working for time avoid stars of very large declination.

For time, bodies should be observed when on or near the prime vertical (Art. 237), and the desirability of this position increases as the latitude increases. In latitudes beyond 66° 30′ an error of 1′ in the altitude will cause an error of at least 10^s of time in the longitude.

Using formula (184), it is not necessary to take out $\frac{1}{2}$ t in arc, then multiply it by 2, and convert it into time. In Table 44, Bowditch, t may be taken directly from the p. m. column corresponding to log sin $\frac{1}{2}$ t in the sine column; and with Table 45, Bowditch, in which log haversines are supplied, the finding of the value of t is further simplified, since

$$\text{hav}\, t = \sin^2 \tfrac{1}{2} t = \frac{\cos S \sin (S-h)}{\cos L \sin p}.$$

In taking out hour angles, take them from the p. m. column, marking them + when the body is West of the meridian, (—) when the body is East of the meridian.

When the body is the sun, *t* its hour angle is local apparent time when the sun is West of the meridian; but if the sun is East of the meridian, its H. A. is (—) *t*, and the local apparent time is 24 hours — *t*. This is astronomical L. A. T. Thus, if the sun's H. A. is + 4 hours, the L. A. T. is 4 p. m.; if the sun's H. A. is (—) 4 the L. A. T. is 20 hours astronomically, or 8 a. m. civil time, that is to say, if the sun's H. A. is (—) 4 hours, the sun will not be on the meridian for 4 hours.

Having the H. A. of the sun which is L. A. T. or 24 hours — L. A. T., to obtain local mean time, the equation of time must be taken out of the Nautical Almanac for the Greenwich instant and applied with its proper sign.

Then the difference between this local mean time and the corresponding Greenwich mean time will be the longitude; West if the Greenwich time is the greater, otherwise East.

Conditions of observation.—A little further on (Art. 237), it will be shown that altitudes for time, whether of the sun, moon, a planet, or a star should be taken when the body is on or near the prime vertical, and certainly more than 45° and less than 135° in azimuth. The altitude should be sufficiently high to eliminate errors of refraction, say above 10°, and especially so when refraction may be affected by fog or mist.

It should be a rule, when observing heavenly bodies, to take several altitudes in quick succession; and the mean of 3 or 5 altitudes, thus taken and so selected that the differences of altitude vary with the differences of time, should be used in preference to a single observation. Whenever the sun is observed for time, its compass bearing should be observed for compass error and the heading of the ship per compass also carefully noted.

Such sights worked for time and longitude are known as "time sights."

227. Rules for working a time sight of the sun.—(1) *Find the Greenwich mean time and date. It is shown in the column marked "Times," in the form for work following, how the G. M. T. of observation is obtained. Applying the chronometer comparison (C—W) to the watch time of observation gives the chronometer time of observation, and if to this is applied the chronometer correction on G. M. T. when leaving port brought up to date for daily loss or gain, the result will be the G. M. T. of observation, but care must be taken to see that this time is astronomical time, and that the date is correct.*

git
gmit

(2) *Reduce the sextant altitude to the true altitude of the center, and take from the Nautical Almanac for the Greenwich mean noon of the given astronomical day the sun's declination, H. D. of declination, equation of time, H. D. of the equation of time, and correct both declination and equation of time for the G. M. T. If the declination is of the same name as the latitude, find $p = 90° - d$; if the declination is of a different name from the latitude, find $p = 90° + d$. Note whether the equation of time is + or (—) to apparent time.*

(3) *Combine h, L and p as required in the equation* (184) *and as per form illustrated in example on page* 467. *Having found the log sin $\frac{1}{2}$ t, look for it in the column of sines (Table 44, Bowditch), and take out the corresponding time from the a. m. or p. m. column according as the sight is an a. m. or p. m. sight. This quantity is the local civil apparent time. The time from the p. m. column is also astronomical time, but 12 hours must be added to the reading from the a. m. column to reduce it to astronomical time. Applying the equation of time, with the proper sign, to the local apparent time gives the local mean time of observation.* (The value of t may be picked out directly from log $\sin^2 \frac{t}{2}$ in Table 45, Bowditch.)

✓

(4) *The difference between the local and Greenwich mean times of observation is the longitude in time; West if the local time is less than the Greenwich time, otherwise East* (Art. 179).

(5) *Or, the G. A. T. may be found by applying the equation of time, with the proper sign, to the G. M. T.; then the longitude will be the difference between the G. A. T. and L. A. T., West if G. A. T. is the greater, East if the L. A. T. is the greater.*

228. Time sights of the moon, a planet, or a star.—When the hour angle determined by the formulæ of Art. 226 is that of any other heavenly body than the sun, that is, of the moon, a planet, or a star, the right ascension of the body must be taken out of the Nautical Almanac for the Greenwich instant; then the algebraic sum of the hour angle and right ascension of this body will give the local sidereal time. Having converted the G. M. T. into the corresponding G. S. T. (Art. 192), the difference between the G. S. T. and L. S. T. will be the longitude, West if the G. S. T. is the greater; East, if the L. S. T. is the greater (Art. 179).

HOUR ANGLE AND LONGITUDE BY A. M. TIME SIGHT OF THE SUN.

Ex. 166.—April 4, 1918, a. m., in Lat. 29° 42′ 30″ S., Long. by D. R. 58° 45′ East, the sextant altitude of the sun's lower limb was 22° 38′. I. C. + 1′. Height of eye 24 feet. W. T. of obs. $8^h\ 10^m\ 30^s$. C—W $7^h\ 51^m\ 10^s$. Chro. slow on G. M. T. $6^m\ 55^s.5$. Required the longitude.

Times.		Altitude.		Corrections to Altitude.		Declination.		H. D.		Eq. of T.		H. D.	
	h m s		° ′ ″		′ ″		° ′				m s		
W.	8 10 30	☉	22 38 00	S. D.	+ 16 01	At 16ʰ,	N 5 21.6		N 1′.0		3 18.4		− 0″.7
C—W	7 51 10	Corr.	+ 10 02	I. C.	+ 1 00	Corr.	N 0.1	G. M. T.	0ʰ.14	Corr.	− .1	G.M.T.	0ʰ.14
C.	4 01 40	-⊖-	22 48 02	D	(−) 4 48	Dec.	N 5 21.7	Corr.	N 0′.14		3 18.3	Corr.	−0″.098
C. C.	+ 6 55.5			p. & R.	(−) 2 11	p	=95° 21′ 42″			(+) to Appt. T.			
G.M.T. (3ᵈ)	16 08 35.5			Corr.	+ 10 02								
	16ʰ.14			Or (Tab. 46, Bowditch	= + 9′ 01″								
				I. C.	= + 1 00								
				Corr.	+10 01								

		° ′ ″		
	h	22 48 02		
	L	29 42 30	........................sec	10.06120
	p	95 21 42	........................cosec	10.00191
	$2s$	147 52 14		
	s	73 56 07	........................cos	9.44205
	h	22 48 02		
	$s-h$	51 08 05	........................sin	9.89133
				2)19.39649
		h m s		
(A. M. Col. Tab. 44) Add 12 hours to obtain ast. time.	L. A. T.	20 00 26.7	...................sin	9.69824
	Eq. of T.	+ 3 18.3		
	L. M. T.	20 03 45		
	G. M. T.	16 08 35.5		
	Long.	3 55 09.5 E		
	Arc	58° 47′ 22″.5 E		

HOUR ANGLE, LONGITUDE, AND AZIMUTH OF SUN, P. M. TIME SIGHT.

Ex. 167.—April 23, 1918, p. m., in Lat. ~~3~~25° 40′ N., Long. by account 84° 30′ W., the sextant altitude of the sun's lower limb was 25° 29′ 50″. I. C.=0. Height of eye 45 feet. W. T. of obs. 4^h 26^m 28^s. C—W 5^h 30^m $55^s.5$. Chro. slow of G. M. T. 8^m $40^s.5$. At the time of observation the sun's center bore (p. s. c.) 268°. Ship's head NE. (p. c.). Required longitude and compass error.

Times.		Altitude.		Corr. to Altitude.		Declination.		H. D.		Eq. of Time.	H. D.	
	h m s		° ′ ″		′ ″		° ′			m s		
W.	4 26 28	☉	25 29 50	S. D.	+15 56	At 10^h	N 12 29.4		N 0′.8	1 41.1		+ $0^s.5$
C—W	5 30 55.5	Corr.	7 26	Dip	− 6 36	Corr. G. M. T.	N .1	G. M. T.	$0^h.1$	Corr. 0.1	G. T.	$0^h.1$
C. C.	+ 8 40.5	⊖	25 37 16	p. & R.	− 1 54	Dec.	N 12 29.5	Corr.	N 0′.08	1 41.2	Corr.	+ $0^s.05$
G. M. T. Apr. 23,	10 06 04 $10^h.1$			Corr.	+ 7 26	p	=77° 30′ 30″			(−) to A. T.		
				Or (Tab. 46, Bowditch)	=+7′ 26″							

		° ′ ″				° ′ ″		
	h	25 37 16			h	25 37 16	sec	10.04495
	L	25 40 00	sec	10.04512	L	25 40 00	sec	10.04512
	p	77 30 30	cosec	10.01041	p	77 30 30		
	$2s$	128 47 46			$2s$	128 47 46		
	s	64 23 53	cos	9.63560	s	64 23 53	cos	9.63560
	h	25 37 16			p	77 30 30		
	$s - h$	38 46 37	sin	9.79678	$s - p$ (−)	13 06 37	cos	9.98853
		h m s	2	$\boxed{19.48791}$			2	$\boxed{19.71420}$
(P. M. Col. Tab. 44)	L. A. T.	4 29 27	sin	9.74396	½Z	43 58 39	cos	9.85710
	Eq. of T.	(−) 1 41.2				2		
	L. M. T.	4 27 45.8	True Bearing		$Z =$	N 87 57 18 W	and Z_N	=272° 02′ 42″
	G. M. T.	10 06 04	Bearing (p. c.)					=268 00 00
	Long.	5 38 18.2 W	Compass error					=+4 02 42
	Arc	84° 34′ 33″ W						

229. Rules for working a time sight of a star.—(1) *When observed for time, a star should be on or near the prime vertical to give the best results. The observation should be made when the horizon is well defined during twilight, or when the moon is shining.*

(2) *Find the correct G. M. T. and date.*

(3) *For this G. M. T. take from the Nautical Almanac the R. A. M.* ⊙ *(Art.* 185), *and convert the G. M. T. into G. S. T.*

(4) *From the N. A., find the star's R. A. and declination to the nearest second. For use in a time sight they require no correction.*

(5) *Reduce the sextant altitude to a true altitude. Substitute h, L, and p in the formula, as per form following, and having found the log sin* $\frac{1}{2}$ *t, look for it in the log sine column of Table* 44, *Bowditch, and abreast of it in the column marked "Hour p. m." will be found the star's H. A. or t in hours, minutes, and seconds of time. The exact value of t may be found by interpolation or by using the table of proportional parts at the foot of the page.*

(6) *If the star is East of the meridian, mark t* (—); *if the star is West of the meridian, mark t* +. *To the hour angle t apply the star's R. A.; the algebraic sum will be the L. S. T., the difference between which and the G. S. T. will be the longitude, West if the G. S. T. is* > *L. S. T., or East if G. S. T. is* < *L. S. T.*

HOUR ANGLE AND LONGITUDE BY OBSERVATION OF A STAR.

Ex. 168.—Jan. 26, 1918, about 6.20 p. m., in Lat. 13° 30′ N., Long. by account 187° 10′ E., the sextant altitude of the star Sirius (*a* Canis Majoris) was 24° 39′. Star East of the meridian. I. C. +1′. Height of eye, 45 feet. W. T. of observation, $6^h\ 22^m\ 30^s$. C—W $2^h\ 57^m\ 57^s$. Chronometer fast of G. M. T. $3^m\ 56^s.5$. Required the longitude.

Times.		✱'s Altitude.		Corr. to ✱'s Alt.		R. A. M. ☉.		✱'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		h m s
W.	6 22 30	✱'s h_s	24 39 00	I. C.	+ 1 00	At G. noon Jan. 25	20 15 55.8	✱'s R. A.	6 41 34.4
C—W	2 57 57	Corr.	− 7 42	Dip	− 6 36				
C.	9 20 27	✱'s h	24 31 18	Ref.	− 2 06	Corr. G. M. T.	3 29.7		° ′
C. C.	(−) 3 56.5			Corr.	− 7 42	R. A. M. ☉	20 19 25.5	✱'s Dec.	S 16 36.4
G. M. T. Jan. 25	21 16 30.5			Or (Tab. 46, Bowditch)	= −8′ 42″				
R. A. M. ☉	20 19 25.5			I. C.	= +1 00				
G. S. T.	17 35 56			Corr.	= −7 42				

° ′ ″

h = 24 31 18

L = 13 30 00 .... sec 10.01217

p = 106 36 24 .... cosec 10.01851

$2s$ = 144 37 42

s = 72 18 51 .... cos 9.48258

h = 24 31 18

$s - h$ = 47 47 33 .... sin 9.86965

2 | 19.38291*

sin 9.69146

h m s

(P. M. Col. Tab. 44) ✱'s t = (−) 3 55 28.7

✱'s R. A. 6 41 34.4

L. S. T. 2 46 05.7

G. S. T. 17 35 56

Long. 9 10 09.7 E

Arc 187° 32′ 25″.5 E

Notice that in the above example the G. M. T. is converted into G. S. T. (Art. 192). The star being East of meridian, mark its H. A. minus; then the L. S. T. equals the algebraic sum of *t* and R. A. of star for the same instant (Art. 173). Longitude = difference of G. S. T. and L. S. T.

* This is the log haversine of *t*, and, therefore, by entering Table 45, Bowditch, with 9.38291, the value of *t* may be found to be $3^h\ 55^m\ 28^s.8$.

230. Rules for working a time sight of the moon.—(1) *Proceed exactly as with a time sight of a star with differences noted below. The moon's right ascension and declination are tabulated for each day and each two hours of G. M. T. with differences for two hours, and are taken out and corrected accordingly. The moon's S. D. and H. P. are similarly tabulated in conjunction with the right ascension and declination in the Nautical Almanac. The augmentation of the moon's S. D. is taken from Table 18, Bowditch, and is + or (−), depending on whether the observed limb was the lower or upper limb of the moon. In refined observations a reduction from Table 19, Bowditch, is applied to the H. P. Since the radius from the center of the earth to an observer in high latitudes is less than the equatorial radius, and the tabulated H. P. is for an observer on the equator, the H. P. should be reduced for the latitude, the reduction being subtractive. In work at sea this reduction is usually omitted.*

(2) *Reduce the sextant altitude of the moon's limb to the apparent altitude of the moon's center by applying the first correction, which consists of (±augmented S. D.±I. C.−Dip). Then with this apparent altitude of the moon's center and the H. P. find from Table 24, Bowditch, the second correction consisting of parallax and refraction. Add this to the apparent altitude and the result will be the true altitude of the moon's center* (see Ex. 152, Art. 212). *To abridge the work at sea, these various corrections, excepting, of course, the index correction, have been consolidated in Table 49, Bowditch.*

(3) *Then proceed as in a time sight of a star.*

HOUR ANGLE AND LONGITUDE BY OBSERVATION OF THE MOON.

Ex. 169.—April 26, 1918, p. m., in Lat. 35° 00′ S., Long. by account 2° 52′ West, the sextant altitude of the moon's lower limb, East of meridian near P. V., was 30° 14′ 30″. I. C. −2′. Height of eye 45 feet. W. T. of obs. $8^h\ 04^m\ 05^s$. C—W $0^h\ 14^m\ 05^s$. Chro. fast of G. M. T. $9^m\ 10^s$. Required the longitude.

Times.		Moon's Alt.		Corr. to Alt.		R. A. M. ☉.		Moon's Dec. and R. A.		H. D.
	h m s		° ′ ″		′ ″		h m s		° ′	
W.	8 04 05	☾	30 14 30	Aug.	+ 0 07	G. M. N.	2 14 42.2	Dec. at 8 hrs.	S 18 28.3	S 7′.6
C—W	0 14 05	1st corr.	+ 6 25	S. D.	+ 14 54	Corr. G. M. T.	1 20.3	Corr. 9 mins.	S 1.1	$0^h.15$
C.	8 18 10	Appt. ☾	30 20 55	I. C.	− 2 00		2 16 02.5	☾'s Dec.	S 18 29.4	S 1′.14
C. C.	− 9 10	2d corr.	45 34	Dip	− 6 36					
									h m s	
G. M. T. Apr. 26,	8 09 00	☾	31 06 29	1st corr.	+ 6 25			R.A. at 8 hrs.	14 31 02	$+124^s.5$
Corr. R. A. M. ☉	2 16 02.5			(Tab. 49, Bowditch =	+53′ 58″	H. P.	54′.7	Corr. 9 mins.	18.7	$0^h.15$
G. S. T.	10 25 02.5			I. C.	− 2 00			☾'s R. A.	14 31 20.7	$+ 18^s.675$
				Corr.	+51 58					

	° ′ ″		
h	31 06 29		
L	35 00 00	sec	10.08664
p	71 30 36	cosec	10.02301
$2s$	137 37 05		
s	68 48 33	cos	9.55808
h	31 06 29		
$s - h$	37 42 04	sin	9.78643
		2	19.45416*
		sin	9.72708

		h m s
Moon's H. A.	$= t -$	4 17 54
Moon's R. A.	=	14 31 20.7
L. S. T.	=	10 13 26.7
G. S. T.	=	10 25 02.5
Long.	=	0 11 35.8 West
Arc	=	2° 53′ 57″ West

---

* This is the log haversine of t, and, therefore, by entering Table 45, Bowditch, with 9.45416, the value of t may be found to be $4^h\ 17^m\ 54^s.2$.

The reliability of a moon sight for time.—The value of any observation for longitude depends upon the correctness of the altitude, the latitude, and the declination. An error in the G. M. T. produces an equal error in the longitude besides causing an error in the declination of any body whose declination changes rapidly. In the case of the moon, not only the declination but the right ascension (an error in which will affect the longitude) changes rapidly, so the G. M. T. should be accurately known to ensure good results. For these reasons a time sight of the moon is not so desirable as one of the sun or of a star.

231. Rules for working a time sight of a planet.—(1) *Proceed exactly as with a time sight of a star, remembering, however, to correct the right ascension and declination of a planet for G. M. T.*

(2) *The planets have a semi-diameter and parallax which, for theoretical reasons, will be used in the corrections to the altitude in the example that follows, though it would be a useless refinement to use them in sea observations. At sea, a planet's altitude should be corrected for I. C., dip, and refraction, and of these, the dip and refraction have been combined into one correction in Table 46, Bowditch.*

(3) *Proceed otherwise as with a time sight of a star.*

232. Haversine formula.—The hour angle t may be found from (157) or that formula transformed into one containing haversines only, but (186) will permit of a more convenient method of solution and a better arrangement of form than either of these. See Exs. 170(a) and 170(b).

For p. m. solar sights the astronomical L. A. T. (or t) is taken from the top of the page of haversine tables; for a. m. solar sights, the L. A. T. (or $24^h - t$) is taken directly from the bottom of the page. In case of the moon, a star, or a planet, the hour angle t is taken from the top of page and marked + or — according as the body is W. or E. of the meridian.

HOUR ANGLE AND LONGITUDE BY A. M. TIME SIGHT OF THE SUN.

***Ex.* 170(*a*).**—Jan. 18, 1918, a. m., in Lat. 40° 15′ N., Long. by D. R. 70° 45′ W., the sextant altitude of the sun's lower limb was 12° 48′ 30″. I. C. +1′. Height of eye, 45 feet. W. T. of obs. $8^h\ 30^m\ 15^s$. C—W $5^h\ 01^m\ 55^s$. Chro. fast of G. M. T. $2^m\ 10^s$. Required the longitude using haversine formula (186).

Times.		Altitude.		Altitude Corrections.		Declination.		H. D.		Eq. of T.		H. D.	
	h m s		° ′ ″		′ ″		° ′						
W.	8 30 15	⊙	12 48 30	S. D.	+16 17	At 0^h,	S 20 38.9		N 0′.5	— to M. T.			$+0^s.8$
C—W	5 01 55	Corr.	+ 6 39	I. C.	+ 1 00	Corr.	N 0.8	G. M. T.	$1^h.5$		m s	G. M. T.	$1^h.5$
				Dip	− 6 36						10 24.4		
C.	1 32 10	⊖	12 55 09	p. & R.	− 4 02		S 20 38.1	Corr.	N 0′.75	Corr.	+ 1.2	Corr.	$+1^s.20$
C. C.	− 2 10	*z*	77 04 51										
				Corr.	+ 6 39						10 25.6		
G. M. T. Jan. 18,	1 30 00			Or (Tab. 46, Bowditch)	= +5′ 39″								
Eq. of T.	− 10 25.6			I. C.	= +1 00								
G. A. T.	1 19 34.4			Corr.	+6 39								

	° ′ ″		
L	40 15 00 N	L. sec	0.11734
d	20 38 06 S	L. sec	0.02879
$L-d$	60 53 06		
s	77 04 51		
$s+(L-d)$	137 57 57	½ L. haver	4.97011
$s-(L-d)$	16 11 45	½ L. haver	4.14881
	h m s		
L. A. T.	20 36 44	L. haver	9.26505
G. A. T.	1 19 34.4		
Long.	4 42 50.4 W		
Arc	70° 42′ 36″ W		

HOUR ANGLE AND LONGITUDE BY OBSERVATION OF A STAR.

Ex. 170(*b*).—April 5, 1918, p. m., Lat. 20° 38′ S., Long. by D. R. 90° 30′ E., the sextant altitude of the star *α* Tauri (Aldebaran), west of the meridian, was 26° 00′ 40″. I. C.—1′. Height of eye 19 feet. W. T. of observation $7^h\ 16^m\ 44^s$. C—W $5^h\ 55^m\ 24^s$. Chro. fast of G. M. T. $1^m\ 38^s$. Required the longitude, using haversine formula (186).

Times.		✱'s Altitude.		Corr. to ✱'s Alt.		R. A. M. ⊙.		✱'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		h m s
W.	7 16 44	✱'s h_s	26 00 40	I. C.	− 1 00	At G. M. N.	0 51 54.6	R. A.	4 31 14.0
C—W	5 55 24	Corr.	− 7 15	Dip	− 4 16	Corr. G. M. T.	11.6		
				Ref.	− 1 59				° ′
C.	1 12 08	✱'s *h*	25 53 25				0 52 06.2	*d*	N 16 20.7
C. C.	− 1 38	✱'s *z*	64 06 35	Corr.	− 7 15				
G. M. T. 5th of April	1 10 30			Or (Tab. 46, Bowditch)	= −6′ 15″				
				I. C.	= −1 00				
R. A. M. ⊙	0 52 06.2								
				Corr.	−7 15				
G. S. T.	2 02 36.2								

	° ′ ″		
L	20 38 00 S	L. sec	0.02879
d	16 20 42 N	L. sec	0.01792
L − *d*	36 58 42		
z	64 06 35		
z + (*L* − *d*)	101 05 17	½ L. haver	4.88768
z − (*L* − *d*)	27 07 53	½ L. haver	4.37025
	h m s		
✱'s *t*	+ 3 33 28.5	L. haver	9.30464
✱'s R. A.	4 31 14.0		
L. S. T.	8 04 42.5		
G. S. T.	2 02 36.2		
Long.	6 02 06.3 East		
Arc	90° 31′ 34″.5 East		

HOUR ANGLE AND LONGITUDE BY OBSERVATION OF A PLANET.

Ex. 170.—March 1, 1918, p. m., in latitude 29° 00′ N., longitude by account 40° 12′ W., the sextant altitude of the lower limb of planet Jupiter, West of the meridian, near P. V., was 40° 35′ 30″. I. C. +1′. Height of eye 45 feet. W. T. of obs. 9ʰ 10ᵐ 10ˢ. C—W 2ʰ 30ᵐ 10ˢ. Chro. slow of G. M. T. 7ᵐ 09ˢ.5. Required the longitude.

Times.		Planet's Alt.		Corr. to Alt.		R. A. M. ☉.		Planet's Dec. and R. A.		H. D.	
	h m s		° ′ ″		′ ″		h m s		° ′		
W.	9 10 10	♃'s h_s	40 35 30	S. D.	+0 19	G. M. N.	22 33 55.2	Dec.	N 20 17.5		N 0′.058
C—W	2 30 10	Corr.	− 6 23	p	+0 2	Corr. G. M. T.	1 56.2	Corr.	N .7	G.M.T.	11ʰ.8
C. C.	+ 7 09.5	♃'s h	40 29 07	I. C.	+1 00		22 35 51.4	Dec.	N 20 18.2	Corr.	N 0′.68
G. M. T. Mar. 1,	11 47 29.5			Dip	−6 36						
Corr. R. A. M. ☉	22 35 51.4			R.	−1 08				h m s		
								R. A.	4 05 43		1ˢ.125
G. S. T.	10 23 20.9			Corr.	−6 23			Corr.	+ 13.3	G.M.T.	11ʰ.8
								R. A.	4 05 56.3	Corr.	+13ˢ.27

	° ′ ″		
h	40 29 07		
L	29 00 00	sec	10.05818
p	69 41 48	cosec	10.02786
$2s$	139 10 55		
s	69 35 28	cos	9.54247
h	40 29 07		
$s-h$	29 06 21	sin	9.68702
			2) 19.31553*

(P. M. col. Tab. 44) $t = +$ 3ʰ 36ᵐ 23ˢ.3 .... sin 9.65777

Planet's R. A. = 4 05 56.3

L. S. T. = 7 42 19.6

G. S. T. = 10 23 20.9

Long. = 2 41 01.3 W

Arc = 40° 15′ 19″.5 W

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* This is the log haversine of t, and, therefore, by entering Table 45, Bowditch, with 9.31553, the value of t may be found to be 3ʰ 36ᵐ 23ˢ.2.

233. Sunrise or sunset time sights.—When the sun's center is in the visible horizon, East or West, its true altitude or h equals — (refraction + dip $\dot{-}$ parallax). The watch time of this instant may be found by noting the watch times when the lower and upper limbs are in the visible horizon, just appearing at sunrise or disappearing at sunset, and taking their mean.

Having the watch time of the instant, the C—W, and the chronometer error on G. M. T., proceed to work a time sight of the sun with a negative altitude which numerically equals (refraction + dip — parallax), the refraction and parallax being for an altitude of 0° and the dip depending on the height of the eye.

However, owing to the difficulty of noting the times at contact of limbs with the horizon and the uncertainties of refraction, the result should be regarded as only approximate; and this method should be used only when fog or cloudy weather has prevented, or may prevent, the navigator from getting more reliable observations of the sun or stars.

234. Time of sunset.—The instant of sunset is when the sun's upper limb is just disappearing below the visible horizon, or when h = — (refraction + dip + S. D. — parallax). For this altitude, a given latitude, and declination, the hour angle of the sun may be found by the time-sight formula. This hour angle will be the civil local apparent time of sunset; (12 hours — the H. A.) will be the L. A. T. of sunrise.

The L. M. T. of sunset may be found by applying the equation of time for the instant to the L. A. T. of sunset.

As the declination and equation of time are tabulated for Greenwich time, to find the declination at the instant of sunset, it will be necessary to either assume, or take from azimuth or sunset tables, an approximate time of sunset, apply the longitude, and obtain an approximate Greenwich time of sunset for which the declination may be found.

For the equation of time it is better to proceed thus: Having found by computation in the time sight the L. A. T. of sunset, find the correct G. A. T. by applying the longitude, and for this G. A. T. take from the Nautical Almanac the equation of time which, if applied to the L. A. T. of sunset, will give the required L. M. T.

The L. M. T. of sunrise and sunset may be found in Table 10, Bowditch, or in the Tide Tables issued by the U. S. C. and G. Survey.

235. To find the duration of twilight.—Twilight lasts till the sun has sunk 18° below the visible horizon, and there will be continual light so long as the sun at its lower transit is not more than 18° below the visible horizon at the place.

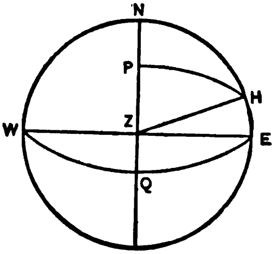


FIG. 113.

The difference of hour angles of the sun obtained by time sights, using altitudes of (—) (ref. + dip + S. D. — p) and of (—) (18° + ref. + dip + S. D. — p) will give the duration of twilight.

236. To find the hour angle of a heavenly body when in the horizon.—In Fig. 113, H is the body in the horizon, its H. A. is $HPZ = t$ and $\angle HPN = 180° - t$, $PN = L$, $PH = 90° - d$, and $\angle PNH = 90°$.

By Napier's rules, $\cos HPN = \tan PN \cot PH$,

or
$$-\cos t = \tan L \tan d, \qquad (187)$$

and if t is < 6 hrs., $2t$ will be < 12 hrs.; also if t is > 6 hrs., $2t$ will be > 12 hrs.

From the above it is apparent that bodies of positive declination (same name as latitude) will be above the true horizon for more than 12 hours, bodies of negative declination will be above the horizon less than 12 hours. This interval

$2t$ is for the sun an interval of apparent time; for a fixed star, an interval of sidereal time.

Length of day and night.—As the length of the day is determined by the length of time the sun is above the horizon of a place, it is evident that since $\cos t = - \tan d \tan L$, at the equator where $L = 0$, $\cos t = 0$ and $t = 6$ hours, so that at the equator the day is 12 hours long in all seasons. At the equinoxes $d = 0$, $\tan d = 0$, and $2t = 12$ hours, so that when the sun is at the equinoxes, the day is everywhere 12 hours.

Within the Arctic Circle, if $d = + 23\frac{1}{2}°$ and $L = 66\frac{1}{2}°$, $\cos t = - 1$, $2t = 24$ hours, and there is no night; this would be the case of midsummer in latitudes beyond $66\frac{1}{2}°$. If $d = -23\frac{1}{2}°$ and $L = + 66\frac{1}{2}°$, $\cos t$ will be $+ 1$, $2t$ will be 0 hours, and there will be no day, only night, as in the case of midwinter in latitudes beyond $66\frac{1}{2}°$. Thus it is plain that 12 hours is the average length of a day throughout a year; on a given date when the sun's declination is positive, the day is $>$ 12 hours, and on a day six months later, when the sun's declination is numerically the same but negative, the day will fall equally short of 12 hours.

237. Effect of small errors in data.—In the solution of the astronomical triangle for time, and hence for longitude, the elements involved are the declination from the Nautical Almanac which may be regarded as accurately known, a measured altitude which is affected by errors of observation, errors of the instrument, and errors of refraction, and a latitude by observation or dead reckoning which depends not only on accuracy of original determination by observation but on the correctness of course and distance run, etc. Thus it is apparent that both the altitude and latitude are liable to error, and it is desirable to consider the effect on the resulting longitude of (1) a small error in altitude, (2) a small error in latitude; (3) to find the position of a body when its altitude changes most rapidly, and then to determine the most favorable position of a heavenly body for observations for time or longitude.

By substitution in the trigonometric formula

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

letting $t = A$, $90° - h = a$, $90° - d = b$, and $90° - L = c$, we have the fundamental equation

$$\sin h = \sin L \sin d + \cos L \cos d \cos t.$$

By differentiation, h and t variables,

$$\cos h dh = -\cos L \cos d \sin t dt,$$

$$\frac{dt}{dh} = -\frac{\cos h}{\cos L \cos d \sin t};$$

but $\sin t = \sin Z \cos h \sec d$; therefore,

$$\frac{dt}{dh} = -\frac{\cos h}{\cos L \cos d \sin Z \cos h \sec d} = -\sec L \operatorname{cosec} Z. \quad (188)$$

Differentiating the fundamental equation, L and t variables, we have $\frac{dt}{dL} = \frac{\sin d \cos L - \cos d \cos t \sin L}{\cos d \cos L \sin t}$.

From trigonometry,

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A.$$

If $t = A$, $B = Z$, $a = 90° - h$, $b = 90° - d$, $c = 90° - L$, by substitution, we have

$$\cos h \cos Z = \sin d \cos L - \cos d \sin L \cos t,$$

and, by rule of sines, $\sin t = \sin Z \cos h \sec d$.

Making these substitutions in equation for $\frac{dt}{dL}$, we have

$$\frac{dt}{dL} = \frac{\cos h \cos Z}{\cos d \cos L \sin Z \cos h \sec d} = \cot Z \sec L. \quad (189)$$

(188) shows $\frac{dt}{dh}$ to be least when $L = 0°$ and $Z = 90°$.

(189) shows $\frac{dt}{dL}$ to be least when $L = 0°$ and $Z = 90°$.

To find the position of a body in azimuth when its altitude changes most rapidly.

In Art. 222, by differentiation, we found formula (167)

$$\frac{dZ}{dh} = \frac{\sin L \cos h - \cos L \sin h \cos Z}{\cos L \cos h \sin Z}, \text{ or}$$

$$\frac{dZ}{dh} = \tan L \operatorname{cosec} Z - \tan h \cot Z.$$

Therefore, $$dh = \frac{dZ}{\tan L \operatorname{cosec} Z - \tan h \cot Z} \qquad (190)$$

(190) shows *dh*, or the change in altitude, to be greatest when $Z = 90°$; or, in other words, when the heavenly body is on the prime vertical.

Conclusions.—(1) Considering the effects of errors in *h* and *L*, sights for longitude are best in low latitudes.

(2) An error in altitude, or an error in latitude, will produce the least change in the hour angle when the heavenly body is on the prime vertical.

(3) The motion of a heavenly body in altitude is most rapid when it is on the prime vertical; its altitude can be taken with greater accuracy; and when the diurnal circle of a body corresponds to the prime vertical, the change of altitude is directly proportional to the change in time, and the mean of a number of altitudes will correspond to the mean of their times of observation.

For these reasons it is better to observe a heavenly body when on or near the prime vertical when observing for time or longitude.

When the latitude and declination are of the same name, there is no difficulty in observing the body near the prime vertical and at an altitude sufficiently great to eliminate the uncertainties of refraction. In low latitudes, when *L* and *d* are of the same name, the body may be on the prime vertical when only a few minutes from the meridian, in the case of the sun near noon, and still be available for time observations.

These remarks should emphasize the fact that the suitability of heavenly bodies for time observations depends more on the azimuths than on the hour angles of such bodies.

When *L* and *d* are of different names, the diurnal circles do not cross the prime vertical above the horizon, and, under such circumstances, bodies are nearest to the prime vertical when in the horizon; and such bodies should be observed, if necessary to observe them for time, as soon as the altitude is sufficiently high to be unaffected by errors of refraction—that is at least 10°.

By the time the sun has reached a proper altitude for observations in winter time, it is so far from the prime vertical that any error in altitude or latitude will produce a larger one in longitude, and this error will increase with the latitude. However, during twilight or moonlight, in winter, there need be no difficulty in finding suitable stars on or near the prime vertical from which to obtain reliable determinations of longitude.

An inspection of the azimuth tables will indicate for a given latitude and declination the hour angle of a body when on, or nearest to, the prime vertical, and from it the local time may be found.

Since when on the prime vertical, a heavenly body is falling or rising most rapidly, and the changes of altitude are proportional to the changes of time, the effect of an error of 1′ in the altitude on the resulting hour angle, and hence longitude, can be gotten by dividing the difference of any two of the recorded times of observation by the difference of the corresponding altitudes in minutes of arc. The result will be in minutes or seconds of time as the differences of times are in minutes or seconds of time.

238. The practical way of finding the effect of an error of 1′ in the latitude, on the resulting longitude, is to find the longitudes for two latitudes differing, say 10′ or 20′, then divide the difference of longitude by the difference of assumed latitudes in minutes of arc. A study of Sumner's method will make this plain.

In Table I of this book and in Table 47, Bowditch, will be found tabulated the changes in longitude for a change of 1′ of latitude, the arguments being the observer's latitude and the body's true azimuth.

239. To find the hour angle of a heavenly body, its true altitude, and azimuth, when nearest to or on the prime vertical, that is nearest in azimuth to 90° (Figs. A and B).

There are seven cases that may be considered:

(1) $+d > L$. It is evident that the azimuth will be greatest when the vertical circle is tangent to the diurnal circle as at M in PZM and at M_1 in PZM_1; $M = 90°$, $M_1 = 90°$, $\cos t = \tan L \cot d$, $\sin h = \sin L \operatorname{cosec} d$, $\sin Z = \cos d \sec L$. The body is circumpolar when d equals or is $>$ co. L as in the case of a body whose diurnal circle is NM.

(2) $+d = L$. Here the diurnal circle is tangent to the prime vertical at the zenith. At the point sought, $Z = 90°$. The body is on the meridian and $t = 0$. The body is in the zenith and $h = 90°$.

(3) $+d < L$. The diurnal circle crosses the prime vertical, and, at the moment of crossing, $Z = 90°$ in triangle PZM_3; $\cos t = \cot L \tan d$, $\sin h = \sin d \operatorname{cosec} L$.

(4) $d = 0$. Here the diurnal circle is in the plane of the equinoctial and passes through the East and West points of the horizon, as does also the prime vertical; those are the points sought. Triangle PZM_4, $Z = 90°$, $\cos t = \cot L \tan d = 0$, and $t = 6$ hours; $\sin h = \sin d \operatorname{cosec} L = 0$, and $h = 0$.

(5) $-d < L$. In this case, d is of a different name from L. The diurnal circle intersects the prime vertical below the horizon. In triangle PZM_5 (Fig. B), $Z = 90°$, $\cos t = \cot L \tan (-d)$, $\cos t$ is negative, and t is $>$ 6 hours; $\sin h = \operatorname{cosec} L \sin (-d)$, h is negative, and the body is below the horizon.

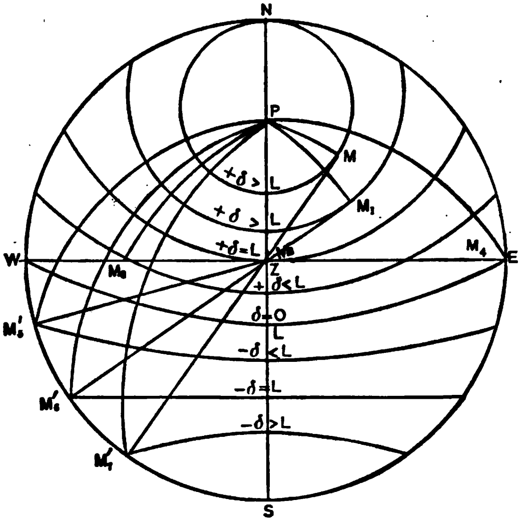

N
P
M
+δ > L
+δ > L
M_1
+δ=L
M_3
M_4
W
E
M_2
Z
+δ<L
δ=O
L
M_5'
−δ<L
−δ=L
M_6'
−δ>L
M_7'
S

FIG. A.

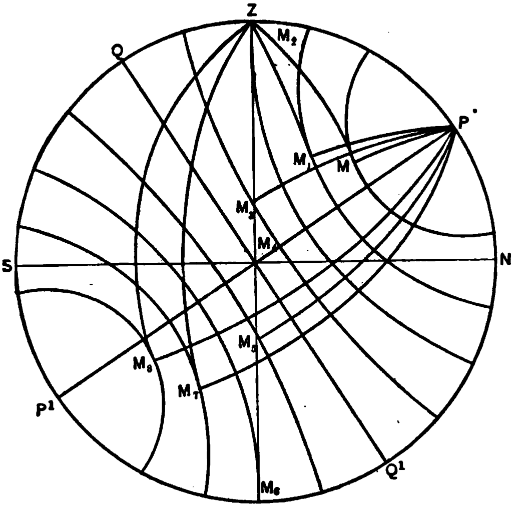

Z
M_2
Q
P
M_1
M
M_3
M_4
S
N
M_5
M_8
M_7
P^1
Q^1
M_6

FIG. B.

The nearest point to the prime vertical at which the body is visible is M'_5 (Fig. A), when the body is in the horizon. From the triangle PNM'_5 (Fig. A), we have $-\cos t = \tan L \tan (-d)$, or $\cos t = \tan L \tan d$ and t is $<$ 6 hours; $\cos NM'_5 = \cos Z = \sec L \sin (-d)$, $\cos Z$ is negative and $Z > 90°$.

(6) $-d = L$. In this case, the diurnal circle is tangent to the prime vertical at the nadir. The triangle is PZM_6 (Fig. B), $Z = 90°$, $\cos t = \cot L \tan (-d) = -1$, $\cos t = -1$ and $t = 12$ hours, $\sin h = \operatorname{cosec} L \sin (-d) = -1$, or $h = -90°$.

The nearest point to the prime vertical at which the body is visible is M'_6 (Fig. A), where the body is in the horizon. From the triangle PNM'_6, $-\cos t = \tan L \tan (-d)$, or $\cos t = \tan L \tan d$, t is $<$ 6 hours; $\cos NM'_6 = \cos Z = \sec L \sin (-d)$, $\cos Z$ is negative, and Z is $> 90°$.

(7) $-d > L$. Triangles PZM_7 and PZM_8 (Fig. B), $Z > 90°$ from the elevated pole. At M_7 and M_8, Z is a minimum, estimated from the depressed pole, at elongation. The vertical circle is tangent to the diurnal circle. $M_7 = 90°$, $M_8 = 90°$. Therefore, in triangles PZM_7 and PZM_8, right angled at M_7 and M_8, $\cos t = \tan L \cot (-d)$, $\cos t$ is negative, and t is $>$ 6 hours; $\sin h = \sin L \operatorname{cosec} (-d)$ and h is negative; $\sin Z = \sec L \cos d$, and should be taken in the second quadrant.

If $-d$ equals or is $>$ co. L, the body is never visible as in triangle PZM_8 (Fig. B). If the body is visible, then $-d$ is $<$ co. L and a point in the horizon will be the nearest point to the prime vertical at which the body is visible, as M'_7, (Fig. A). In the triangle PNM'_7, t is $<$ 6 hours and Z is $> 90°$, as in cases (5) and (6) when the body is visible.

Recapitulation for hour angle when the body is visible and on or nearest to the prime vertical.

When d is of the same name as L.

If d is $< L$, use formula $\cos t = \cot L \tan d$. (194)

If d is $> L$, use formula $\cos t = \tan L \cot d$. (195)

When d and L are of different names, and if the body is ever above the horizon, it will be nearest to the prime vertical on rising or setting, then use formula

$$\cos t = - \tan L \tan d. \tag{196}$$

When $+ d = L$, the diurnal circle passes through the zenith, so that when the declination is equal to, or nearly equal to, the latitude, observations for time may be made within a few minutes of meridian passage, the mean of altitudes corresponding to a mean of the times.

However, such is not the case when the body is near the meridian in azimuth, at which time the changes of altitude are proportional to the squares of the hour angles.

The azimuth tables may be used for finding the time when a body is on or nearest to the prime vertical, and will give results sufficiently accurate for all practical purposes.

In finding the t corresponding to a given log cos, in examples 171 to 173, when $\cos t$ is $+$, use the p. m. column, dividing through by 2. When $\cos t$ is $-$, use the a. m. column, adding 12 hours to the reading and then dividing by 2 to obtain t.

For a body whose declination and right ascension are changing sufficiently rapidly to require correction for G.M.T., the approximate time of being on the prime vertical must be known in order to get these elements for use, the declination for substitution in the proper formula, and the right ascension to be applied to the hour angle in finding the L. S. T. to be converted into L. M. T. To get the approximate time, when the body is on the prime vertical, assume the hour angle or take it from the azimuth tables and apply it to the L. M. T. of local transit from the Nautical Almanac.

Ex. 171.—April 26, 1918, astronomical time, at Cape Town, South Africa, Lat. 33° 56′ 04″ S., Long. 18° 28′ 40″ East, find the local mean time when the star *α* Virginis (Spica) is in the true horizon setting; also when the star *α* Argus (Canopus) is in the true horizon rising.

α Virginis (Spica).				*α* Argus (Canopus).			
	° ′ ″				° ′ ″		
L	33 56 04 S	tan	9.82792	*L*	33 56 04 S	tan	9.82792
d	10 44 18 S	tan	9.27794	*d*	52 39 18 S	tan	10.11746
	h m s				h m s		
✱'s *t*	+ 6 29 19.5	− cos	9.10586	✱'s *t*	−10 07 26.8	− cos	9.94538
✱'s R. A.	13 20 55.5			✱'s R. A.	6 22 07.6		
L. S. T.	19 50 15.0			L. S. T.	20 14 40.8		
			h m s				h m s
R. A. M. ⊙ at G. M. N. April 26,			2 14 42.2	R. A. M. ⊙ at G. M. N. April 26,			2 14 42.2
Corr. λ E Tab. III			− 0 12.2	Corr. λ E Tab. III			− 0 12.2
Sid. time local 0 hrs.			2 14 30.0	Sid. time local 0 hrs.			2 14 30.0
L. S. T.			19 50 15.0	L. S. T.			20 14 40.8
Sid. int. from noon			17 35 45	Sid. int. from noon			18 00 10.8
Red. Tab. II			− 2 53	Red. Tab. II			− 2 57.0
L. M. T. April 26			17 32 52	L. M. T. April 26			17 57 13.8
or April 27 (a. m.)			5 32 52	or April 27 (a. m.)			5 57 13.8

Ex. 172.—April 26, 1918, astronomical time, at Cape Town, South Africa, find the local mean times when the stars α Virginis (Spica) and α Argus (Canopus) are on or nearest to the prime vertical East and West.

α Virginis (will be on P. V.).
$+d<L$. Therefore $\cos t=\cot L \tan d$.

	° ′ ″	
L	33 56 04 S	cot 10.17208
d	10 44 18 S	tan 9.27794
$t=\pm 4^h\ 54^m\ 31^s.1$		cos 9.45002

	h m s	h m s
✱'s $t=$	+ 4 54 31.1	− 4 54 31.1
✱'s R. A.	13 20 55.5	13 20 55.5
L. S. T.	18 15 26.6	8 26 24.4
R. A. M. ⊙ at G. M. N.	2 14 42.2	2 14 42.2
Corr. λ E	− 0 12.2	− 0 12.2
Sid. time local 0 hrs.	2 14 30.0	2 14 30.0
L. S. T.	18 15 26.6	8 26 24.4
Sid. interval	16 00 56.6	6 11 54.4
Red. Tab. II	− 2 37.4	− 1 00.9
L. M. T. April 26	15 58 19.2 When West	6 10 53.5 When East

of the meridian.

α Argus (will not be on P. V.).
$+d>L$. Therefore $\cos t=\tan L \cot d$.

	° ′ ″	
L	33 56 04 S	tan 9.82792
d	52 39 18 S	cot 9.88254
✱'s $t\pm 3^h\ 56^m\ 26^s.2$		cos 9.71046

	h m s	h m s
✱'s $t=$	+ 3 56 26.2	− 3 56 26.2
✱'s R. A.	6 22 07.6	6 22 07.6
L. S. T.	10 18 33.8	2 25 41.4
R. A. M. ⊙ at G. M. N.	2 14 42.2	2 14 42.2
Corr. λ E	− 0 12.2	− 0 12.2
Sid. time local 0 hrs.	2 14 30.0	2 14 30.0
L. S. T.	10 18 33.8	2 25 41.4
Sid. interval	8 04 03.8	0 11 11.4
Red. Tab. II	− 1 19.3	− 0 01.8
L. M. T. April 26	8 02 44.5 When West	0 11 09.6 When East

of the meridian.

Ex. 173.—Feb. 15, 1918, astronomical time, in Lat. 32° N., Long. 28° W., find the L. M. T. when the planet Jupiter is on the prime vertical West; also the planet's true altitude at that time.

Here $+d$ is $<L$; hence use formulæ $\cos t = \cot L \tan d$, and $\sin h = \operatorname{cosec} L \sin d$.

From the Nautical Almanac the approximate time of transit across the upper branch of the local meridian is $6^{h}\ 21^{m}$. From the azimuth tables the H. A. when $Z = 90°$ is found to be approximately $3^{h}\ 38^{m}$ (which may be assumed to be an interval of sidereal time). This, reduced to a mean time interval, is approximately $3^{h}\ 37^{m}$.

Approximate Times.		Declination.		H. D.		Right Ascension.		H. D.	
	h m		° ′				h m s		
Approx. L. M. T. of transit	6 21	At G. M. noon	N 20 00.4		0′.042	At G. M. noon	4 00 42		$0^{s}.708$
Approx. H. A. when on P. V.	3 37	Corr. G. M. T.	0.5	G.M.T.	$11^{h}.83$	Corr. G. M. T.	8.4	G.M.T.	$11^{h}.83$
App. L. M. T. when on P. V.	9 58	Dec.	N 20 00.9	Corr.	0′.49	R. A.	4 00 50.4	Corr.	$8^{s}.375$
Longitude West	+ 1 52								
Approx. G. M. T. Feb. 15,	11 50 $=11^{h}.83$								

$L =$	32° N	cot	10.20421	cosec	10.27579
d	20° 00′ 54″ N	tan	9.56142	sin	9.53435
	h m s				
t	+ 3 37 22	cos	9.76563		
♃'s R. A.	4 00 50.4	$h =$	40° 18′ 48″	sin	9.81014
L. S. T.	7 38 12.4				

	h m s
R. A. M. ⊙, Feb. 15,	21 38 43.5
Reduction for λ W	+ 18.4
S. T. of local 0 hrs.	21 39 01.9
L. S. T.	7 38 12.4
Sid. interval	9 59 10.5
Red. to a M. T. interval	− 1 38.1
L. M. T. when Jupiter is on P. V.	9 57 32.4

CHAPTER XVII.

LATITUDE.

240. By definition, **the latitude of a place** is its angular distance North or South of the equator, measured on the meridian passing through the place. From Art. 142 we know that latitude is the declination of the zenith of the place or the altitude of the elevated pole at the place, and the astronomical work of finding the latitude consists in finding one or the other of these arcs, the position of the body determining which arc should be found.

In working for latitude the elements involved are *h, d,* and *t,* and the relation between them is shown in the fundamental trigonometric equation,

$$\sin h = \sin L \sin d + \cos L \cos d \cos t.$$

Now, if $t = 0$, as when the heavenly body is at its upper transit,

$$\sin h = \cos z = \sin L \sin d + \cos L \cos d,$$
$$\cos z = \cos (L - d),$$
$$z = L - d \text{ and } L = z + d.$$

$L = z + d$ is the general formula for latitude from altitudes of bodies on the meridian. The value of L depends on the values of z and d and the method of their combination. This method of finding latitude by observations of bodies on the meridian is the simplest as well as the most accurate one, the results being independent of the time for all practical purposes (except in the cases of the moon and planets where an error in time affects the declination), and having no greater errors due to altitude than the error in the altitude itself. The declinations of the sun and stars do not change rapidly, hence latitude from observations of these bodies is but little affected by errors in longitude or time.

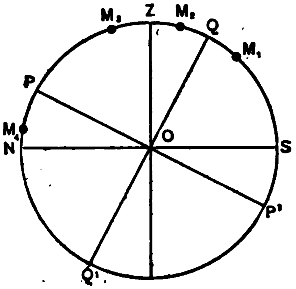


FIG. 114.

When a heavenly body is on the upper branch of a meridian, its declination, zenith distance, and latitude are all measured by arcs of the same great circle, the proper combination of any two arcs to produce the third being shown in the illustrations of the four cases considered.

Let Fig. 114 be a projection of the celestial sphere on the plane of the meridian.

NS is the horizon; *N* the North, *S* the South point.

Z the observer's zenith.

QQ' the equator.

PP' the axis of the sphere.

P the elevated pole.

P' the depressed pole.

QZ the declination of the zenith equals the latitude.

NP the altitude of the elevated pole equals the latitude.

Let M_1, M_2, M_3, M_4 be the four positions of the body to illustrate the four cases.

(1) Case of M_1 whose declination is of a different name from the latitude, or negative.

QM_1 is the declination. M_1Z is the zenith distance $= 90° - h = z$.

$$\text{Then } QZ = M_1Z - QM_1,$$

$$L = z - d. \qquad \textbf{(197)}$$

(2) Case of M_2, declination + and $< L$.

$$QZ = M_2Z + QM_2,$$

$$L = z + d. \tag{198}$$

(3) Case of M_3, declination + and $> L$.

$$QZ = QM_3 - M_3Z,$$

$$L = d - z. \tag{199}$$

In all these cases L is +, d is + or (—) as it is of the same or a different name from the latitude. In (1) and (2) d is $< L$ and the body bears towards the depressed pole, but in both cases z is +; therefore it is to be marked the opposite of the bearing of the body. In (3) d is $> L$ and the body bears towards the elevated pole, but in the formula z is (—); therefore in this case also mark z the opposite of the body's bearing. In other words, give d its proper mark N. or S. If the body bears North, mark z S.; if it bears South, mark z N. The latitude will be the algebraic sum, with the name of the greater, N. or S.

(4) Case of M_4, a heavenly body at its lower culmination.

$$\text{Then } PN = M_4N + PM_4 = M_4N + (90° - Q'M_4)$$

$$L = h + p = h + 90° - d. \tag{200}$$

Formula (199) is also correct for this case, provided we use $180° - d$ instead of d.

Writing (200) thus, $h = L - p$, it is evident that the polar distance of a body must be less than the latitude of the place in order that the body may be visible on the meridian below the pole. A body visible at its lower transit in any latitude is termed circumpolar for that latitude. The sun's maximum declination is about 23½°, or the polar distance a minimum of about 66½° from the North pole in June, or the South pole in December; so that the latitude of an observer must be in excess of about 66½° to see the sun at its lower transit, in North latitude, at the time of the sun's nearest approach to the North pole, as in June; or in South latitude, at the time of the sun's nearest approach to the South pole, as in December. On the supposition that altitudes under 10° are not reliable, owing to the uncertainties of refraction, the latitude would have to be at least 75° and of the same name as the declination to justify meridian observations of the sun below the pole. However, stars are available in all latitudes above 10°, for observations under favorable conditions at their lower transit, and, in North latitude, the pole star is available at all times when visible and if of sufficient altitude.

Since a heavenly body cannot be seen at its lower transit, unless its declination is positive, or of the same name as the latitude, it follows that the latitude resulting from an observation of a body crossing the lower branch of the meridian will be of the same name as the declination.

From the formula in each case it is apparent that an error in a meridian altitude produces an equal error in the resulting latitude.

In all meridian observations the declination of the heavenly body must be taken from the Nautical Almanac for the instant of transit; in the case of the sun, at upper transit the declination is corrected for the Greenwich ~~apparent~~ Mean time of noon, which in West longitude equals + λ of the given date, and in East longitude equals — λ of the given date or (24 hours — λ) of the day before; at lower transit the declination must be corrected for (12 hours + λ) if λ is West, or for (12 hours — λ) if λ is East, in either case for the instant of local apparent midnight.

In the case of stars, the declinations do not change with sufficient rapidity to require corrections, and hence when conditions are favorable, as in morning or evening twilight, observations of stars on or near the meridian are desirable.

In the case of the moon, the time should be accurately

✓

known, owing to the very rapid changes of declination; for this reason the moon is not so well adapted for observation as the sun or stars.

In the case of any body observed on the meridian for latitude, the sextant altitude must be reduced to the true altitude of the center (Art. 212).

Polaris is on the meridian when (Mizar) ζ Ursæ Majoris, the second star in the handle of the "Dipper," is vertically above or below the pole star, and since Polaris changes its altitude very slowly when crossing the meridian, such times are the best for observation of that star. However, the latitude may be found without appreciable error from Polaris at any time when the conditions for observation are favorable (see Art. 254).

241. Work preparatory to observing a meridian altitude of the sun.—It is customary at sea to find the watch time of local apparent noon (Art. 198), to begin observations 10 to 15 minutes before and to take continuous observations till the watch shows noon, the altitude at that time being taken as the meridian altitude. However, observations should be taken till the sun ceases to rise, or dips, in case it is not stationary when the watch indicates noon, the maximum altitude being the meridian altitude, subject to the remarks of Art. 246.

Before going on deck the navigator should have found the value of $\Delta_0 h$; observing the sun by watch at 15, 10, or a few minutes before apparent noon, and applying the correction for $\Delta_0 ht^2$ (see Art. 251), he knows very closely, minutes ahead of time, what the meridian altitude and the latitude will be. He should also have prepared a constant which, if properly applied to the meridian altitude by sextant, will give the latitude at once. The latitude and longitude being approximately known by D. R., find in order the declination and the approximate altitude, for which take out the parallax in altitude and refraction.

Calling the algebraic sum of the I. C., dip, refraction, parallax, and semi-diameter c, the sextant meridian altitude h_s, we have:

For M_1, dec $(-)$, $L=(90°-d-c)-h_s$.
For M_2, $d+$and$<L$, $L=(90°+d-c)-h_s$.
For M_3, $d+$and$>L$, $L=h_s-(90°-d-c)$.
For M_4, lower transit, $L=h_s+(9C°-d+c)$
or $h_s+(p+c)$.

The quantities in brackets are called constants, are computed beforehand, and entered in the navigator's note-book. The constant is applied to the sextant altitude as indicated for each particular case.

242. To find the latitude from the sun's meridian altitude: (a) at upper transit; (b) at lower transit.

(a) The local apparent time of transit is $0^h\ 0^m\ 0^s$ of the day at the ship. Find the G. A. T. of local apparent noon by applying the longitude, + if West, – if East; and then find the G. M. T. of local apparent noon by applying the equation of time.

(2) Take the sun's declination from the Almanac for the proper month and day, and correct it for the G. M. T., marking it, as it should be, N. or S. (Art. 185).

(3) Reduce the sextant altitude to the true altitude of the center (Art. 202). Subtract the latter from 90° to get the zenith distance. If the body bears N., mark the zenith distance S.; if it bears S., mark the zenith distance N.

(4) The latitude is the sum of the declination and zenith distance, if they are of the same name, and is marked like them; the difference of the two, if of different names, and marked with the name of the greater.

(b) At lower transit the local apparent time is 12 hours. Find the corresponding G. M. T. and for it the declination of the sun, then the polar distance$=90°-d$.

(2) Reduce the sextant altitude as above explained.

(3) Add the altitude and polar distance; the result is the latitude marked like the declination.

CASE M_1, DECLINATION NEGATIVE.

Ex. 174.—Jan. 2, 1918, in Long. 30° W., Lat. by D. R. 45° N., the sextant altitude of the sun's lower limb on the meridian bearing S. was 21° 57′. I. C.—1′. Height of eye 30 feet. Find latitude.

Altitudes, &c.		Altitude Corrections.	
	° ′ ″		′ ″
Sextant alt. or $\underline{\odot}$	21 57 00	S. D.	+16 18
Corr. to center	+ 7 40	I. C.	− 1 00
		Dip	− 5 22
True alt. of center } or $\ominus$	22 04 40 S	p. & R.	− 2 16
	90	Corr.	+ 7 40
		Or (Tab. 46, Bowditch)	=+8′ 40″
Zen. Dist. or z	67 55 20 N		
Dec. or d	22 57 42 S	I. C.	=−1 00
Lat. or L	44 57 38 N	Corr.	=+7 40

	h m s	
G. A. T. of local app. noon	=2 00 00	Eq. t. 2^h, $=3^m 57^s$
Equation of time	+ 3 57	(+ to app. time)
G. M. T. of local app. noon	2 03 57 $=2^h.07$	

Sun's Declination.		H. D. of Dec.	
	° ′		
At 2^h,	S 22 57.7		N 0′.2
Corr.	00.0	G. M. T.	$0^h.07$
At L. A. noon	S 22 57.7	Corr.	N 0′.014

To find the constant $K=(90°-d-c)$; $L=K-$sextant altitude $\underline{\odot}$. First find the approximate altitude, then the constant.

To Find the Approximate Altitude.			
	° ′		′ ″
$\odot$'s dec. at 2^h	S 22 57.7	S. D.	16 18
Corr.=N $0'.2\times0^h.07$	00.0	I. C.	− 1 00
Corrected dec. at L. A. N.	S 22° 57′ 42″	D.	− 5 22
	90	Approx. corr.	+ 9 56
$90°-d$	67 02 18	p. & R.	− 2 16
Approx. corr.	9 56		
$90°-d-$approx. c	66 52 22	Corr.	+ 7 40
Latitude	N 45 00 00		
Approx. alt. $\underline{\odot}$	21 52 22		

To Find the Constant.	
	° ′ ″
	90 00 00
Corrected dec.	22 57 42
$90°-d$	67 02 18
Corrections to $\underline{\odot}$	7 40
Constant	66 54 38
Sextant alt. $\underline{\odot}$	21 57
Latitude	44 57 38 N

Ex. 175.—April 7, 1918, in longitude 49° 30′ W., approximate latitude 41° 10′ N., the sextant altitude of the sun's lower limb on the meridian bearing S. was 55° 20′. I. C. +2′. Height of eye 45 feet. Find the latitude.

	h m s	
G. A. T. of local app. noon	3 18 00	Eq. t. 3^h 18^m
Eq. of time	+ 2 18	$=2^m$ 18^s
		(+ to app. time)
G. M. T. of local app. noon	3 20 18	
	$=3^h.3$	

Altitudes, &c.	° ′ ″	Altitude Corrections.	′ ″
Sextant alt. or $\underline{\odot}$	55 20 00 S	S. D.	+16 00
Corr. to center	+ 10 49	I. C.	+ 2 00
		Dip	− 6 36
True alt. of center } or -⊖-	55 30 49 S	p. & R.	− 0 35
	90	Corr.	+10 49
		Or (Tab. 46, Bowditch)	=+ 8′ 49″
Zen. Dist. or *z*	34 29 11 N		
Dec. or *d*	6 40 36 N	I. C.	=+ 2 00
Latitude or *L*	41 09 47 N	Corr.	+10 49

Sun's Declination.	° ′	H. D. of Dec.	
At 2^h,	N 6 39.4		N 0′.9
Corr.	= N 1.2	G. M. T.	$1^h.3$
At L. A. noon	N 6 40.6	Corr.	1′.17

To find the constant $K=(90°+d-c)$ before going on deck for the noon observation; $L=K-$sextant altitude $\underline{\odot}$.

To Find the Approximate Altitude.

	° ′ ″		′ ″
⊙'s dec. at 2^h	N 6 39.4	S. D.	+ 16 00
Corr. = N 0′.9 × $1^h.3$	N 1.2	I. C.	+ 2 00
Corrected dec. at L. A. N.	N 6° 40′ 36″	Dip	− 6 36
	90	Approx. corr.	+ 11 24
90° + *d*	96 40 36	p. & R.	− 0 35
Approx. correction	11 24	Corr.	+ 10 49
90° + *d* − approx. *c*	96 29 12		
Approx. lat.	N 41 10		
Approx. alt. $\underline{\odot}$	55 19 12		
Use approx. alt. $\underline{\odot}$ to find p. & R.			

To Find the Constant.

	° ′ ″
	90 00 00
Corrected dec.	6 40 36
90 + *d*	96 40 36
Corrections to $\underline{\odot}$	10 49
Constant	96 29 47
Sextant alt. $\underline{\odot}$	55 20
Latitude	41 09 47 N

CASE M_3, d + AND > L.

Ex. 176.—April 26, 1918, in Long. 65° E., Lat. by D. R. 1° 30′ N., the sextant altitude of the sun's lower limb on the meridian bearing N. was 77° 58′ 10″. I. C. −1′. Height of eye 24 feet. Find the latitude.

Altitudes, &c.	° ′ ″	Altitude Corrections.	′ ″
Sextant alt. or ⊙̲	77 58 10 N	S. D.	+15 55
Corr. to center	+ 9 57	I. C.	− 1 00
		Dip	− 4 48
True alt. of center } or ⊖	78 08 07 N	p. & R.	− 0 10
	90	Corr.	+ 9 57
Zen. Dist. or z	11 51 53 S	Or (Tab. 46, Bowditch)	= +10′ 58″
Dec. or d	13 16 48 N	I. C.	− 1 00
Latitude or L	1 24 55 N	Corr.	= + 9 58

	h m s	
G. A. T. of local app. noon April 25,	19 40 00	Eq. of time, 2^m 08^s (− to app. time)
Eq. of time	− 2 08	
G. M. T. of local app. noon	19 37 52	
	$=19^h.63$ Apr. 25	

Sun's Declination.	° ′	H. D. of Dec.	
At 18^h,	N 13 15.5		N 0′.8
Corr.	= N 1.3	G. M. T.	$= 1^h.63$
At L. A. noon	N 13 16.8	Corr.	N 1′.30

To find the constant $K=(90°-d-c)$ and $L=$sextant alt. ⊙̲$-K$.

To Find the Approximate Altitude.

	° ′ ″		′ ″
⊙'s dec. at 18^h, Apr. 25,	N 13 15.5	S. D.	15 55
Corr. = N $0'.8\times1^h.63$	N 1.3	I. C.	− 1 00
Corrected dec. at L. A. N.	N 13° 16′ 48″	Dip	− 4 48
	90	Approx. corr.	+ 10 07
$90°-d$	76 43 12	p. & R.	− 10
Approx. corr. to center	10 07	Corr.	+ 9 57
$90°-d-$approx. c	76 33 05		
Approx. lat.	N 1 30		
Approx. alt. ⊙̲	78 03 05		

To Find the Constant.

	° ′ ″
	90 00 00
Corr. dec. at L. A. N.	13 16 48
$90°-d$	76 43 12
Correction to ⊙̲	9 57
Constant	76 33 15
Sextant alt. ⊙̲	77 58 10
Latitude	1 24 55 N

CASE M_4, BODY ON MERIDIAN BELOW THE POLE.

Ex. 177.—On Jan. 1, 1918, in Long. 150° W., the sextant altitude of sun's lower limb at lower transit was 7° 55′. I. C. +1′. Height of eye 25 feet. Required the latitude.

This culmination is at local apparent midnight and the G. A. T. of the instant is $12^h+\lambda=$ Jan. 1, 22^h or Jan 2, $(-)\ 2^h$.

Altitudes, &c.	° ′ ″
Sextant alt. or $\underline{\odot}$	7 55 00
Corr. to center	+ 5 56
True alt. of center or $\ominus$	8 00 56
Sun's polar dist. or p	67 01 24
Latitude	75 02 20 S

Altitude Corrections.	′ ″
S. D.	16 18
I. C.	+ 1 00
Dip	− 4 54
p. & R.	− 6 28
Corr.	+ 5 56
Or (Tab. 46)	= +4′ 56″
I. C.	= +1 00

	h m s
G. A. T. of local apparent midnight	22 00 00
Eq. t. 22^h	+ 3 52
G. M. T. of local apparent midnight	22 03 52
	$=22^h.06$

Sun's Declination.	° ′	H. D. of Dec.	
At 22^h,	S 22 58.6		0′.2
Corr.	0.0	G. M. T.	$0^h.06$
At L. A. midnight	22 58.6	Corr.	−0′.012
p	=67° 01′ 24″		

To find the constant $K=p+c$ before getting the observation. $L=K+$ sextant altitude $\underline{\odot}$.
Approximate latitude 75° 03′ S.

To Find the Approximate Altitude.	° ′ ″		′ ″
$\odot$'s dec. at 22^h or L. A. midnight	S 22 58 36	S. D.	+ 16 18
$\odot$'s polar distance	67 01 24	I. C.	+ 1 00
Approx. latitude	S 75 03	Dip	− 4 54
Approx. alt. of $\ominus$	8 01 36	Approx. corr.	+ 12 24
Approx. corr. to center	12 24	p. & R.	− 6 32
Approx. alt. $\underline{\odot}$	7 49 12	Corr.	+ 5 52

To Find the Constant.	° ′ ″
p	= 67 01 24
Corr. c	= + 5 52
Constant	67 07 16
Sextant alt. $\underline{\odot}$	7 55 00
Latitude	75 02 16 S

243. To find the latitude from the meridian altitude of a fixed star.

Rules. (1) *Find by computation beforehand the local and watch time of transit for the ship's position at the approximate time of transit (Arts. 196 and 198), and observe at that time. Altitudes should be obtained before and after the watch time of transit and times noted for use in case the meridian altitude is missed. See Art. 151 on star observations.*

Where a selection can be made about a given time, select a star, if possible, which has a comparatively low altitude for observation on or near the meridian, because such a star will change its altitude slowly when in that position.

(2) *The change of declination of a fixed star is so slow that it may be neglected; so take the declination direct from the Nautical Almanac.*

(3) *Reduce the altitude to a true altitude by correcting for I. C., dip, and refraction, unless the star is observed with an artificial horizon, in which case proceed as in Art. 154.*

(4) *Find the true zenith distance and combine it with the declination as in the case of the sun.*

Ex. 178.—At sea, January 21, 1918, the sextant altitude of the star α Tauri (Aldebaran) on the meridian bearing S. was 69° 30′ 10″. I. C. −1′. Height of eye 45 feet. Find the latitude.

Altitudes, &c.		Altitude Corrections.		✳'s Declination.
	° ′ ″		′ ″	
✳'s sextant alt.	69 30 10 S	I. C.	−1 00	✳'s Dec. N 16° 20′.8
Corr. to alt.	− 7 58	Dip	−6 36	
		Ref.	−0 22	
✳'s true alt.	69 22 12 S			
✳'s true z	20 37 48 N	Corr.	−7 58	
✳'s declination	16 20 48 N	Or (Tab. 46, Bowditch) =	−6′ 58″	
Latitude	36 58 36 N	I. C. =	−1 00	
		Corr.	−7 58	

Ex. 179.—January 1, 1918, the sextant altitude of the star α Eridani (Achernar) on the meridian bearing S. was 35° 22′. I. C. + 2′. Height of eye 20 feet. Find the latitude.

Altitudes, &c.		Altitude Corrections.		✱'s Declination.
	° ′ ″		′ ″	
✱'s sextant alt.	35 22 00 S	I. C.	+2 00	✱'s Dec. S 57° 39′.4
Corr. to alt.	— 3 45	Dip	−4 23	
✱'s true alt.	35 18 15 S	Ref.	−1 22	
✱'s true *z*	54 41 45 N	Corr.	−3 45	
✱'s declination	57 39 24 S	Or (Tab. 46, Bowditch) = −5′ 45″		
Latitude	2 57 39 S	I. C.	= + 2 00	
		Corr.	−3 45	

Ex. 180.—April 19, 1918, the sextant altitude of the star α Aurigæ (Capella) on the meridian below the pole was 11° 20′. I. C. + 2′. Height of eye 40 feet. Find the latitude.

Altitudes, &c.		Altitude Corrections.		✱'s Dec. and Polar Dist.	
	° ′ ″		′ ″		° ′
✱'s sextant alt.	11 20 00	I. C.	+2 00	✱'s *d* =	N 45 55.1
Corr. to alt.	— 8 55	Dip	−6 12	✱'s *p* =	44 04.9
✱'s true alt.	11 11 05	Ref.	−4 43		
✱'s polar distance	44 04 54	Corr.	−8 55		
Latitude	55 15 59 N	Or (Tab. 46, Bowditch) = −10′ 55″			
		I. C.	= +2 00		
		Corr.	− 8 55		

The latitude has the same name as the star's declination, otherwise the star would not be visible on the meridian below the pole.

244. To find the latitude by the meridian altitude of the moon.—The moon being comparatively near to the earth, its changes of declination and semi-diameter are more rapid than in the case of other bodies; besides, its parallax is quite large. These elements require careful correction, and the great liability to error, due to error of time, render observations of the moon less desirable than those of other bodies. When the moon is near the equator its declination changes most rapidly, and at such times the maximum altitude may differ considerably from the meridian altitude. A movement of the zenith due to high speed of observer's ship, especially in the direction of the meridian, will intensify such a discrepancy (Art. 246); hence it is better to calculate the time of meridian passage of the moon and consider the altitude observed at that time as the meridian altitude.

Rules. (1) *Find the Greenwich mean time and date of the moon's local meridian passage (Art.* 188).

(2) *For this G. M. T. find the moon's declination S. D. and H. P. (Art.* 185).

(3) *Reduce the sextant altitude to the true altitude of the center (Art.* 212).

(4) *After which, proceed as with the sun.*

245. To find the latitude by the meridian altitude of a planet.

(1) *Find from the Nautical Almanac the G. M. T. of Greenwich transit. To this apply the retardation or acceleration for the longitude, and the result will be the L. M. T. of local transit (Art.* 189), *the retardation or acceleration per hour being one-twenty-fourth of the difference of times of transits on two successive days as indicated in the Nautical Almanac.*

(2) *From the L. M. T. of local transit and the error of observer's watch on L. M. T. find the watch time of local transit and observe the planet's altitude at that time. Reduce the sextant altitude to a true altitude by applying the I. C., dip, and refraction, neglecting for sea observations S. D. and parallax.*

(3) *To the L. M. T. of local transit apply the longitude and obtain the G. M. T. of local transit. For this G. M. T. find the planet's declination; after which, proceed as in the case of the sun.*

meridian bearing N. was 60° 15′ 00″. I. C.+3′. Height of eye [illegible] feet. Find the latitude.

Meridian Passage.	Retardation for λ.	Altitudes.	1st Corr.	Moon's Dec.	H. D.
h m		° ′ ″	′ ″	° ′	
Jan. 25, G.M.T. of Gr. tr. 11 16	H.D. 2^{m}	☾ 60 15 00 N	I. C. + 3 00	At 18^{h} N 18 5.2	S 8′.2
Corr. for λ West + 13	λ +6^{h}.7	1st corr. + 13 19	S. D. +15 06	Corr. S 1.6	G.M.T. 0^{h}.2
			Aug. + 13		
L. M. T. of local transit 11 29	Corr. 13^{m}.4	Appt. ☾ 60 28 19 N	Dip − 5 00	Dec. N 18 55.6	Corr. S 1′.64
Long. West + 6 42		2d corr + 26 43	1st corr. 13 19	Semi-D. =15′.1 | Hor. Par. =55′.3	
G.M.T. of local tr., Jan. 25, 18 11		☾ 60 55 02 N	Total corr. (Tab. 49, Bowditch)		
=18^{h}.2		z 29 04 58 S	=+37′ 04″		
		d 18 55 36 N	I. C. =+ 3 00		
		Lat. 10 09 22 S	Corr. +40 04		

Ex. 182.—March 31, 1918, a. m., in Long. 150° East, the sextant altitude of the moon's upper limb on the meridian bearing S. was 29° 00′. I. C.+2′. Height of eye 45 feet. Find the latitude.

Meridian Passage.	Retardation for λ.	Altitudes.	1st Corr.	Moon's Dec.	H. D.
h m		° ′ ″	′ ″	° ′	
Mar. 30, G.M.T. of Gr. tr. 14 26	H.D. 2^{m}.08	☾ 29 00 00 S	S. D. −14 54	At 4^{h} S 18 47.6	S 7′.45
Corr. for λ East − 21	λ −10^{h}	1st corr. − 19 37	Aug. − 07	Corr. S 0.7	0^{h}.1
			I. C. + 2 00		
L. M. T. of local tr. 14 05	Corr. −20.8	Appt. ☾ 28 40 23 S	D. − 6 36	d = S 18 48.3	S 0′.745
Long. East − 10		2d corr. + 46 09	1st corr. −19 37	Semi-D. =14′.9 | Hor. Par. =54′.6	
G.M.T. of local tr., Mar. 30, 4 05		☾ 29 26 32 S	Total corr. (Tab. 49, Bowditch)		
=4^{h}.1		z 60 33 28 N	=+24′ 32″		
		d 18 48 18 S	I. C. =+ 2 00		
		Lat. 41 45 10 N	Corr. +26 32		

Ex. 183.—Feb. 22, 1918, about 6 p. m., in longitude 30° W., the sextant altitude of the planet Jupiter on the meridian bearing S. was 52° 20′. I. C. −1′. Height of eye 45 feet. Find the latitude.

Meridian Passage.		Acceleration.		Altitude.		Corrections to Altitude.		Declination.		H. D.	
	h m		m		° ′ ″		′ ″		° ′		
Feb. 22, G. M. T. of G. tr.	5 56	For 24 hrs.	4.0	♃'s h_s	52 20 00 S	I. C.	−1 00	At G. M. noon	N 20 08.2		N 0′.054
Acceleration for λ W	− 0.3	For. λ hrs.	0.3	Corr.	− 8 21	Dip.	−6 36			G. M. T.	$7^h.9$
						Ref.	−0 45	Corr.	N .4		
L. M. T. of local tr.	5 55.7			♃'s h	52 11 39 S					Corr.	N 0′.427
Long. West	+ 2 00			z	37 48 21 N	Corr.	−8 21	d	N 20 08.6		
				d	20 08 36 N	Or (Tab. 46, Bowditch)	= −7′ 21″				
G.M.T. of local tr., Feb.22,	7 55.7										
				Lat.	57 56 57 N	I. C.	= −1 00				
						Corr	−8 21				

246. Maximum and minimum altitudes.—The maximum altitude of a heavenly body occurs at the instant of upper transit, provided the body is of an unchanging declination, as for instance a fixed star, and is observed from a fixed position. However, if the body is changing its declination, and if the observer is on a ship in motion, changing his horizon and zenith, the maximum altitude is not at the instant of meridian passage, but after, if the body and zenith are approaching, before, if they are separating; for, if approaching, the body will continue to rise after its upper transit till its downward velocity equals that of the approach of the zenith (or observer), at which time the maximum altitude is reached; if the body and zenith are separating, the body reaches the maximum altitude before the meridian passage and at a time when the velocity of rising equals that of separation.

Let Δd be the change of declination in 1 minute, if expressed in seconds of arc; or, in 1 hour, if expressed in minutes of arc.

Let ΔL be the change of the zenith with respect to the body in the meridian, in 1 minute, if expressed in seconds of arc; or, in 1 hour, if expressed in knots or minutes of arc per hour.

Let Δc be the combined action of Δd and ΔL, causing approach or separation, expressed in the same units as Δd and ΔL, or the velocity of approach or separation.

Let $\Delta_0 h$ be the change in altitude in 1 minute from meridian passage due solely to diurnal rotation (see Art. 251) expressed in seconds of arc; Δh, the correction to be applied to the maximum altitude to reduce it to the meridian altitude, in seconds of arc, sign of application minus.

Let t be the H. A. of the body at the instant of highest altitude, easterly when the body and zenith are separating; westerly, when approaching. It will be apparent or sidereal time, depending upon the body observed, and must be converted into a mean time interval if required in mean time units.

The value of t is affected by the change of longitude made by the ship. The correct H. A. is obtained by increasing an easterly H. A., or decreasing a westerly H. A., for a westerly change in longitude, and the reverse for an easterly change in longitude.

Then, Δc being the velocity of approach or separation,

Δct will be the change in altitude produced by changes of declination and latitude in the time t;

$\Delta_0 ht^2$ (see Art. 251), the diminution of altitude due to diurnal rotation;

and $\Delta h = \Delta ct - \Delta_0 ht^2$.

Now, since the velocity of the body on the meridian is zero, and its change of altitude near the meridian varies as t^2, its motion near the meridian is uniformly accelerated or retarded, according as t is $+$ or $(-)$.

Therefore, $\Delta_0 ht^2 = \frac{1}{2} at^2$ where a is the acceleration or retardation, and

$$a = 2\Delta_0 h. \qquad (201)$$

From the formula for velocity of uniformly accelerated bodies $V = v + at$, we have, since the velocity at transit is zero, for the velocity when the H. A. is t,

$$V = 2\Delta_0 ht. \qquad (202)$$

Now, at the moment of maximum altitude, the body is stationary, its velocity in altitude equals that of approach or separation.

$$\left.\begin{array}{l} \text{Therefore, } 2\Delta_0 h t = \Delta c, \text{ and } t = \dfrac{\Delta c}{2\Delta_0 h} \\ \text{and, by substitution,} \\ \quad \Delta h = \dfrac{\Delta^2 c}{2\Delta_0 h} - \dfrac{\Delta^2 c}{4\Delta_0 h} = \dfrac{\Delta^2 c}{4\Delta_0 h} \end{array}\right\} \qquad (203)$$

At the lower transit, the minimum altitude occurs after the meridian passage, if the body and zenith are separating; and before, if approaching. The method of applying the change of longitude to the H. A. is exactly the reverse of that at the upper transit.

In view of the many inaccuracies of observations at sea, this correction is not of any importance except for observations taken on board very swift steamers on courses near the meridian.

The following example will illustrate the preceding remarks:

Ex. 184.—April 29, 1918, in latitude 50° N., longitude 25° 30′ W., observed from a ship steaming 225° (true) 20 knots per hour the maximum altitude of the sun's lower limb to be 54° 15′. I. C. + 1′. Height of eye 45 feet. Corrected declination of the sun N. 14° 19′.1. H. D. N. ′.8. Required the time, a. m. or p. m., of maximum altitude, and the correction to reduce this altitude to the meridian altitude.

Course.	Distance.	S	W	$D = 22'$ W 4^s
225°	20	14.1	14.1	$D = 88^s$ W

Observer's change in latitude 14′.1 S, per hour, or 14″.1 S per minute.
Change in the sun's declination ′.8 per hour, or.... 0 .8 N "
Δc = combined velocity of approach.............. 14″.9 "
For Lat. 50° N. and Dec. $14\frac{1}{3}$° N. (Table 26, Bowditch) $\Delta_0 h = 2''.1$.
From (203), $t = \dfrac{\Delta c}{2\Delta_0 h} \therefore t = \dfrac{14''.9}{2 \times 2''.1} = \dfrac{14''.9}{4''.2} = 3^m.55 = 3^m\ 33^s$.

As observer's zenith and body approach, t is a westerly H. A. As the ship changes her longitude to the westward, at the rate of 88^s per hour or 5^s in $3^m.55$, the corrected H. A. is $+3^m\ 28^s$, and the time of maximum altitude is $3^m\ 28^s$ after meridian transit or $3^m\ 28^s$ p. m.

$$\Delta h = \frac{\Delta^2 c}{4\Delta_0 h} = 26''.$$

247. Finding latitude by observations of bodies out of the meridian.—It may often happen that the sun is not visible at noon, making an observation for latitude desirable at its earliest appearance after noon; or from threatening appearances in the morning, the indications may be that the sun, when on the meridian, will not be visible. Under such circumstances forenoon observations should be taken. In fact a careful navigator will always take them and practically know his noon position ahead of time. Besides, during morning or evening twilight, or moonlight, there are many stars favorably situated and available for latitude observations; so methods for finding latitude from altitudes of bodies out of the meridian are necessary.

There are five methods in use:

(1) A rigorous method, known as the $\phi''\phi'$ method, available for bodies within three hours of the meridian and whose azimuth, Z_N, is between 315° and 45° or 135° and 225°, and which is independent of latitude.

(2) An approximate method involving both latitude and longitude.

(3) By reduction to the meridian, a special case of the second method.

(4) By altitude of the pole star, using modified formulæ of the first method.

(5) Chauvenet's method by two altitudes near noon, time unknown.

(6) Prestel's method by rate of change of altitude near the prime vertical (only an approximation).

248. Effect of errors in data on the latitude.—In the solution of the astronomical triangle for latitude from an observation of a heavenly body out of the meridian, all the elements involved, except the declination, may be considered as liable to error, and it is desired to find the effect on the latitude of (1) an error in altitude, (2) an error in t, and (3) to determine the most favorable position of a heavenly body for observations for latitude.

(1) **To determine the effect of an error in altitude on the resulting latitude.**—Differentiating the fundamental equation

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

regarding h and L as variables, we have

$$\cos h dh = \sin d \cos L dL - \cos d \cos t \sin L dL$$

$$\frac{dL}{dh} = \frac{\cos h}{\sin d \cos L - \cos d \cos t \sin L};$$

but, from trigonometry,

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A,$$

and by substitution,

$$\cos h \cos Z = \sin d \cos L - \cos d \cos t \sin L.$$

Therefore,

$$dL = \frac{\cos h dh}{\cos h \cos Z} = dh \sec Z. \qquad (204)$$

(2) **To determine the effect of an error in t on the latitude.**—Differentiating the same equation, L and t variables, we have from equation (189), Art. 237,

$$\frac{dt}{dL} = \cot Z \sec L; \text{ therefore, } \frac{dL}{dt} = \cos L \tan Z \ (dt \text{ in arc}).$$

Now, since dt (in arc) equals $15dt$ (in time), if dt is given in time,

$$dL = 15dt \cos L \tan Z. \qquad (205)$$

(204) and (205) show that the maximum effect of errors in altitude and time are produced when $Z = 90°$, the mini-

mum effect when $Z = 0°$ or $180°$, the inference being that positions on or near the meridian are better for observations for latitude, and that observations near the prime vertical for latitude should be avoided.

The hour angle used in the various methods is very liable to error at sea, either from error in the original determination of the longitude, or error in run to time of observation for latitude.

If the sign of dL due to dt is positive when the body is on one side of the meridian, it will be negative for the same azimuth on the other side; hence the error may be eliminated by taking the mean of results from observations on both sides of the meridian.

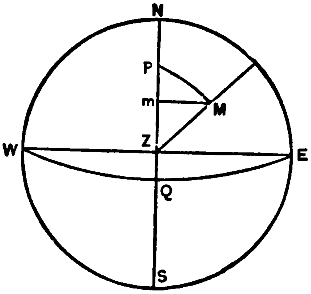


FIG. 115.

249. To find latitude by an altitude out of the meridian. First method.—To find the latitude of a place at any time, given the sextant altitude of a heavenly body, the Greenwich mean time of observation, and the longitude of the place.

Reduce the sextant altitude to the true altitude of the center. Find the body's declination and hour angle (Arts. 212 and 185).

Then in the astronomical triangle PZM, there are given $ZPM = t$, $PM = 90° - d$, $ZM = 90° - h$, and it is required to find $PZ = 90° - L$.

In Fig. 115, let fall from M a perpendicular Mm on the meridian. Let $Pm = \phi$ and $Zm = \phi'$.

By Napier's rules, we have

$$\left.\begin{aligned} \tan\phi &= \cot d \cos t, \\ \cos\phi' &= \cos\phi \sin h \operatorname{cosec} d, \\ L &= 90° - (\phi \pm \phi'). \end{aligned}\right\} \qquad (206)$$

Following Chauvenet's methods, the above can be put into a more convenient form.

If for ϕ in the above, 90° — ϕ'' be substituted, then

$$\left.\begin{aligned} \tan\phi'' &= \tan d \sec t, \\ \cos\phi' &= \sin\phi'' \sin h \operatorname{cosec} d, \\ L &= \phi'' \mp \phi'. \end{aligned}\right\} \qquad (207)$$

Case of M, $d +$ and $> L$.

From the figure it is seen that

$\phi = Pm$, the polar distance of m, the foot of the perpendicular;

$\phi'' = Qm$, the declination of m, the foot of the perpendicular;

$\phi' = mZ$, the zenith distance of m, the foot of the perpendicular;

$L = QZ$, the declination of the zenith.

But $QZ = Qm - mZ$ or $L = \phi'' - \phi'$.

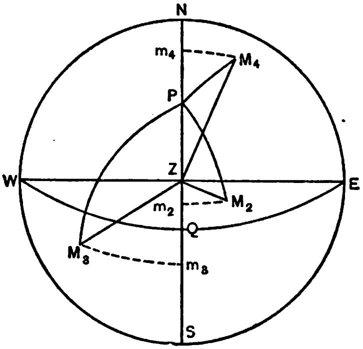


FIG. 116.

As shown in the figure (115), the declination of M is positive and $> L$, and finding the latitude in this case resolves itself into the finding of the declination and zenith distance on the meridian of the foot of the perpendicular and combining them by the rules applying to a similar case in finding latitude from the meridian altitude of a body.

Case of M_2, $d +$ and $< L$.

In triangle PZM_2 (Fig. 116), $\phi'' = Qm_2$ and $\phi' = m_2Z$.

$$QZ = Qm_2 + m_2Z \text{ or } L = \phi'' + \phi'.$$

Case of M_3, d is negative.

In triangle PZM_3 (Fig. 116), $\phi'' = Qm_3$ and $\phi' = m_3Z$.

$$QZ = m_3Z - Qm_3 \text{ or } L = \phi' - \phi''.$$

Case of M_4, $t > 6$ hours, $d +$.

In triangle PZM_4, $\phi'' = Qm_4$ and $\phi' = m_4Z$.

$$QZ = Qm_4 - m_4Z = \phi'' - \phi'.$$

In this case of M_4, t is > 6 hours and d is $+$; therefore, Qm_4 is taken out $+$, same sign or name as d, but in the second quadrant—or same quadrant as t.

Now,
$$\begin{aligned} Qm_4 &= \phi'' = 90° + Pm_4 = 90° + p, \\ \underline{m_4Z} &\underline{= \phi'\ = 90° - Nm_4 = 90° - h,} \\ QZ = Qm_4 - m_4Z &= \phi'' - \phi' = NP = p + h = L. \end{aligned}$$

Therefore this case corresponds to that of a body observed on the meridian below the pole.

As ϕ' is found from its cosine it may be either $+$ or $-$, thus giving two values of L, differing largely from each other, unless ϕ' is small. However, the latitude is approximately known, and no trouble need be experienced in determining how to mark ϕ'.

The following rules, closely attended to, will prevent any error in the proper marking, or the method of combining ϕ'' and ϕ' to obtain the latitude.

(1) *The mere fact of* t *being* $+$ *(W.), or* $-$ *(E.), has no influence on the signs of the functions. If* t *is* > 6 *hours* sec t *is* ($-$) *and* ϕ'' *is in the second quadrant. Therefore,*

(2) *In formulæ (207) ϕ'' is taken out in the same quadrant as t and is marked N. or S. like the declination. ϕ', being the zenith distance of m, is marked like the zenith distance of the body in a meridian observation for latitude; that is, if the body bears northerly, mark the zenith distance S., if it bears southerly, mark the zenith distance N. Then combine ϕ'' and ϕ' algebraically according to their names. The result will be the latitude.*

Under the following conditions this method is not conducive to accuracy, or fails entirely.

(1) When ϕ' is very small, that is, when Z is near 90° and the body is near the prime vertical, it cannot be found accurately from the cosine.

(2) When d is 0, ϕ'' is 0, ϕ' is indeterminate, and the latitude cannot be found by this method.

Observations of the sun, planets, or fixed stars, worked for latitude by this method give excellent results; owing to the very rapid changes of the moon's elements, and the uncertainties of the hour angle due to the uncertainties of longitude, observations of the moon are not recommended.

Rules for Working a $\phi''\phi'$ Sight.

(1) *Find the G. M. T. of observation for which in the case of the sun, take from the Nautical Almanac the declination and equation of time; or, in the case of any other body, its right ascension and declination and the right ascension of the mean sun, and also, if the moon has been observed, its semi-diameter and horizontal parallax; and reduce the sextant altitude to the true altitude of the center.*

(2) *Find the body's t, then having t, d, and h, proceed by substitution in formulæ (207) to find ϕ'' and ϕ', paying particular attention to the rules preceding regarding the sign of t, and the naming and combining ϕ'' and ϕ' to obtain the latitude.*

Ex. 185.—April 7, 1918, a. m., in longitude 45° 30′ W., the sextant altitude of the sun's lower limb, bearing N^d^ and E^d^ was 33° 48′ 40″. W. T. of obs. $10^h\ 12^m\ 06^s$. C—W $3^h\ 10^m\ 06^s$. Chro. fast of G. M. T. $5^m\ 56^s$. I. C.—1′. Height of eye 21 feet. Required the latitude ($\phi''\ \phi'$).

Times.		Altitude.		Altitude Corrections.		Sun's Declination.		H. D.		Equation of Time.		H. D.	
	h m s		° ′ ″		′ ″		° ′			— to M. T.			
W.	10 12 06	⊙	33 48 40	S. D.	+16 00	At G.M.N.	N 6 37.5		N 0′.9		m s		—$0^s.7$
C—W	3 10 06	Corr.	+ 9 12	I. C.	— 1 00	Corr.	N 1.1	G. M. T.	$1^h.27$		2 20.1	G.M.T.	$1^h.27$
				Dip	— 4 29					Corr.	— .9		
C.	1 22 12	⦵	33 57 52	p. & R.	— 1 19	Dec.	N 6 38.6	Corr.	N 1′.14			Corr.	$0^s.889$
C. C.	— 5 56										2 19.2		
				Corr.	+ 9 12								
G. M. T. April 7	1 16 16			Or (Tab. 46)	+10′ 12″								
Eq. of T.	— 2 19.2			I. C.	— 1 00								
G. A. T.	1 13 56.8			Corr.	+ 9 12								
Long. W	8 02				° ′ ″								
				h=	33 57 52			sin	9.74717				
L. A. T.	22 11 56.8			d=	6 38 36 N	tan	9.06624	cosec	10.93669				
t	E. 1 48 03.2			t=	27 00 48	sec	10.05017						
	=27° 00′ 48″												
				ϕ''=	7 26 55 N	tan	9.11641	sin	9.11273				
				ϕ'=	51 14 34 S			cos	9.79659				
				Lat.=	43 47 49 S								

Ex. 186.—January 19, 1918, a. m., in longitude 54° 56′ 21″ W., during moonlight, the sextant altitude of the star α Canis Minoris (Procyon) bearing S^d^ and W^d^ was 48° 55′ 30″. I. C. + 2′. Height of eye 45 feet. W. T. of obs. $12^h\ 43^m\ 24^s$. C—W $3^h\ 48^m\ 16^s$. Chro. fast of G. M. T. $2^m\ 25^s$. Required the latitude (φ″ φ′).

Times.		Altitudes.		Altitude Corrections.		R. A. M. ⊙.		Star's R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		h m s
W.	12 43 24	✱'s h	48 55 30	I. C.	+ 2 00	At G. M. N.	19 48 19.9	✱'s R. A.	7 35 03.2
C—W	3 48 16	Corr.	− 5 27	Dip	− 6 36	Corr. G. M. T.	+ 2 42.5		° ′
				R.	− 0 51			✱'s Dec.	N 5 26.0
C.	4 31 40	✱'s h	48 50 03				19 51 02.4		
C. C.	− 2 25			Corr.	− 5 27				
				Or (Tab. 46, Bowditch)	= − 7′ 27″				
G. M. T. Jan. 18,	16 29 15			I. C.	= + 2 00				
R. A. M. ⊙	19 51 02.4								
				Corr.	− 5 27				
G. S. T.	12 20 17.4				° ′ ″				
λ W	3 39 45.4			h=	48 50 03			sin	9.87669
				d=	5 26 00 N	tan	8.97825	cosec	11.02371
L. S. T.	8 40 32.0			t=	16 22 12	sec	10.01797		
✱'s R. A.	7 35 03.2			φ″=	5 39 40 N	tan	8.99622	sin	8.99410
				φ′=	38 20 30 N			cos	9.89450
t	+ 1 05 28.8								
or	16° 22′ 12″			Lat.	44 00 10 N				

250. Second method.—When the latitude is approximately known.

Taking the fundamental equation

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

and, substituting for cos t its equivalent $1-2 \sin^2 \frac{1}{2} t$, we have

$$\begin{aligned}\sin h &= \sin L \sin d + \cos L \cos d - 2 \cos L \cos d \sin^2 \tfrac{1}{2} t\\ &= \cos (L \sim d) - 2 \cos L \cos d \sin^2 \tfrac{1}{2} t,\end{aligned}$$

but $(L \sim d) = z_0 = 90° - h_0$ where h_0 represents the meridian altitude of the body at some place in the same latitude as the observer at the same instant when the body's declination is d.

$$\text{Therefore, } \sin h = \sin h_0 - 2 \cos L \cos d \sin^2 \tfrac{1}{2} t$$

$$\text{and } \sin h_0 = \sin h + 2 \cos L \cos d \sin^2 \tfrac{1}{2} t. \qquad (208)$$

The approximate latitude is used in finding the value of $2 \cos L \cos d \sin^2 \frac{1}{2} t$, and then from the formula an approximate value of the meridian altitude is computed. Having found the declination for the Greenwich instant of observation, the latitude is then found as in the case of a meridian observation (Art. 240).

The nearer the body is to the meridian at the time of observation, the more correct will be the resulting latitude. Observing at equal altitudes on opposite sides of the meridian will eliminate effect of errors in time.

If the computed latitude differs largely from the assumed approximate latitude, repeat the computation, using in the formula the computed latitude. It is seldom necessary to repeat the computation more than once. In the example that follows, 30′ more of assumed latitude would have increased the computed latitude only 27″.

Reference has been made in Art. 219 to the use of a formula involving haversines in finding Z, and in Art. 226 to a similar formula for finding t and thence the longitude. Since

Ex. 187.—April 5, 1918, a. m., in longitude 187° 30′ E., and approximate latitude 31° 20′ S., the sextant altitude of the sun's lower limb bearing northward and eastward, was 51° 35′. I. C. −2′ 20″. Height of eye 21 feet. W. T. of obs. 11h 20m 27s. C—W 2h 55m 00s. Chro. fast of G. M. T. 2m 20s. Find the latitude (2d method).

Times.		Altitudes.		Altitude Corrections.		Sun's Dec.		H. D.		Equation of Time.		H. D.	
	h m s		° ′ ″		′ ″		° ′			— to M. T.			
W.	11 20 27	⊙	51 35 00	S. D.	+16 01	At 14h	N 5 42.6		N 1′.0		m s		—0s.7
C—W	2 55 00	Corr.	+ 8 32	I. C.	− 2 20	Corr.	N 0.2	G. M. T.	0h.22	At 14h	3 02.1	G. M. T.	0h.22
				Dip	− 4 29					Corr.	− .2		
C.	2 15 27	⦵	51 43 32	p. & R.	− 0 40	Dec.	N 5 42.8	Corr.	N 0′.22			Corr.	—0s.154
C. C.	− 2 20									Eq. of T.	3 01.9		
				Corr.	+ 8 32								
				Or (Tab. 46)									
G. M. T. April 4	14 13 07				= +10′ 50″								
				I. C.	= − 2 20								
or	14h.22				° ′ ″								
G. M. T.	14 13 07			$\frac{t}{2}$=	4 59 21.8	sin	8.93938			2 sin	7.87876		
Eq. of T.	− 3 01.9			2						log	0.30103		
G. A. T.	14 10 05.1			L=	31 20 S					cos	9.93154		
λ East	− 9 10			d=	5 42 48 N					cos	9.99783		
							.01286			log	8.10916		
L. A. T.	23 20 05.1			h=	51 43 32	Nat sin	.78506						
	h m s												
t	E. 0 39 54.9			h_0=	52 55 57 N	Nat sin	.79792						
$\frac{t}{2}$	4° 59′ 21″.8			z_0=	37 04 03 S								
				d=	5 42 48 N								
				Lat.=	31 21 15 S								

navigators are now supplied with haversines in Table 45, Bowditch, the following formula for latitude is recommended as being simpler than the one above used, by one step in the calculation.

Taking the fundamental formula,

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$

and substituting for cos t its equivalent $1 - \text{versin } t$, and for sin h, its equivalent cos z, we have

$$\cos z = \cos (L \sim d) - \cos L \cos d \text{ versin } t;$$

but $\cos (L \sim d) = \cos z_0$ when z_0 is the meridian zenith distance; therefore,

$$\cos z = \cos z_0 - \cos L \cos d \text{ versin } t.$$

Taking each side from unity,

$$1 - \cos z = 1 - \cos z_0 + \cos L \cos d \text{ versin } t,$$

or $$\text{versin } z = \text{versin } z_0 + \cos L \cos d \text{ versin } t,$$

$$\text{versin } z_0 = \text{versin } z - \cos L \cos d \text{ versin } t,$$

or $$\text{haver } z_0 = \text{haver } z - \cos L \cos d \text{ haver } t.$$

Now if we let

$$\text{haver } \theta = \cos L \cos d \text{ haver } t \text{ (to determine } \theta), \qquad (209)$$

then $$\text{haver } z_0 = \text{haver } z - \text{haver } \theta. \qquad (210)$$

Having found the meridian zenith distance, proceed as above.

251. Third method.—Reduction to the meridian.—When an observation of a heavenly body is taken very near the meridian, and t, the hour angle of the body East or West, is known, the observed altitude may be reduced to what would be the meridian altitude at the same place by applying a correction called "The Reduction to the Meridian," the declination of the body being assumed the same at the time of observation and when on the meridian. In the American naval service the method is known as that of "The Reduction to the Meridian"; in the British navy as that of "The Ex-Meridian."

Ex. 187a.—With the data of example 187, it is required to find latitude by the haversine formulæ (209 and 210).

$$\text{haver } \theta = \cos L \cos d \text{ haver } t,$$
$$\text{haver } z_0 = \text{haver } z - \text{haver } \theta,$$
$$L = z_0 + d.$$

Times.		Altitude.		Altitude Corrections.		Sun's Dec.		H. D.		Equation of Time.		H. D.	
	h m s		° ′ ″		′ ″		° ′						
W.	11 20 27	⊙	51 35 00	S. D.	+16 01	At 14^h,	N 5 42.6		N 1′.0	(−) to M. T.			(−) 0^s.7
C—W	2 55 00	Corr.	+ 8 32	I. C.	− 2 20	Corr.	N 0.2	G. M. T.	0^h.22		m s		0^h.22
				Dip	− 4 29						3 02.1		
C.	2 15 27	⊖	51 43 32	p. & R.	− 0 40	Dec.	N 5 42.8	Corr.	N 0′.22	Corr.	− .2	Corr.	−0^s.154
C. C.	(−) 2 20												
				Corr.	+ 8 32						3 01.9		
G. M. T.				Or (Tab. 46,									
April 4,	14 13 07			Bowditch)									
or	14^h.22				+10′ 50″								
G. M. T.	14 13 07			I. C.	− 2 20								
Eq. of T.	− 3 01.9												
				Corr.	+ 8 30								
G. A. T.	14 10 05.1				° ′ ″								
Long. E.	9 10 00				$L=$ 31 20 S		cos		9.93154				
					$d=$ 5 42 48 N		cos		9.99783				
L. A. T.	23 20 05.1				$t=$ 9 58 43.5		haver		7.87875				
$t=$	E 0 39 54.9												
$t=$	E 9° 58′ 43″.5				θ		haver		7.80812	Nat haver	.00643		
					$z=(90-h)=$ 38 16 28					Nat haver	.10748		
					$z_0=$ 37 04 15 S					Nat haver	.10105		
					$d=$ 5 42 48 N								
					Lat. = 31 21 27 S								

To find the Reduction to the Meridian.

From (208)

$$\sin h_0 = \sin h + 2 \cos L \cos d \sin^2 \tfrac{1}{2} t,$$

$$\text{or } \sin h_0 - \sin h = 2 \cos L \cos d \sin^2 \tfrac{1}{2} t.$$

From trigonometry,

$$\sin x - \sin y = 2 \cos \tfrac{1}{2} (x + y) \sin \tfrac{1}{2} (x - y).$$

Therefore,

$$\cos \tfrac{1}{2} (h_0 + h) \sin \tfrac{1}{2} (h_0 - h) = \cos L \cos d \sin^2 \tfrac{1}{2} t.$$

Now h_0 and h differ but little; therefore, $\cos \tfrac{1}{2} (h_0 + h)$ may be considered as $\cos h_0 = \sin (L \sim d)$.

Letting Δh be $h_0 - h$, which is the "Reduction to the Meridian" desired, we have, by substitution,

$$\sin \tfrac{1}{2} \Delta h = \cos L \cos d \sin^2 \tfrac{1}{2} t \operatorname{cosec} (L \sim d).$$

Since the body is near the meridian, Δh and Δt are assumed to be small, and we have

$\sin \tfrac{1}{2} \Delta h = \tfrac{1}{2} \Delta h'' \sin 1''$ (Δh expressed in seconds of arc),

$\sin \tfrac{1}{2} t \quad = \tfrac{1}{2} (15t) \times \sin 1''$ (t expressed in seconds of time);

and, by substitution in the preceding equation,

$$\Delta h'' = 112.5\, t^2 \cos L \cos d \operatorname{cosec} (L \sim d) \sin 1''.$$

In the above, t is in seconds of time, but if we wish the hour angle to be expressed in minutes of time, we must substitute $60t$ for t in the equation.

Therefore,

$$\Delta h'' = 112.5\, (60t)^2 \times .000004848 \cos L \cos d \operatorname{cosec} (L \sim d).$$

$$\Delta h'' = 1''.96349\, t^2 \cos L \cos d \operatorname{cosec} (L \sim d). \qquad (211)$$

If t is one minute,

$$\Delta_0 h'' = 1''.96349 \cos L \cos d \operatorname{cosec} (L \sim d). \qquad (212)$$

Then the meridian altitude is

$$h_0 = h + \Delta h'' = h + \Delta_0 h'' t^2, \qquad (213)$$

in which h_0 is the meridian altitude of the center of the body;

h, the true altitude at observation;

$\Delta h''$, the "Reduction to the Meridian" in seconds of arc;

$\Delta_0 h$, the change of altitude expressed in seconds of arc for one minute of time from the meridian;

t, the body's hour angle expressed in units of its own time.

The sign of application of Δh is always positive to an observed altitude near upper transit, negative to one near lower transit.

To find the Hour Angle *t*.

Find the watch time of the body's transit (Art. 198); subtracting from this the watch time of observation, the result will be the mean time interval between transit and observation. The difference between apparent and mean time intervals is so small that the mean time interval may be taken as the sun's hour angle without correction. As a fixed star's hour angle is expressed in sidereal units, in the case of a star observation, the above mean time interval must be converted to a sidereal time interval to give the star's hour angle at the time of observation.

In the case of a planet, we may disregard the slight change in right ascension and take the sidereal interval as its hour angle.

In the case of the moon, owing to the rapid change of its right ascension, the above method will not do, and the hour angle must be found in lunar units. To do this, find the G. M. T., then the L. S. T., corresponding to the watch time of observation (see Art. 199 (b)); the difference between this L. S. T. and the moon's right ascension for the Greenwich instant of observation will be the required hour angle of the moon. However, owing to the rapid changes of the moon's right ascension and declination, observations of the moon for reduction to the meridian are not recommended.

To find $\Delta_0 h$.

It may be found in Table 26, Bowditch, where it is tabulated for each degree of latitude from 0° to 60° and each degree of declination from 0° to 63°, there being a table for the case when the declinations are of the same name, also of a different name from the latitude. No values are given when $L \sim d$ is $< 4°$, or, in other words, when the altitudes are above 86°, as the method is inapplicable when the body transits so near the zenith. Furthermore, no values are given in those cases where a body's altitude would be less than 6°, as such altitudes themselves, owing to the uncertainties of refraction, are unreliable.

It may also be found in Table III of the Ex-Meridian Tables of Brent, Walter, and Williams, under the designation "C."

Table 26 gives $\Delta_0 h$ to the nearest tenth of a second of arc only; if a closer approximation is desired, $\Delta_0 h$ must be computed by the formula

$$\Delta_0 h = 1''.96349 \cos L \cos d \operatorname{cosec} (L \sim d).$$

If d is of a different name from L, $(L \sim d)$ becomes numerically the sum of the two.

If $\Delta_0 h$ is desired for any case not tabulated, it may be computed by the above formula.

To find Δh.

In (211), $\Delta h''$ is in seconds of arc and represents the change in the altitude near the meridian for t minutes of hour angle expressed in time.

The value of Δh expressed in minutes and seconds of arc may be found from Table 27, Bowditch, the arguments being

t (in minutes and seconds of time) and $\Delta_0 h$ (in seconds of arc). The larger the value of $\Delta_0 h$, the smaller is the limit of t; thus for a value of $\Delta_0 h = 2''.0$, Δh is given for t as great as 26 minutes, while for a value of $\Delta_0 h = 28''$, it is given only for values of t of and less than 5 minutes. It may also be found from Table IV of Brent, Walter, and Williams' Ex-Meridian Tables.

Limit of H. A.—The following most excellent rule is given in "Wrinkles in Navigation": "The hour angle of the sun or time from noon in minutes should not exceed the number of degrees in the sun's meridian zenith distance." This rule is made general by saying, "The hour angle in minutes from the meridian of a body observed for latitude by reduction to the meridian should not exceed the number of degrees in its meridian zenith distance." Chauvenet has shown that so long as the zenith distance is not greater than 10°, the reduction computed as above may amount to as much as 4′ 30″ without being in error more than 1″. The rule should not be made applicable to circumpolar stars in whose cases, the limits of H. A. may be greatly extended.

As a practical rule, in all cases when the time from the meridian transit exceeds the limit laid down in Table 27, Bowditch, it would be better to find the latitude by the $\phi''\phi'$ *method.*

Limits of Table 27.—The limits of this table are the limits within which the method may be used with a fair degree of confidence in the accuracy of results.

The use of the table depends on the object sought; for instance, in determining latitude, when surveying, the hour angles of bodies observed should be so small that the value of Δh itself should not exceed 1′; on the high seas, the reduction obtained under conditions in which even the limits of the table are used will be sufficiently exact. Again the table may be used at sea beyond its limits in the following way, if this use is desired: Suppose $\Delta_0 h = 4''.0$ and $t = 24$ minutes; since the table does not give Δh for t beyond 21 minutes, find the reduction for 12 minutes and multiply it by 4 to obtain Δh for 24 minutes. Therefore, the required $\Delta h = (9' \, 36'') \times 4 = 38' \, 24''$. Proof of this is seen from the fact that

$$\Delta_0 h t^2 = \Delta_0 h \left(\frac{t}{2}\right)^2 \times 4.$$

Hence, if the hour angle is greater than the tabulated one for the given value of $\Delta_0 h$, *take out the correction* Δh *for one-half the H. A. and multiply it by 4; the result will be the required* Δh.

Restrictions in the tropics.—In the tropics, where at transit the body's altitude may approach 90°, the factor

$$\operatorname{cosec}\,(L \sim d)$$

will be so large as to make Δh too great for the assumption made in the deduction, that $\sin \frac{1}{2}\,\Delta h = \frac{1}{2}\,\Delta h \sin 1''$. For such cases the value of $\Delta_0 h$ is not tabulated. In those regions, therefore, in summer time, this method is not applicable; however, it is not much needed owing to the strong probability of the sun being visible when on the meridian.

To Find the Declination and Latitude.

The declination to be combined with the meridian altitude h_0 should properly be corrected for the G. M. T. of observation; in the case of the moon this is essential; in the cases of a planet or the sun, it is sufficiently accurate to use the declination at the instant of meridian transit, except when the hour angle is large, and in the case of the sun, therefore, the declination may be corrected for the longitude at upper transit and for (12 hours $+ \lambda$) at the lower transit.

Having found h_0 and d, the latitude is found as in the case of a meridian altitude (Art. 240).

It must not be forgotten that the latitude thus found is for the instant of observation, and that the latitude at the time *of transit (or in the case of a sun sight, near the upper meridian, the latitude at noon) may be found by applying the run in the interval.*

Errors in H. A.—The effect of errors in H. A. may be minimized by observing the body at practically the same altitude and with small hour angles on opposite sides of the meridian, reducing the latitudes found to noon and taking the mean of results for the true noon latitude.

Various "Reduction" or "Ex-Meridian Tables."—The Tables of Bowditch and of Brent, Walter, and Williams, which are practically identical, have been referred to. The arguments in these Tables are L, d, and t; so the navigator, having set his watch to L. A. T., may have in his note-book the corrections to be applied to altitudes to be observed at certain times by the watch to obtain the meridian altitudes, in fact have ready a constant allowing for the run to noon (see Art. 253), so that the noon latitude may be found at once by applying this constant to the observed altitude. Besides, the above-mentioned tables are applicable to bodies of a declination as large as 63°.

Towson's Tables are also issued to the American navy. They are not applicable to bodies whose declination may be greater than 23° 20′. The arguments used in these tables are d, h, and t, so that the correction is taken out after the altitude has been observed; a matter of delay if not of inconvenience.

The "Ex-Meridian Tables" of Captain Armistead Rust, U. S. N., are very convenient, as they give the value of Δh at a single opening, the arguments being latitude, declination and hour angle. These tables cover latitudes 0° to 65° and declinations 0° to 71° and, by the use of a second correction obtained from a simple diagram, are made available over a much wider range of hour angles than the tables of Bowditch.

There have been various graphic and automatic methods for finding the value of the "Reduction to the Meridian," the best of which perhaps is the invention of Wm. Hall, Naval Instructor, R. N., and which consists of two calculating slides for automatic calculation. It is known as "Hall's Nautical Slide Rule."

Rules.—(1) *Find the watch time of transit* (*Art. 198*), *and the H. A. from the meridian, remembering it is to be in sidereal time for a fixed star, and that for the sun the mean time interval may be used.* (2) *Take from the Nautical Almanac the declination for the instant of transit, or in the case of the sun, for local apparent noon* (*if sun was observed near lower transit, for local apparent midnight*). (3) *From Table 26, Bowditch, take out* $\Delta_0 h$ *and from Table 27,* Δh. (4) *Reduce the sextant altitude to a true altitude of the center and apply* Δh*; adding, when the body was observed near upper transit, subtracting, for an altitude near lower transit. The result will be the meridian altitude.* (5) *Then proceed to find latitude as in Art. 240.*

Attention is called to the fact that formula (*212*) *is made applicable to the case of a body near lower transit either by substituting* $180° - d$ *for* d*, or, by substituting* $-L$ *for* L *since the lower transit of the meridian in a given latitude is the upper transit of the same meridian at the antipode; hence for a body below the pole take* $\Delta_0 h$ *from that part of Table 26 in which* L *and* d *are of different names* (*see bottom of last three pages of Table 26*).

Ex. 185.—Jan. 1, 1918, in Long. 64° 06′ 30″ W., Lat. by D. R. 16° 45′ N., the sextant altitude of the sun's lower limb, near noon, bearing Southerly, was 49° 55′ 30″. I. C.+1′ 00″. Height of eye 45 feet. W. T. of obs. $11^h\ 49^m\ 00^s$. C—W $4^h\ 18^m\ 52^s$. Chro. slow at G. M. T. $2^m\ 05^s.3$. Find the latitude.

W. T. of L. A. Noon and H. A.		Altitude, &c.		Corrections to Altitude.		Declination.		H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″		° ′			
λ=G.A.T. of L.A. noon	4 16 26	$\underline{\odot}$	49 55 30	S. D.	+16 18	At 4^h, Jan. 1	S 23 02.3	N 0′.2	+toApp.T	$+1^s.2$
Eq. of time	+ 3 31.3	Corr.	+ 9 59	I. C.	+ 1 00	Corr.	N .1	G.M.T. $0^h.33$	m s 4^h=3 31.0	$0^h.27$
				Dip	− 6 36				+ .3	
G. M. T. of noon	4 19 57.3	⊖	50 05 29	p. & R.	− 0 43	At L.A. noon	S 23 02.2	Corr.		$0^s.324$
Chro. slow	− 2 05.3	Δ_0ht^2	+ 4 32					N 0′.066	3 31.3	
				Corr.	+ 9 59					
C. T. of noon	4 17 52	h_0	50 10 01 S	Or (Tab. 46, Bow-						
C—W	4 18 52	z_0	39 49 59 N	ditch) =	+8′ 59″					
		d	23 02 12 S	I. C.	+1 00					
W. T. of noon	11 59 00									
W. T. of obs.	11 49 00	Lat.	16 47 47 N	Corr.	+9 59					
$t=-$	10									

$\Delta_0h=2''.72$ (Tab. 26)

$\Delta_0ht^2=4'\ 32''$ (Tab. 27)

Ex. 189.—April 20, 1918, p. m., during moonlight, in Long. 40° 30′ W., Lat. by D. R. 54° N., the sextant altitude of the star α Leonis (Regulus) near the meridian, bearing Southerly, was 48° 25′ 10″. I. C. + 1′ 00″. Height of eye 45 feet. W. T. of obs. $7^h\ 59^m\ 25^s$. C—W $2^h\ 37^m\ 56^s$. Chro. slow of G. M. T. $3^m\ 02^s.3$. Required the latitude.

W. T. of Transit and *t*.		Altitudes, &c.		Altitude Corrections.		✳'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s
R. A. M. ⊙ at G. M. N.	1 51 02.9	Sextant alt. ✳	48 25 10	I. C.	+ 1 00	✳'s R. A.	10 04 03.0
Red. for λ, Tab. III	+ 26.6	Corr.	− 6 28	Dip	− 6 36		° ′
				Ref.	− 0 52	✳'s *d*	12 21.9 N
Sid. time local 0 hrs.	1 51 29.5	✳'s alt.	48 18 42 S				
✳'s R. A. = L. S. T.	10 04 03.0	$\Delta_0 ht^2$	+ 4 41	Corr.	− 6 28		
				or (Tab. 46, Bow-			
Sid. int. from noon	8 12 33.5	✳'s h_0	48 23 23 S	ditch)	= −7′ 28″		
Reduction, Tab. II	− 1 20.7	✳'s z_0	41 36 37 N	I. C.	= +1 00		
		✳'s *d*	12 21 54 N				
L. M. T. April 20,	8 11 12.8			Corr.	−6 28		
Long. W.	+2 42 00	Lat.	53 58 31 N				
G. M. T. of local tr.	10 53 12.8						
Chro. slow	− 3 02.3						
C. T. of transit	10 50 10.5						
C—W	2 37 56						
W. T. of transit	8 12 14.5						
W. T. of obs.	7 59 25						
t mean time	− 12 49.5						
t sidereal time	− 12 51.6						

$\Delta_0 h = 1''.7$, $\Delta_0 ht^2$ (Tab. 27) 4′ 41″.

Ex. 190.—April 4, 1918, a. m., during morning twilight, in Long. 60° E., the sextant altitude of *a* Argus (Canopus) near lower transit was 8° 50′ 30″. I. C.+2′ 00″. Height of eye 45 feet. Lat. by D. R. 46° S. W. T. of obs. $5^h\ 27^m\ 12^s$. C—W $8^h\ 05^m\ 13^s$. Chro. fast of G. M. T. $2^m\ 01^s.6$. Required the latitude.

W. T. of Transit and *t*.		Altitudes, &c.		Altitude Corrections.		✳'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s
April 3, R. A. M. ☉ at G. M. N.	0 44 01.5	Sextant alt. ✳	8 50 30	I. C.	+ 2 00	✳'s R. A.	6 22 08.4
Red. for λ East, Tab. III	− 39.4	Corr.	− 10 35	Dip	− 6 36	12 hrs.+R. A.	18 22 08.4
				Ref.	− 5 59		° ′
Sid. time local 0 hrs.	0 43 22.1	✳'s *h*	8 39 55			✳'s *d*	S 52 39.4
✳'s R. A.+12 hrs.=L. S. T.	18 22 08.4	$\Delta_0 ht^2$	− 0 25	Corr.	−10 35	✳'s *p*	37 20.6
				or (Tab. 46)			
Sid. int. from noon	17 38 46.3	✳'s h_0	8 39 30		−12′ 35″		
Red., Tab. II	− 2 53.5	✳'s *p*	37 20 36	I. C.	+ 2 00		
L. M. T. of lower transit	17 35 52.8	Lat.	46 00 06 S	Corr.	−10 35		
Long. East	−4 00 00						
G. M. T. of lower transit	13 35 52.8						
Chro. fast	+ 2 01.6						
C. T. of lower transit	1 37 54.4						
C—W	8 05 13						
W. T. of lower transit	5 32 41.4						
W. T. of observation	5 27 12						
t { mean time	− 5 29.4						
t { sidereal time	− 5 30.3						
$\Delta_0 h$ 0″.83, $\Delta_0 ht^2$ 0′ 25″ (Tab. 27).							

The L. S. T. of lower transit is 12 hrs.+ ✳'s R. A. The latitude must have the same name as ✳'s declination for the star to be seen at the lower culmination.

252. To find the latitude from a number of altitudes of a heavenly body observed very near the meridian, the longitude and Greenwich times being known.—Very near the meridian, the change of altitude varies nearly as the square of the hour angle, so that the mean of the altitudes cannot be taken as corresponding to the mean of the times, but each altitude may be reduced to the meridian by the principles of Art. 251, and the mean of these used in finding the latitude, hence the term

Circummeridian Altitudes.

Let $h_1, h_2, h_3 \ldots\ldots h_n$ be the several true altitudes;

$t_1, t_2, t_3 \ldots\ldots\ldots t_n$ be the corresponding hour angles in minutes of time at the times of observations;

$\Delta_1 h, \Delta_2 h, \Delta_3 h \ldots \Delta_n h$ be the several reductions to the meridian;

where $\Delta_1 h = \Delta_0 h t^2{}_1$, $\Delta_2 h = \Delta_0 h t^2{}_2$, and $\Delta_n h = \Delta_0 h t^2{}_n$.

Then for n observations, the mean value of the meridian altitude will be

$$h_0 = \frac{h_1 + h_2 + h_3 \ldots\ldots h_n}{n} + \frac{\Delta_1 h + \Delta_2 h + \Delta_3 h \ldots\ldots \Delta_n h}{n}.$$

Substituting the values of the reductions as above,

$$h_0 = \frac{h_1 + h_2 + h_3 \ldots\ldots h_n}{n} + \left(\frac{t^2{}_1 + t^2{}_2 + t^2{}_3 \ldots\ldots t^2{}_n}{n}\right) \Delta_0 h. \tag{214}$$

From this value of h_0, the meridian altitude, the latitude is computed. The principles involved in this method suppose the declination not to change from the time of observation till the meridian passage, and as the declination for that instant is wanted to combine with what would be the meridian altitude, it is better to find the Greenwich time of passage (see Art. 196), and for this take out the body's declination. Then find the watch time of passage (Art. 198), the differences between which and the watch times of observation will be the hour angles of the body expressed in mean time. The mean time interval differs from the apparent time interval only by the change in the equation of time in the interval; so in the case of the sun the mean time intervals need not be reduced.

In the case of a fixed star, they must be converted into sidereal time intervals.

The values of refraction and parallax for the various altitudes will differ so slightly, that it will be sufficient to reduce the mean of the sextant altitudes to a true altitude, to which the reduction will be applied to give the true meridian altitude.

When possible, altitudes at about the same hour angles should be taken on both sides of the meridian in order to eliminate errors due to the time.

The best results are gotten when two stars culminating at about the same altitude are observed on opposite sides of the zenith; for, by taking a mean of the two latitudes thus obtained, personal and instrumental errors, if the instruments are used in the same way and under like conditions, are eliminated. In using this method on shore, if prismatic effect is suspected in the roof of the artificial horizon, it would be better to take two sets of observations, the roof being reversed between the sets.

At sea, single observations near the meridian are sufficient and $\Delta_0 h$ from the tables are accurate enough; but for refined determinations of latitude on land, it is better to take a number of observations near transit, on both sides of the meridian, using a bright star in preference to the sun, and computing the value of $\Delta_0 h$.

The barometer and thermometer should be noted during observations ashore, and a correction (Tables 21 and 22, Bowditch), dependent upon the instrumental indications, should be applied to the mean refraction.

Ex. 191.—Jan. 10, 1918, p. m., at Crocodile Cove, Isle of Pines, Long. $5^h\ 32^m\ 15^s$ W., approximate latitude 21° 30′ N., observed the following circummeridian altitudes of the star α Canis Majoris (Sirius), bearing S. for latitude, using artificial horizon and sextant. I. C. −3′. Watch times as noted below in form. C—W $5^h\ 33^m\ 30^s$. Chro. fast of G. M. T. $3^m\ 06^s.7$. Required the latitude.

W. T. of Transit.		W Ts. of Obs.	Mean Time Interval t.	Sidereal Time Interval t_1.	Tab. 27 $\Delta_0 h t_1^2$.	Double Altitudes.	Mer. Alt. and Lat.		Star's R.A. & Dec.	
	h m s	h m s	m s	m s	′ ″	° ′ ″		° ′ ″		h m s
Jan. 10 R. A. M. ⊙	19 16 47.4	11 16 22	− 7 30	− 7 31	2 39	103 51 20	✳'s $2h_s$	103 50 50	✳'s R. A.	6 41 34.3
Tab. III Red. λ	+ 54.6	11 19 52	− 4 01	− 4 01	0 46	103 55 10	I. C.	− 3 00	✳'s d	16° 36′.3 S
		11 22 22	− 1 30	− 1 30	0 06	103 56 20	2	103 47 50		
Sid. time local 0 hrs.	19 17 42.0	11 26 02	+ 2 10	+ 2 10	0 13	103 56 10	✳'s h'	51 53 55		
R. A. of Sirius = L. S. T.	6 41 34.3	11 27 57	+ 4 05	+ 4 06	0 48	103 55 00	Ref	− 46		
		11 30 54	+ 7 02	+ 7 03	2 20	103 52 00				
Sid. interval	11 23 52.3	11 34 22	+10 30	+10 32	5 12	103 46 10	✳'s h	51 53 09 S		
Tab. II Red. to M. T. I.	− 1 52.0	11 39 12	+15 20	+15 23	11 06	103 34 30	$\Delta_0 h t^2$	+ 2 54		
		$\Delta_0 h = 2''.82$.		Mean val. $\Delta_0 h t_1^2$	2 54	103 50 50				
L. M. T. of transit	11 22 00.3						✳'s h_0	51 56 03 S		
Long. West	+ 5 32 15						✳'s z_0	38 03 57 N		
							✳'s d	16 36 18 S		
G. M. T. of transit	16 54 15.3									
Chro. fast on G. M. T.	+ 3 06.7						Lat.	21 27 39 N		
C. T. ✳'s transit	4 57 22									
C—W	5 33 30									
W. T. of ✳'s transit	11 23 52									

Ex. 192.—Jan. 30, 1918, in longitude $5^h\ 05^m\ 56^s.5$ West, latitude about 39° N., C—W $5^h\ 22^m\ 15^s$, chro. fast of G. M. T. $2^m\ 04^s.8$, I. C. $-3'$, observed the following circummeridian altitudes $\frac{\odot}{\odot}$ using artificial horizon, sun's separating at the beginning, approaching near end of observations. Required the latitude.

W. T. of L. A. Noon.		W. Ts. of Obs.	Time from Noon $=t$.	Tab. 27 $\Delta_0 ht^2$.	Altitudes.	
	h m s	h m s	m s	′ ″		° ′ ″
$\lambda=$G. A. T. of local noon	5 05 56.5	11 43 40	− 15 30	6 55	$\frac{\odot}{\odot}$	65 54 50
Equation of time	+ 13 23.7	11 48 40	− 10 30	3 10		66 02 10
		11 54 55	− 4 15	0 31		66 07 30
G. M. T. of noon	5 19 20.2	12 01 40	+ 2 30	0 10		66 08 20
Chro. fast	+ 2 04.8	12 06 20	+ 7 10	1 29		66 05 40
		12 10 25	+ 11 15	3 40		66 01 30
C. T. of L. A. noon	5 21 25			6 | 15 55		
C—W	5 22 15	$\Delta_0 h=1''.73$.	Mean val. of $\Delta_0 ht^2$	2 39	Mean	66 03 20
W. T. of L. A. noon	11 59 10				Sextant alt $\underline{\odot}$	33 01 40
					Corr.	+ 13 24
					Mean $-\!\ominus\!-$	33 15 04
					Mean $\Delta_0 ht^2$	+ 2 39
					Mean h_0	33 17 43
					z_0	56 42 17 N
					d	17 44 24 S
					Lat.	38 57 53 N

Altitude Corrections.		Declination.		H. D.		Eq. of Time.		H. D.	
	′ ″		° ′				m s		
S. D.	+16 16	At 4^h	S 17 45.3		N 0′.7		13 23.8		$+0^s.4$
½ (I. C.)	− 1 30	Corr.	N .9	G. M. T.	$1^h.32$	Corr.	+ .4	λ	$+1^h.099$
p. & R.	− 1 22								
Corr.	+13 24	At L. A. noon	S 17 44.4	Corr.	N 0′.92		13 23.7	Corr.	$+0^s.43$

253. To find a constant for latitude by circummeridian altitudes of the sun near upper transit. Before going on deck for the meridian observations, the navigator should previously have obtained the following data for use in determining the noon position; viz., the longitude at noon corresponding to a given latitude and the change in longitude for 1′ of latitude; so that, by a slight mental calculation, he can obtain the noon longitude as soon as he has determined the true noon latitude (see Art. 301) and be able to report both latitude and longitude when he reports twelve o'clock.

To obtain the latitude ahead of time, he should know not only the constant for latitude by meridian altitude (Art. 240), but the constants which will give the noon latitude, if properly applied to sextant altitudes of the sun at given hour angles from noon; these are gotten from the former by applying a correction consisting of two parts: first, a reduction to the meridian for the hour angle from noon at which the observation is taken, the sign of application to the noon constant being the same as that of the altitude; second, a correction representing the difference of latitude for the run for the interval of time in the hour angle, the sign of which is + if of the same name as the latitude for forenoon observations, (—) for afternoon observations; (—) if of a different name from the latitude for forenoon observations, + for afternoon observations. In getting this latter correction it is well to remember that if for a given speed of the ship, the difference of latitude for one hour is x', then for one minute it is x''.

In preparing the "constant," or "constants," to be used for the noon latitude, the required longitude and the hour angles for the reduction of observations to the meridian are obtained in the following way:

Find the change in longitude from noon of the preceding day till noon of the given day by using the run till 11 a. m.

NOTE.—The "correction for difference of latitude," with its sign as determined above, is applied algebraically to the "constant for latitude by meridian altitude," the sign of this constant being regarded as + when the sextant altitude is subtracted from it, and (—) when it is subtracted from the altitude to give the latitude.

from the log and estimating the run from 11 o'clock to noon; this change of longitude, expressed in time, giving the amount that the deck clock must be set ahead or back in order that it may be correct at meridian. The setting of the clock is usually done after 11 a. m., and this shortening or lengthening of the last hour affects the ship's run for that hour.

Then find the longitude at local apparent noon from the a. m. longitude and the run in longitude during the interval, and with that longitude find the watch time of noon as in Art. 198a. Or, having learned the watch error on local apparent time at a. m. sight (body perhaps on or near the prime vertical at that time), apply to it the difference of longitude in time from sight to noon, thus getting the watch error at noon on local apparent time, and hence the watch time of noon. From the watch time of noon and the watch times of observation, the hour angles are found.

Having found the watch time of noon, reset the clock, if necessary, to make it show 12 o'clock when the watch indicates local apparent noon.

Ex. 193.—On April 10, 1918, a vessel's position at 8 a. m. (ship's time) was Lat. 33° 57′ 30″ N., Long. 146° 38′ 18″ E., and by sight the watch was slow on L. A. T. $6^m\ 45^s$. The ship ran till apparent noon 23° (true) 12 knots per hour, it being estimated that the clock would be set ahead about 9 minutes.

It is required to find the longitude at noon by D. R., the W. T. of noon, the constant for latitude by meridian altitude, given the I. C. + 1′ and height of eye 45 feet.

It is also required to find the constants which will give the noon latitude when applied to the sextant altitudes observed at the following intervals of time before noon, 20^m, 15^m, 10^m, 8^m, 6^m, 4^m, 2^m; also the W. T. of the first observation and the noon latitude, if at the first observation the sextant altitude of the sun's lower limb was 62° 26′ 50″.

D. R. 8 a. m. to noon.

The clock being set ahead 9 minutes, the distance run will be 12 × 3.85 = 46.2 miles.

True Course.	Distance.	Diff. Lat.	Dep.	Diff. Long
23°	46.2	42.5 N	18.1 E	21′.9 E

	° ′ ″
Lat. 8 a. m.	33 57 30 N
Diff. of lat.	42 30 N
Lat. in at noon by D.R.	34 40 00 N
	L_0 = 34°.3 N

	° ′ ″
Long. at 8 a. m.	146 38 18 E
Diff. of long.	21 54 E
Long. in at noon by D. R.	147 00 12 E
	h m s
G. A. T. at noon	=14 11 59.2
Eq. t.	+ 1 36.5
G. M. T. of noon	14 13 35.7
	or $14^h.23$

(1) To find the W. T. of noon.

At 8 a. m. watch slow on L. A. T.	$6^m\ 45^s$
Change in time due to Diff. of Long. 21′.9 E	1 27 .6
At noon watch slow on L. A. T.	$8^m\ 12^s.6$

Therefore, W. T. of local apparent noon is $11^h\ 51^m\ 47^s.4$ a. m.
and W. T. of first observation was 11 31 47 .4 a. m.

To Find the Approximate Altitude.				To Find the Constant.	
	° ′		′ ″		° ′ ″
☉'s dec. at 14h April 9	N 7 35.6	S. D.	+ 15 59	Corrected dec.	N 7 35 48
Corr.=N 0′.9×0h.23	S .2	I. C.	+ 1 00		90
		Dip	− 6 36		
Corrected dec. at L. A. N.	N 7 35.8			90°+*d*	97 35 48
	90	Approx. corr.	+ 10 23	Corr.	9 57
		p. & R.	− 26		
90°+*d*	97 35 48			Constant	97 25 51
Approx. correction	10 23	Corr.	+ 9 57		
90°+*d*−approx. *c*	97 25 25				
Approx. lat.	N 34 40				
Approx. alt. ☉	62 45 25				

From Table 26, $\Delta_0 h = 3''.5$.

On course $C_N = 23°$ (true) at 12 knots per hour, Diff. Lat. per hour equals 11′ and Diff. Lat. per minute equals 11″.

Table of Corrections to Noon Constant.

Intervals from Noon.	Table 27.	Diff. Lat.	Combined Corr.	Constant.	Lat. from Constant.	
	′ ″	′ ″	′ ″	° ′ ″		° ′ ″
20 min.	− 23 20	+ 3 40	− 19 40	97 06 11	Constant	97 06 11
15 "	− 13 07	+ 2 45	− 10 22	97 15 29	Sextant ☉	62 26 50
10 "	− 5 50	+ 1 50	− 4 00	97 21 51		
8 "	− 3 44	+ 1 28	− 2 16	97 23 35	Noon lat.	N 34 39 21
6 "	− 2 06	+ 1 06	− 1 00	97 24 51		
4 "	− 0 56	+ 0 44	− 0 12	97 25 39		
2 "	− 0 14	+ 0 22	+ 0 08	97 25 59		

254. Fourth method.—By altitude of the pole star. The given quantities are t, d, and h, the required one is L.

Formulæ (206) apply here, but owing to a very small polar distance in the case of Polaris, they can be simplified.

$$\tan\phi = \cot d \cos t,$$
$$\cos\phi' = \cos\phi \sin h \operatorname{cosec} d,$$
$$90^\circ - L = \phi \pm \phi'.$$

As before, ϕ is the polar distance of foot of perpendicular,
ϕ' is the zenith distance of foot of perpendicular.

Now ϕ and p are so small, that having substituted $90^\circ - p$ for d, $90^\circ - z$ for h, we may consider $\cos\phi$ and $\sec p$ each unity, also $\tan\phi = \phi \tan 1'$ and $\tan p = p \tan 1'$; hence the above will become

$$\phi = p \cos t,$$
$$\cos\phi' = \cos z \text{ or } \phi' = z,$$
$$90^\circ - L = \phi \pm \phi' = \phi \pm z = 90^\circ - (h - \phi) \text{ or } \phi - 90^\circ + h,$$
$$L = h - \phi \text{ or } L = 180^\circ - \phi - h.$$

Therefore, $$L = h - p \cos t, \tag{215}$$

where h is the true altitude of Polaris;
p, the polar distance of the star at the instant of observation;
t, its hour angle.

Close attention must be given to the sign of $\cos t$ as it affects the sign of application of ϕ. If $t < 6$ hours or > 18 hours, $\cos t$ is $+$; if $t > 6$ hours and < 18 hours, $\cos t$ is —, and $L = h + p \cos t$.

The second value $L = 180^\circ - \phi - h$ is inadmissible as it exceeds 90°.

Since by definition the latitude equals the altitude of the elevated pole equals PN, in position a, $L = N\text{a} - P\text{a} = h_1 - p$; at position b, $L = PN = N\text{b} + \text{b}P = h_2 + p$. The mean of these two will give excellent determinations, that is, the mean of the latitudes from observations at upper and lower transits. (See Fig. 117.)

Let M be any position of Polaris when t is $<$ 6 hours. Let $ZM = Zd$. Let Mm be a perpendicular to the meridian, and regard PMm as a plane triangle, then ϕ is the polar distance of m and equals p cos t. By the above formulæ $L = h - p \cos t$; in other words, Nm is assumed equal to Nd or HM, the star's altitude. For any other position as M_1, when t is $>$6 hours, $L = h + p \cos t$ and Nm_1 is assumed equal to Nd_1 or H_1M_1. Though these assumptions are a source of a slight error, the above method is sufficiently exact for all nautical purposes. It is available at all times when the horizon is distinctly seen, and the star Polaris is visible and of sufficient altitude to eliminate the errors of refraction. Its application is limited to the northern hemisphere.

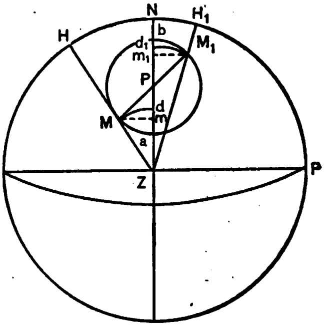


FIG. 117.

In Table I of the Nautical Almanac, there are set down, for intervals of 10 minutes of L. S. T. throughout the 24 hours, values of the correction to be applied to the true altitude of Polaris, in order to find the approximate latitude.

In Art. 176 of Chauvenet's Astronomy, a rigorous formula is deduced, from which the latitude by altitude of the pole star may be found with great accuracy.

It is

$$\left.\begin{aligned}L=h-p\cos t+\tfrac{1}{2}\,p^2\sin 1''\sin^2 t\tan h\\ -\tfrac{1}{3}p^3\sin^2 1''\cos t\sin^2 t+\tfrac{1}{8}\,p^4\sin^3 1''\sin^4 t\tan^3 h\end{aligned}\right\}\quad(216)$$

Ex. 194.—Jan. 26, 1918, p. m., in Long. 108° 36′ W., the sextant altitude of Polaris was 27° 48′. W. T. of obs. $8^h\ 10^m\ 20^s$. C—W $7^h\ 23^m\ 10^s$. Chro. fast of G. M. T. $7^m\ 29^s$. I. C.+2′. Height of eye 30 feet. Required the latitude.

Times.		Alt. and Lat.		Corrections to Altitude.		R. A. M. ☉.		✳'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		
W.	8 10 20	✳'s h_s	27 48 00	I. C.	+ 2 00	At G. M. N.	20 19 52.4	✳'s R. A.	$1^h\ 30^m\ 42^s$
C—W	7 23 10	Corr.	− 5 12	D.	− 5 22	Corr. G. M. T.	2 32.1		
				R.	− 1 00			✳'s d	88° 52′.5 N
C.	3 33 30	✳'s h	27 42 48			At sight	20 22 24.5		
Chro. fast	− 7 29	ϕ	= + 47 02	Corr.	− 5 12				
				or (Tab. 46, Bowditch)	= −7′ 12″				
Jan. 26, G. M. T.	15 26 01	Lat.	26 55 46 N	I. C.	= +2 00				
R. A. M. ☉	20 22 24.5								
G. S. T.	11 48 25.5			Corr.	−5 12				
Long. W	7 14 24								
L. S. T.	4 34 01.5								
✳'s R. A.	1 30 42.0								
✳'s t	+ 3 03 19.5								
✳'s t	45° 49′ 53″.5								

$p=67'.5$ log 1.82930

$t=45°\ 49'\ 53''$ cos +9.84310

$\phi=+47'.033$ log +1.67240

$\phi=+47'\ 02''$ and $L=h-\phi$

Ex. 195.—April 12, 1918, p. m., in Long. 132° 30′ E., the sextant altitude of Polaris was 36° 26′. I. C. +2′. Height of eye 45 feet. W. T. of obs. $8^h\ 05^m\ 09^s$. C—W $3^h\ 04^m\ 51^s$. Chro. slow of G. M. T. $7^m\ 59^s$. Required the latitude.

Times.		Alt. and Lat.		Corrections to Altitude.		R. A. M. ⊙.		✱'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		
W.	8 05 09	✱'s h	36 26 00	I. C.	+ 2 00	At G. M. N.	1 15 33.9	✱'s R. A.	$1^h\ 29^m\ 59^s.0$
C—W	3 04 51	Corr.	— 5 55	Dip	— 6 36	Corr. G. M. T.	3 49.7		
				R.	— 1 19			✱'s d	88° 52′.2 N
C.	11 10 00	✱'s h	36 20 05			At sight	1 19 23.6		
C. C.	+ 7 59	ϕ	= — 33 14	Corr.	— 5 55				
				or (Tab. 46)					
April 11, G. M. T.	23 17 59	Lat.	36 53 19 N		—7′ 55″				
R. A. M. ⊙	1 19 23.6			I. C.	+2 00				
G. S. T.	0 37 22.6			Corr.	—5 55				
Long. E	8 50								
L. S. T.	9 27 22.6								
✱'s R. A.	1 29 59.0								
✱'s t	+7 57 23.6								
✱'s t	119° 20′ 54″								

$p = 67'.8$ log 1.83123

$t = 119° 20' 54''$ cos −9.69030

$\phi = -33'.230$ log −1.52153

$\phi = -33'\ 13''.8$ and $L = h - (-\phi)$

The sum of the last three terms in equation 216, page 539, represents dm in Fig. 117, also dm_1, etc.

The last two terms may be omitted with no greater error in the latitude than 1″. If $p \cos t$ is the only correction applied, the error will amount to only about 1′ when $t = 6$ hours and $h = 54°$, and a maximum of 3′ when $t = 6$ hours and $h = 68° \; 30'$.

255. Fifth Method, called Chauvenet's Method. This consists in finding the latitude by two altitudes near the meridian when the time is not known.

It frequently happens that the time is uncertain, or the deck watch has not been compared with the chronometers, enabling the navigator to get the correct hour angle at observation; under such circumstances this method is of great use to the practical navigator.

Let h_1 and h_2 be the true altitudes of the body at the first and second observations;

W_1 and W_2 be the corresponding watch times of observation;

x and y be the unknown hour angles of the body, respectively, at the first and second observations;

T be the interval of time between the observations, then $T = W_2 - W_1$;

$x \sim y$ be the difference of hour angles of the body at the two observations. For the sun, it is an interval of apparent time and without error may be represented by T. For a star $x \sim y$ is an interval of sidereal time which equals T when T is reduced by Table III to a sidereal interval.

As in Art. 251, let h_0 represent the true meridian altitude of the body, and $\Delta_0 h$ the change in altitude in 1^m from the meridian.

Then, by formula (213),

$$h_0 = h_1 + \Delta_0 h x^2,$$
$$h_0 = h_2 + \Delta_0 h y^2.$$

Taking the half sum of the above equations, we have

$$h_0 = \frac{h_1 + h_2}{2} + \frac{x^2 + y^2}{2} \Delta_0 h; \tag{217}$$

but

$$\frac{x^2 + y^2}{2} = \left(\frac{x+y}{2}\right)^2 + \left(\frac{x-y}{2}\right)^2 = \left(\frac{x+y}{2}\right)^2 + \left(\frac{T}{2}\right)^2;$$

therefore,

$$h_0 = \frac{h_1 + h_2}{2} + \left(\frac{T}{2}\right)^2 \Delta_0 h + \left(\frac{x+y}{2}\right)^2 \Delta_0 h. \tag{218}$$

Taking the difference of the same equations, we have

$$\begin{aligned} h_2 - h_1 = (x^2 - y^2) \Delta_0 h &= \left(\frac{x+y}{2}\right) (x-y)\, 2\, \Delta_0 h \\ &= \left(\frac{x+y}{2}\right) 2 T \Delta_0 h \end{aligned}$$

and

$$\frac{x+y}{2} = \frac{h_2 - h_1}{2 T \Delta_0 h} = \frac{\frac{1}{4}(h_2 - h_1)}{\left(\frac{T}{2}\right) \Delta_0 h}. \tag{219}$$

Substituting this value of $\frac{x+y}{2}$ in (218),

$$h_0 = \frac{h_1 + h_2}{2} + \left(\frac{T}{2}\right)^2 \Delta_0 h + \frac{\left[\frac{1}{4}(h_2 - h_1)\right]^2}{\left(\frac{T}{2}\right)^2 \Delta_0 h}. \tag{220}$$

Therefore, to obtain the meridian altitude by this method, two corrections must be added to the mean of the body's two true altitudes. The first is of the form of the reduction to the meridian, using one-half the elapsed time in place of the hour angle. The second is the square of one-fourth the difference of the altitudes divided by the first correction, care being taken to have both terms of this fraction in the same unit, usually seconds of arc.

The second correction is the larger of the two, as a general thing, and, as this depends largely on the difference of altitudes, the accuracy of the resulting latitude will depend on the precision with which the altitudes have been measured, since errors due to the tabulated dip, refraction and constant instrumental errors affect both altitudes alike.

Having found h_0, proceed as in Art. 251 to find the latitude.

When $h_2 = h_1$, the second correction reduces to zero; therefore, the most favorable case is that of equal altitudes observed on each side of the meridian.

The value of the hour angles may be obtained approximately thus,

From (219),
$$\frac{x+y}{2} = \frac{\frac{1}{4}(h_2 - h_1)}{\left(\frac{T}{2}\right)\Delta_0 h},$$

but
$$\frac{x-y}{2} = \frac{T}{2};$$

therefore,
$$x = \frac{\frac{1}{4}(h_2 - h_1)}{\left(\frac{T}{2}\right)\Delta_0 h} + \frac{T}{2} \tag{221}$$

and
$$y = \frac{\frac{1}{4}(h_2 - h_1)}{\left(\frac{T}{2}\right)\Delta_0 h} - \frac{T}{2}. \tag{222}$$

Restrictions.—The restrictions of this method are the same as those limiting the reduction of a single altitude to the meridian. It must be remembered, however, that the observations in this method are not made at the same place. A slight change of the observer's zenith, which would result from a small interval between observations, would produce but a slight error and especially so when the course is at a right angle to the bearing of the body. When the interval is comparatively large, and the distance run also of consequence, the first altitude must be reduced for the run (see Art. 213) to what it would have been, if observed at the same time at the second position. The value of T will not change.

The latitude found will be that at the instant of the second observation; and to obtain the noon latitude, allowance must be made for the change in latitude during the run from the time of the second observation to noon.

It is not necessary to reduce each altitude to a true altitude and then take the mean. It will be sufficiently accurate for practical purposes at sea to take the mean of the sextant altitudes, and reduce it to the true altitude of the center.

Ex. 197.—April 4, 1918, in Long. 58° E., Lat. about 30° 15′ S., the following sextant altitudes of the sun's lower limb were taken near noon. $h_1=53° 58' 10''$. W$_1$ 11^h 47^m 30^s. $h_2=54° 03' 10''$. W$_2$ 11^h 53^m 00^s. Sun bearing northerly. I. C.+1′. Height of eye 45 feet. Required the latitude.

Times and $\frac{T}{2}$.		Difference of Sextant Alts., etc.		Mean of Alts. and Lat.		Altitude Corrections.		G. A. T. of local app. noon April 3, Eq. t. G. M. T. of local app. noon	h m s 20 08 00 + 3 15 20 11 15 =20^h.19		
	h m s		° ′ ″		° ′ ″		′ ″	Declination.		H. D.	
W$_2$	11 53 00	h_2	54 03 10	$\frac{h_2+h_1}{2}$	54 00 40	S. D.	+ 16 01		° ′		
W$_1$	11 47 30	h_1	53 58 10			I. C.	+ 1 00	At 20^h	N 5 25.4	N	1′.0
				Corr.	+ 9 48	Dip	− 6 36	Corr.	N .2	G.M.T.	0^h.19
T	5 30	h_2-h_1	= 5 00			p. & R.	− 37				
					54 10 28				N 5 25.6	N	0′.19
$\frac{T}{2}$	2 45	$\frac{1}{2}(h_2-h_1)$	= 1 15								
$\Delta_0h=$	2″.92	$(\frac{1}{2}(h_2-h_1))^2$	=5625″	1st corr.	+ 0 22	Corr.	+ 9 48		5625″ log 3.75012		
1st corr. (Tab. 27)		$\frac{h_2+h_1}{2}$	54 00 40	2d corr.	+ 4 16	Or (Tab. 46, Bowditch) = +8′ 47″			22″ log 1.34242		
$\left(\frac{T}{2}\right)^2\Delta_0h=0' 22''$				h_0	54 15 06 N	I. C.	+1 00				
				z_0	35 44 54 S			2d corr.	255″.68 log 2.40770		
				d	5 25 36 N	Corr.	+9 47	2d corr. 4′ 15″.68			
				Lat.	30 19 18 S						

256. Sixth method.—To find the latitude by the rate of change of altitude near the prime vertical (called Prestel's method). In Art. 237, by differentiation of the fundamental formula of the astronomical triangle,

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

regarding h and t as variables, we found formula (188), from which, expressing dt in time, we have

$$dt = -\frac{dh}{15} \sec L \operatorname{cosec} Z,$$

in which dh is a small change of altitude in seconds of arc, occurring in a very brief interval in seconds of time.

If the altitude is increasing, as when the body is East of the meridian, the hour angle is diminishing or dt is (—); if the altitude is diminishing, as when the body is West of the meridian, the reverse holds true.

Let w_1 be the noted time when the body is at the altitude h_1,
w_2 that when the body's altitude is h_2;

then $$T = -(w_1 - w_2) = -\frac{h_1 - h_2}{15} \sec L \operatorname{cosec} Z,$$

and $$T = w_2 - w_1 = \frac{h_2 - h_1}{15} \sec L \operatorname{cosec} Z.$$

$$\cos L = \frac{h_2 - h_1}{15T} \operatorname{cosec} Z. \tag{223}$$

When Z is near 90°, its cosecant varies very slowly and when $Z = 90°$ we have

$$\cos L = \frac{h_2 - h_1}{15T}. \tag{224}$$

The accuracy of this method depends on the precision with which the altitudes are measured and the care with which times are noted.

As the latitude is found from its cosine, the method is more precise in high than in low latitudes. Though the result may be only approximate, it may be useful in restricting the ship's position to a limited portion of a Sumner line.

The time when a body is on the prime vertical can be found from the azimuth tables, or from Art. 239, or sufficiently near by compass if its error is known, or by Table C of N. A.

In case the body is within 2° of the P. V., measure the altitudes and note the times carefully, not letting T be $> 8^m$; use formula (224) and for high latitudes the result may be found within a limit of error that would still make it desirable. However, only an emergency will justify the use of this method. Chauvenet recommends bringing one reflected limb of the sun into contact with the sea horizon, the time being noted; then, keeping the sextant clamped, note the time when contact of the other limb occurs; beginning in the forenoon with the upper limb, in the afternoon with the lower limb; dh will be the sun's diameter in seconds from the Almanac.

In case the body is more than 2° from the P. V., use formula (223).

Ex. 198.—April 24, 1918, in approximate latitude 43° 20′ N., longitude 30° 10′ W., about 5 p. m., the sun bearing true West, the sun's reflected lower limb was brought tangent to the horizon. W. T. $4^h\ 59^m\ 03^s$. The sextant being kept clamped, when the upper limb made contact with the horizon, the watch read $5^h\ 01^m\ 58^s$. Find the latitude.

(Semi-diameter of sun from Ephemeris.)

Formula $\cos L = \frac{h_3 \sim h_1}{15T}$	$h_2 \sim h_1 = 1911''.6$	log	3.28140
	15	colog	8.82391
$h_2 \sim h_1 =$ sun's diameter	$T = 175^s$	colog	7.75696
$= (15'\ 55''.80) \times 2$	$L = 43°\ 15'\ 40''$ N	cos	9.86227
$= 1911''.60$			

257. Reduction of latitude.—In the previous articles of this chapter, we have assumed the earth to be a sphere, implying that a plumb-line at any point of the earth's surface, if extended, passes through the earth's center, and that the altitude of an observed body, after the usual corrections have been applied, is referred to the center.

The earth is not a sphere but a spheroid, and the vertical line at any point of the surface as *O'L* in Fig. 118, which corresponds exactly with the normal drawn at that point, does not coincide with the earth's radius passing through the same point excepting at the equator and at the poles.

The point *Z* where the vertical line *O'L* prolonged meets the celestial sphere is the geographical zenith and the angle *ZO'Q* is the geographical latitude of the point *L*, as determined by observations at sea. The point *Z'* in which the radius *OL* prolonged meets the celestial sphere, is the geocentric zenith and the angle *Z'OQ* is the reduced or geocentric latitude.

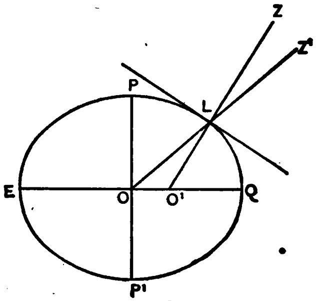


FIG. 118.

The geocentric latitude is smaller than the geographical latitude at all places except at the equator and poles where they are equal; the difference between the two being the angle *OLO'* called the "angle of the vertical" or the "reduction of the latitude."

Though necessary in certain refined observations ashore, it is not necessary to consider the reduction at sea where extreme precision is unattainable.

CHAPTER XVIII.

CHRONOMETER ERROR, CORRECTION, AND RATE.—LONGITUDE ASHORE AND AT SEA.

258. It has already been shown in Chap. X that the chronometer is the navigator's means of getting the Greenwich mean time of any desired instant or observation. Though constructed with the greatest care and at much expense it is far from perfect, seldom indicating the exact time of the prime meridian and seldom running with regularity for any length of time.

However, a sidereal or a mean time chronometer is said to be regulated to local or Greenwich time, when its error on that particular time, the amount by which it is fast or slow of that time, and its rate, or daily gain or loss, are known.

Both the error and rate are positive, if the chronometer is fast and gaining; otherwise, negative; the sign of the error being the sign of application to the correct standard of time to get the chronometer reading. It is preferable, however, to regard the error as a correction to be applied to the chronometer reading to obtain the desired true time, and to consider the rate as a daily change. Both are positive or plus when the chronometer is slow and losing.

259. To find the rate.—The rate is found by taking the algebraic difference (that is, the numerical difference when of the same name, the numerical sum when of a different name) of the errors on two different days and dividing it by the elapsed time in days and decimals of a day. The interval should be at least 5 to 7 days. When the errors are determined at two different places, the times of observation should be reduced to one (say Greenwich) meridian and the interval found from the two reduced times.

The rate will be gaining when both errors are fast and the last one is the greater, when both errors are slow and the last one the lesser of the two, or when the error changes from a slow to a fast one; otherwise, the rate will be a losing one.

Ex. 199.—The error of a given chronometer on G. M. T. on April 15 at noon was $+5^m\ 32^s.5$; at noon on April 25 it was $+5^m\ 35^s.8$. Required the daily rate.

Error at noon April 15,	+	$5^m\ 32^s.5$
Error at noon April 25,	+	5 35 .8
Change for 10 days	+	$3^s.3$
Daily rate	+	0 .33

260. To find the error on a given date, knowing the error on another date and the daily rate.—Multiply the daily rate by the number of days elapsed since the determination of the error and this, applied with proper sign to the original error, will give the error on the required date.

Ex. 200.—With the data of the above example, find the error of the same chronometer on G. M. T. at noon April 30.

Error April 25,	$+5^m\ 35^s.8$	Daily rate	$+0^s.33$
Change	+ 1 .65	No. of days	5
Error April 30,	$+5^m\ 37^s.45$	Change	$+1^s.65$

261. Sea rate.—Ordinarily the error and rate of a chronometer are determined entirely from shore determinations, and that error is brought up by its rate to the instant of later observations at sea in working for longitude. Now this rate found in port may be very different from the actual sea rate, even at the same temperature.

Vessels equipped with radio outfits may obtain a sea rate directly in localities where it is possible to receive the wireless time signals explained in Art. 265, and these localities now comprise a large share of the most frequented waters of the globe.

In case an error is determined just before leaving port and again after return to the same port, the difference of errors divided by the elapsed time will give the sea rate.

Again, a vessel on a voyage may stop at many places whose longitudes are well known, these having been determined perhaps by direct or indirect telegraphic connection with Greenwich or some place of known longitude.

Say the error of the chronometer on G. M. T. is found at place *A*, of known longitude; by applying the longitude to the local time of determination of the error, the Greenwich time of the determination is gotten. At place *B*, obtain the same data, the chronometer error on G. M. T., and the Greenwich instant corresponding. The algebraic difference of the errors, divided by the elapsed Greenwich time, will be the sea rate, the rate being regarded as uniform.

Ex. 201.—On April 2, 1918, at Southampton, in longitude 1° 18′ W., a time ball was dropped at $0^h\ 00^m\ 00^s$ L. M. T. At this instant the error of a chronometer *A* was found to be slow $14^m\ 52^s$ on G. M. T.

On April 28, 1918, at Lisbon, in longitude 9° 12′ W. by single altitudes in the forenoon, using artificial horizon, the error of the same chronometer was found to be slow of G. M. T. $8^m\ 50^s.005$. C. T. of observation $9^h\ 15^m\ 36^s.2$. Required the sea rate.

	h m s		h m s		m s
April 2, L.M.T.	0 00 00	April 28, C. T.	9 15 36.2	Error April 2, slow	14 52
Long. W	5 12	C. C.	+ 8 50	Error April 28, slow	8 50.005
G.M.T. April 2,	0 05 12	April 27, G.M.T.	21 24 26.2	Gain	= 6 01.995
		April 2, G.M.T.	0 05 12		= 361^s.995

Elapsed time $25^d\ 21^h\ 19^m\ 14^s.2$
$= 25^d.888$

$$\text{Rate or Daily Gain} = \frac{361^s.995}{25.888} = 13^s.983.$$

262. Irregular rate.—In case the rate is not constant, as shown by getting two different rates by observations for two different intervals, the change in the rate itself may be regarded as uniform and the rate interpolated to the middle instant between the two intervals. This will permit the use of the formula for uniformly accelerated or retarded motion in finding the change in error for a given interval of time. The first rate is taken as correct at the middle instant of the first interval.

At the second series of observations, the second rate is taken as at the middle instant of the second interval.

Ex. 202.—The error of a chronometer *B* at noon April 1, 1918, at St. Nicholas Mole, Hayti, was found to be fast of G. M. T. 11^{m} 42^{s}.5; at same place, at noon April 11 fast of G. M. T. 11^{m} 51^{s}.

The chronometer was then carried to a point on the South coast of Cuba where observations for longitude were made in the forenoon and afternoon of April 16.

On return to the Mole, the error at noon April 22 of the same chronometer was found to be fast of G. M. T. 11^{m} 56^{s}.4, and, again, on April 30 fast of G. M. T. 11^{m} 58^{s}.4. Required the error at noon April 16.

		m s			m s
Error April 11,	+	11 51	**Error April 30,**		11 58.4
Error April 1,	+	11 42.5	**Error April 22,**		11 56.4
Change in 10 days	+	8.5	**Change in 8 days**	+	2
Daily rate	+	0.85	**Daily rate**	+	0.25

These rates are assumed correct at the middle instant of the period during which they were determined.

April 6, rate	$+0^{s}.85$
April 26, rate	$+0\ .25$
Change of rate in 20 days	$-0^{s}.60$
Daily change of rate	$-0\ .03$
Rate April 11,	$+0\ .70$

The problem now resolves itself into this, "On April 11 the error of a chronometer was + 11^{m} 51^{s}, the daily rate + 0^{s}.70, retardation of the rate 0^{s}.03. Find the error April 16."

Taking the formulæ

$$S = (V_0 + \tfrac{1}{2}\,at)\,t \text{ and } E = E_0 + S, \qquad (225)$$

where S is the change in error in time t,

V_0 is the initial rate April 11 $+ 0^s.70$,

a the daily retardation equals $0^s.03$,

t the elapsed time equals 5 days,

E_0 the initial error April 11 equals $+ 11^m\ 51^s$,

E the required error April 16.

$$S = +\left(0^s.70 - \frac{0^s.03 \times 5}{2}\right) 5 = +(0^s.70 - 0^s.075)\, 5 = +3^s.125$$

$E = 11^m\ 51^s + 3^s.125 = + 11^m\ 54^s.125$; or at noon April 16, the chronometer is fast of G. M. T. $11^m\ 54^s.125$.

As the error on April 16 is to be used in the determination of longitude, it should also be determined by working back from April 22, and the two values combined by giving weights to each inversely proportional to its interval of time from the original determination.

From the rate on April 26 find the rate on April 22, call it V_0, and let E_0 be the error April 22; then, by substitution as before, we shall obtain an error of the chronometer, on April 16, of $11^m\ 53^s.64$.

There is a discrepancy in the two errors arising from the fact that the actual change of rate is not uniform as assumed.

If E_1 is an error brought forward t_1 days;

E_2, an error carried back t_2 days;

E_x, the probable error at the given time;

then, by the method of "Least Squares,"

$$E_x = \frac{E_1 t_2 + E_2 t_1}{t_1 + t_2} = E_1 + \frac{t_1(E_2 - E_1)}{t_1 + t_2}$$

Substituting in the above equation

$E_1 = + 11^m\ 54^s.125$, $t_1 = 5$, $E_2 = + 11^m\ 53^s.64$, $t_2 = 6$, we have

$$E_x = + 11^m\ 54^s.125 + \frac{5(-0^s.485)}{11} = + 11^m\ 53^s.905.$$

Therefore the chronometer is fast of G. M. T. $11^m\ 53^s.905$ on

April 16; that is to say the probable C. C. is (—) $11^m\ 53^s.905$ on that date.

However, as chronometer rates are greatly affected by variations in temperature, the theory of this method, that of a uniform change of rate, is not tenable when there have been erratic changes of temperature between successive ratings.

263. Finding the chronometer error.—Before leaving port, the navigator must ascertain the error and rate of each of his chronometers, and, if he has sufficient data, construct a temperature curve.

The methods of obtaining chronometer error and rate may be considered under three general heads:

(1) Observatory methods, as by transits.
(2) By time signals.
(3) By the navigator's own observations.

264. (1) **Observatory methods.**—The most accurate method of finding chronometer error is by noting the chronometer time of transit of a heavenly body across each wire of a transit instrument well adjusted in the meridian. The mean of these times, reduced to the time of meridian passage by applying the proper corrections in case the middle wire is not exactly in the meridian, will give the chronometer time of transit.

At the instant of transit of a star over the upper branch of the meridian, its right ascension equals the local sidereal time; if over the lower branch of the meridian, the L. S. T. equals the star's right ascension plus 12 hours. If the chronometer is a sidereal chronometer, the difference between its reading at the instant of the star's transit and the star's right ascension will be the error on L. S. T. If the star is observed at the lower transit, then the error on L. S. T. will be the difference between the chronometer reading and the sum of the star's right ascension plus 12 hours.

If the chronometer is a mean time chronometer, convert the local sidereal time of transit into local mean time, apply the longitude to obtain the G. M. T.; the difference between which and the chronometer time of the transit will be the error of the chronometer on G. M. T.

The time of upper transit of the sun's center is $0^h\ 00^m\ 00^s$ of apparent time; the corresponding local sidereal time must be found, if the time of transit was marked by a sidereal chronometer to find its error on L. S. T.; or, the corresponding G. M. T. must be found, if the time of transit was marked by a mean time chronometer whose error on G. M. T. is desired.

Transits of stars are preferred to transits of the sun. In this, as in all other cases of finding time, observers are advised not to use the moon.

By repeating the observations, at a subsequent time, the rate will be found as in Art. 259. Though the error may not be good when the instrument is somewhat out of adjustment, the rate will be good, if the second error is determined by the same instrument, without change of position or adjustment, and under like circumstances.

265. (2) **By time signals.**—At nearly all important sea ports of the world a time signal is made at a specified instant of time. This instant will be found in the sailing directions of the locality and in the daily papers; the latter usually publish the day following any failure or error in the signal. The general form of signal is a time ball or gun-fire. The ball is usually painted black and, a few minutes before the instant of dropping, is hoisted to the top of a high pole conspicuously located so as to be seen from all parts of the harbor, and is dropped by electricity, usually by signal from an observatory. The observer notes the time by his chronometer the moment the ball starts from the top; the difference between the chronometer face and the L. M. T of fall is the error on L. M. T., from which the error on G. M. T is at once found by applying the longitude.

In cases where the signal is gun-fire, the gun is usually fired electrically from an observatory. The flash is the moment to be timed by the chronometer; but if not seen, listen for the report, making an allowance for the velocity of sound at the rate of 1090 feet per second at a temperature of 32° F., with an increase or decrease for each degree above or below 32° F. at the rate of 1.15 of a foot per degree. Dividing the known distance between the observer and the gun by the proper velocity of sound per second, we find the correction in seconds to be subtracted from the chronometer reading at hearing the signal to give the chronometer face at the flash or time of signal.

Ex. 203.—At Southampton, England, a time ball was dropped on April 10, 1918, at $1^h\ 00^m\ 00^s$ G. M. T., a chronometer at the instant reading $11^h\ 45^m\ 30^s$. What was the chronometer error?

G. M. T.	$1^h\ 00^m\ 00^s$
Chronometer face	11 45 30
Chronometer error −	$1^h\ 14^m\ 30^s$

In other words, the chronometer is slow on G. M. T. $1^h\ 14^m\ 30^s$; or, the C. C. is $+1^h\ 14^m\ 30^s$.

Standard time (Art. 177) is now being used in many countries; Great Britain, Belgium, France, Portugal, and Spain keep Greenwich time; Germany, Austria, Italy, and the Scandinavian countries have adopted the standard time of the meridian of 15° E. The standard times adopted in the United States and Canada are explained in Art. 177; Egypt, South Africa, and Asia Minor keep the time of the meridian of 30° E.

Clocks in business houses, hotels, and schools, when electrically controlled, are thrown into circuit with the local telegraph lines, and are corrected electrically at noon.

Navigators, being at the telegraph office when the time signals are being made, can find with ease and certainty the error of a chronometer on the standard time being received, by noting the chronometer face when the signal is made. The difference between the chronometer face and the time of the signal will be the error on that time. The error on G. M. T. may be obtained by applying to this error the proper difference of longitude.

A list of the places at which time signals are made, together with a description of the signals, is kept current in the issues of the British Admiralty List of Lights and Time Signals.

The United States System of Time Signals.

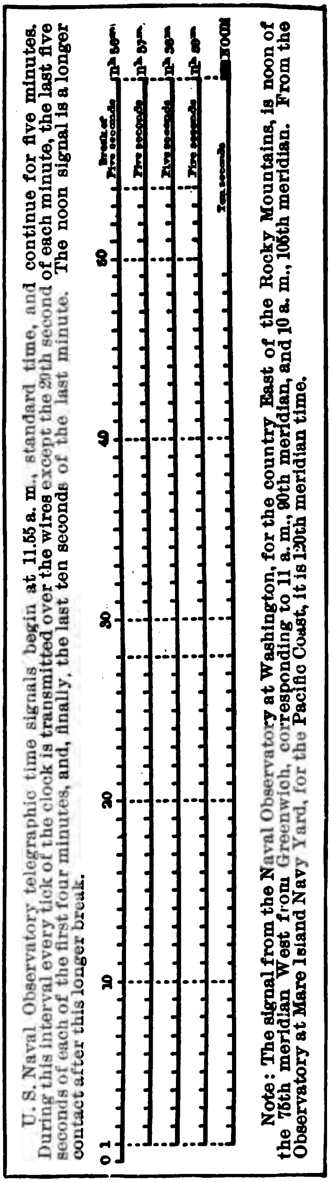
In the United States and some other countries time signals are sent not only over telegraph wires to the various towns and cities of the country, but from the Government wireless stations along the coast to vessels at sea. The general scheme of transmission is explained in the following extract from the Annual Report of the Superintendent U. S. Naval Observatory for the fiscal year ending June 30, 1902:

Telegraphic Time Signals.

Sent out at noon daily, except Sundays and holidays, by the U. S. Naval Observatory.

The entire series of noon signals sent out daily over the wires is shown graphically in the accompanying diagram. This represents the signals as they would be recorded on a chronograph, where a pen draws a line upon a sheet of paper moving along at a uniform rate beneath it and is actuated by an electro-magnet so as to make a jog at every tick of the transmitting clock. The electric connections of the clock are such as to omit certain seconds, as shown by the breaks in the record. These breaks enable anyone who is listening to a sounder in a telegraph or telephone office to recognize the middle and end of each minute, especially the end of the last minute, when there is a longer interval that is followed by the noon signal. During this last long interval, or 10-second break, those who are in charge of time balls and of clocks that are corrected electrically at noon throw their local lines into circuit so that the noon signal drops the time balls and corrects the clocks.

This series of noon signals is sent continuously over the wires all over the United States for an interval of five minutes immediately preceding noon. The transmitting clock that sends out the signals is corrected very accurately, shortly before noon, from the mean of three standard clocks that are rated by star sights with a meridian transit instrument. The noon signal is seldom in error to an amount greater than one or two-tenths of a second, although a tenth more may be added by the relays in use on long telegraph lines. Electric transmission over a continuous wire is practically instantaneous. For time signals at other times than noon, similar signals can be sent out by telegraph or telephone from the same clock that sends out the noon signal.


U. S. Naval Observatory telegraphic time signals begin at 11.55 a. m., standard time, and continue for five minutes. During this interval every tick of the clock is transmitted over the wires except the 29th second of each minute, the last five seconds of each of the first four minutes, and, finally, the last ten seconds of the last minute. The noon signal is a longer contact after this longer break.

Note: The signal from the Naval Observatory at Washington, for the country East of the Rocky Mountains, is noon of the 75th meridian West from Greenwich, corresponding to 11 a. m., 90th meridian, and 10 a. m., 105th meridian. From the Observatory at Mare Island Navy Yard, for the Pacific Coast, it is 120th meridian time.

Ex. 204. On board a battleship at the New York Navy Yard on April 1, 1918, a comparison of a Hack chronometer *H* was made with a standard *A* as follows, *H* $10^h\ 40^m\ 17^s$. *A* $4^h\ 39^m\ 47^s$. The navigator then landed to get a "tick" and when the 75th meridian mean noon signal was received, the chronometer *H* read $11^h\ 05^m\ 47^s$. A comparison after return on board was *H* $11^h\ 18^m\ 32^s$, *A* $5^h\ 18^m\ 03^s$. On April 21, by the same means, *A* was found to be fast of G. M. T. $4^m\ 22^s\ 5$. Find the error of *A* on April 1, and then the rate.

Comparisons.

	h	m	s
Before going ashore,			
A	4	39	47
H	10	40	17
A−*H*	5	59	30
After return on board,			
A	5	18	03
H	11	18	32
A−*H*	5	59	31

		h	m	s
1st Comparison *H*		10	40	17
2d Comparison *H*		11	18	32
Interval			38	15
Change in *A*−*H*	+			1
1st Comparison *H*		10	40	17
Time of signal *H*		11	05	47
Interval			25	30
Change in *A*−*H*	+			0.67
A−*H* at signal		5	59	30.67

		h	m	s
G. M. T. 75th mer. M. N.		5	00	00
Reading of Hack at time of signal		11	05	47
G. M. T. — *H*		5	54	13
A — *H* at signal		5	59	30.67
A — G. M. T. or *A* fast on G. M. T. April 1,	+		5	17.67
A fast April 21,	+		4	22.5
Difference for 20 days				55.17
Rate losing				2.7585
or	—			$2^s.76$

266. (3) **By the navigator using his own observations.**—The navigator, thrown on his own resources, may rate his chronometers by one of the following methods, using the navigational instruments ordinarily provided him: (a) By single altitudes. (b) Double altitudes. (c) Equal altitudes.

In all the methods that follow, the longitude of the observation spot must be accurately known to get the chronometer error on G. M. T. The latitude of the spot should also be accurately known, though, if the body is observed on the prime vertical, the errors due to uncertainties of latitude will be a minimum. In single or double altitudes, the closest possible approximation should be made to the chronometer error, and this used in finding the G. M. T. for which to take out the body's declination. In the absence of an artificial horizon, a fair error may be found from a good sea horizon by single or double altitudes. Altitudes of any heavenly body may be used, but only those of the sun, and of certain stars, are recommended.

(a) Single Altitudes.

267. With the sextant and artificial horizon at a place of known latitude and longitude, the navigator takes a series of altitudes of the sun or a star, when the body is near or on the prime vertical, noting the time of each observation. It is recommended to take a series of five altitudes at regular intervals of 10′ of sextant arc; making the limbs in case of the sun overlap, and noting, by chronometer or a watch compared with a chronometer of known error, the instant of separation of the lower limb of the sun and the upper limb of its image in the horizon during a. m. observations; or, by a reverse operation, noting the instant of contact during p. m. observations. The mean of the altitudes and the mean of the times should be taken.

Using an approximate chronometer correction, an approximate G. M. T. of observation is obtained, the declination of the body is taken out, and the altitude reduced to a true altitude of the center.

The navigator then has given *h, d,* and *L* from which to find the body's hour angle as in Art. 226. In case of a star, the local sidereal time obtained from the star's hour angle is reduced to L. M. T., to which the known longitude is applied; the difference between the resulting G. M. T. and the chronometer reading (or chronometer face) at the instant of observation will be the error of the chronometer on G. M. T.

In case the body is the sun, its hour angle, reckoned positively, is the L. A. T., and the following procedure is recommended: Apply the known longitude to the L. A. T., finding the G. A. T. Take the difference between the longitudes of the place of observation and of Washington and apply this difference of longitude to the L. A. T. obtaining the Washington apparent time for which take out from the Ephemeris the equation of time. Apply the equation of time with its proper sign to the G. A. T.; the result will be the G. M. T., the difference between which and the chronometer reading at the instant of observation (C. F.) will be the error of the chronometer on G. M. T., fast or slow, according as the chronometer face is greater or less than the G. M. T.

If an artificial horizon is used, as it should be when possible, two sets of observations should be made with a different end of the roof in each set, and a mean of the two resulting errors taken as the correct error, thus eliminating errors due to a possible want of parallelism of the faces of the glass.

Such observations and the results are liable to the same errors as are similar observations for longitude at sea.

To determine the rate.—On a subsequent date, repeat the observations, find a second error, take the algebraic difference between this and the first error, and divide by the interval in days to obtain the rate.

The observations should, as far as possible, be taken under similar conditions as to body, hour angle, altitude, instrument, and atmosphere. If like conditions exist, each error will be similarly affected, and the rate should be reliable.

Hence, it may be laid down as a general rule that the rate should be determined by a comparison of a. m. sights with a. m. sights, or p. m. sights with p. m. sights, and never the reverse, unless conditions absolutely demand it. Under such circumstances, the same observer, using the same instrument, will find that assumed errors of latitude, constant instrumental and personal errors will but slightly affect the rate.

Ex. 205.—Jan. 8, 1918, a. m., at Cay Sal, Cuba, Lat. 23° 56′ 30″ N., Long. 80° 27′ 51″ W., observed altitudes of sun for chronometer error, using artificial horizon, limbs separating. Mean of double altitudes by sextant of sun's lower limb 34° 36′. I. C.—2′. W. T. of obs. $8^h\ 08^m\ 30^s$. C—W $5^h\ 37^m\ 05^s$. Chronometer fast of G. M. T. (approximately) $6^m\ 50^s$. Required chronometer error on G. M. T. (Use data from Ephemeris.)

Times.		Altitudes.		Altitude Corrections.		Declination.		H. D.	Eq. of Time.		H. D.	
	h m s		° ′ ″		′ ″		° ′ ″			m s		
W.	8 08 30	2 $\odot$	34 36 00	S. D.	+16 18		S 22 18 54.9	N 19″.69	At Wash. app. noon, Jan. 8,	6 42.20		+1ˢ.067
C—W	5 37 05	I. C.	— 2 00	p.&R.	— 2 57	Corr.	N 32.4	G.M.T. $1^h.645$			W.A.T.	$-3^h.6$
		2	34 34 00									
C. face	1 45 35	$\underline{\odot}$	17 17 00	Corr.	+13 21		S 22 18 22.5	N 32″.390	Corr.	— 3.84		$-3^s.841$
Approx. C. C.	— 6 50	Corr.	+ 13 21									
									Eq. t. (+ to apparent)	6 38.36		
Jan. 8 G.M.T. approx.	1 38 45	$\ominus$	17 30 21									
or Jan. 8	$1^h.645$											

	° ′ ″				h m s		h m s
h	17 30 21			L. A. T.	20 10 09.37	Long. place	=5 21 51.4
L	23 56 30	sec	10.03908	Eq. t.	+ 6 38.36	Long. Wash.	=5 08 15.8
p	112 18 23	cosec	10.03378				+ 13 35.6 W. of Wash.
				L. M. T.	20 16 47.73	L. A. T.	20 10 09.4
$2s$	153 45 14			λ W	5 21 51.4	λ W. of Wash.	+ 13 35.6
s	76 52 37	cos	9.35612			Wash. A. T.	20 23 45.0 Jan. 7.
h	17 30 21			G. M. T.	1 38 39.13	or Jan. 8,	$-3^h\ 36^m\ 15^s = -3^h.6$
$s-h$	59 22 16	sin	9.93474	C. face	1 45 35		
		2	9.36372				
L. A. T. $20^h\ 10^m\ 09^s.37$		sin	9.68186	Chro. fast on G. M. T.	6 55.87		

(b) By Double Altitudes or Altitudes on Opposite Sides of the Meridian.

268. Instead of relying on a single determination of the chronometer error from altitudes on one side of the meridian, it is better to observe the same body on both sides of the meridian, and, if possible, at about the same altitude. The error of the chronometer having been found from each set of sights, the mean is taken as the correct error, and this mean will probably be nearer the true error than the result from either set; the effect of the constant errors of latitude, instrument, and observer, being opposite in the two cases, will be eliminated by taking the mean.

(c) By Equal Altitudes; Deduction of the Equation of Equal Altitudes.

269. If a heavenly body is observed at a given altitude on one side of the meridian, and, again, at the same altitude on the other side of the meridian, the chronometer times of each observation being noted, the mean of the two, or the middle chronometer time, will be the time of the body's transit, provided its declination has not changed in the interval. The difference between this C. T. of transit and the actual time of transit, found independently, will be the chronometer error on that particular time.

Fixed stars are bodies whose change of declination is so slight that it may be neglected, and to these the above remarks apply.

For a body whose declination changes in the interval between observations, the hour angles at the same altitude, East and West of the meridian, are not numerically equal, the East or West hour angle being the larger according to circumstances, the difference being due to the change of declination in the interval. Half the difference of these hour angles, called the "Equation of Equal Altitudes," is the hour angle of the body at the middle chronometer time; or, in other words, the correction to be applied to the middle chronometer time to obtain the chronometer time when the body's center is on the local meridian.

Let Fig. 119 represent a projection on the plane of the horizon. *NS* is the meridian. *EQW* is the equator. *PM'*, *PM"*, *PM*, and *Pm* are hour circles. *MnM"M'* is a parallel of altitude. *dMcM'w* is the diurnal circle of a star or body whose declination does not change, so that if the altitude of the star is observed at *M*, East of the meridian, and again at *M'*, when it is on the same parallel of altitude West of the meridian, the hour angle *MPZ* equals *M'PZ*, and if the times be noted when the star is at *M* and *M'*, the mean of these, ignoring the rate of the time piece, will be the time of transit at *c*. When the star is at *c* on the upper branch of the meridian, its R. A. is the L. S. T. Knowing the longitude, this L. S. T. can be converted into G. M. T., the difference

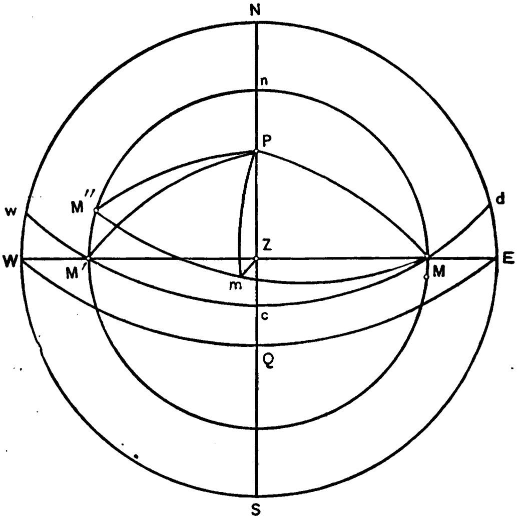


FIG. 119.

between which and the chronometer time of transit will be the error of the chronometer on G. M. T.

If the star is West of the meridian at the first observation, the mean of the times will correspond to the instant when the star is on the lower branch of the meridian, at which time the L. S. T. = R. A. of the star plus 12 hours, and this L. S. T. can be converted into G. M. T., and the error of chronometer found as above explained.

If the heavenly body be one, as the sun for instance, whose declination changes during the interval between the observations, the hour angles at the same altitudes East and West of the meridian will not be numerically equal.

In the figure, a case is assumed in which the declination of the sun is of the same name and less than the latitude, and the sun is moving toward the elevated pole. If the sun is observed when it is at *M*, its altitude being *ME*, in its diurnal path it will not follow *dcw*, but will follow the circle *dmM″* and will reach the parallel of altitudes at *M″* instead of at *M′*. The western hour angle will evidently differ from the eastern one by *M″PM′*. This, then, is the change in the western hour angle during the local apparent time represented by the eastern and western hour angles t' and t'', due to the total change in declination in those times. If the times had been noted by a mean time chronometer when the sun was at *M* and *M″*, the mean of these times, ignoring as before the rate and in addition the change in the equation of time during the interval, will correspond not to the time of transit but to the instant when the sun is at *m* and when its hour angle is *mPZ*, which is clearly equal to one-half *M″PM′*; or, in other words, one-half the change in the western hour angle during the interval between the observations, due to a change in the declination.

If this small angle be reduced to the same unit as that of the chronometer and applied to the mean of the chronometer times, the result will be the chronometer time of transit or local apparent noon. If the error in the western hour angle is positive or increases that angle, the small angle mPZ is to the westward of the meridian and its sign of application as a correction is minus. If the error is negative or decreases the western hour angle, its sign as a correction is plus; or, in other words, its sign of application is opposite from that gotten by differentiation.

Let L be the latitude, always plus;

d, the declination of sun, plus if of same name as L;

dd, the hourly change of declination at L. A. noon, plus if body is moving toward the elevated pole;

t', the eastern hour angle;

t'', the western hour angle;

dt'', the hourly change in H. A. due to dd in western H. A.;

$2t$, the elapsed time by chronometer.

As $M'PM''$ is the error produced in the western H. A. by the entire change in declination between observations, we assume the declination to vary uniformly and regard d and t as variable in the general equation for the western H. A., t''.

$$\sin h = \sin L \sin d + \cos L \cos d \cos t'',$$

$$0 = \sin L \cos d\, dd - \cos L \cos t'' \sin d\, dd$$
$$- \cos L \cos d \sin t'' dt'',$$

$$dt'' = + \frac{dd \tan L}{\sin t''} - \frac{dd \tan d}{\tan t''}.$$

Since dd is taken as the hourly change of declination at the approximate middle instant, in other words, at L. A. noon, dt'' is the change in the H. A. in one hour, but the complete change takes place in L. A. times t' and t'', expressed in hours; therefore, the angle $M'PM''$ is

$$(t' + t'')\, dt'' = + \frac{dd\,(t' + t'') \tan L}{\sin t''} - \frac{dd\,(t' + t'') \tan d}{\tan t''}.$$

The hours $(t' + t'')$ are apparent time, but it is more convenient to use the elapsed time $(2t)$, as shown by chronometer between observations, and to substitute t for t'' or t'. This involves a practically inappreciable error due to change in equation of time and in rate of chronometer during interval; hence we have

$$2t\,dt = \frac{2t\,dd \tan L}{\sin t} - \frac{2tdd \tan d}{\tan t}.$$

As this error has been shown to be twice the H. A., *mPZ*, at the middle instant, we divide the above by 2, and, to express it in time, divide the right-hand member by 15, and as this is an error whose sign of application as a correction has been shown to be the reverse, we change the signs of the right-hand member and have

$$tdt = -\frac{tdd \tan L}{15 \sin t} + \frac{tdd \tan d}{15 \tan t}, \tag{226}$$

which is the equation of equal altitudes.

Rules.

L is always positive.

d is positive if of same name as latitude or of elevated pole.

dd is positive if body is moving towards elevated pole.

t is always positive.

tdt in seconds is applied with its proper sign to the middle chronometer time, giving chronometer time of L. A. noon, or error of chronometer fast on L. A. T.

To this, the equation of time is applied as to mean time, giving chronometer time of local mean noon, or error of chronometer fast on L. M. T. at L. A. noon.

To this is applied the longitude, adding when East, subtracting when West, giving error of chronometer fast of G. M. T. at L. A. noon.

d, dd, and equation of time are taken from the American Ephemeris for the instant of local apparent noon. The hourly change of the equation of time and declination are corrected for second differences.

In p. m. and a. m. equal altitudes, it is evident that when equal zenith distances are observed in a latitude *L,* their supplements may be considered as equal zenith distances observed at the antipode in latitude $-L$ on the same meridian. Hence the formula will give the equation for noon at the antipode, or for midnight at the place of the observer, by substituting $-L$ for L in the first term of the equation, which, therefore, becomes

$$tdt = +\frac{tdd \tan L}{15 \sin t} + \frac{tdd \tan d}{15 \tan t}. \qquad (227)$$

In p. m. and a. m. equal altitudes of the sun, or, in other words, when the first observation is West of the meridian, the "equation of equal altitudes," *tdt,* is applied with its proper sign to the middle chronometer time, giving the chronometer time of local apparent midnight.

To this, the equation of time is applied as to mean time, giving the chronometer time of local mean midnight, or error of chronometer fast on L. M. T. at local apparent midnight.

To the C. T. of local mean midnight is applied the longitude, adding when East, subtracting when West; the result is the error of chronometer fast of G. M. T. at local apparent midnight.

d, dd, and the equation of time are taken from the American Ephemeris for the instant of local apparent midnight (12 hours$+\lambda$).

The hourly change of equation of time and of declination are corrected for second differences.

Ex. 206.—April 4, 1918, p. m., in latitude 38° 59′ N., longitude $5^h\ 05^m\ 56^s.5$ W., observed equal altitudes of α Leonis (Regulus), noting time by a mean time chronometer. C. T. ✱ East of meridian $11^h\ 04^m\ 26^s$. C. T. ✱ West of meridian $3^h\ 32^m\ 30^s$. Find the C. C. on G. M. T.

	h	m	s
C. T. ✱ East of mer.	11	04	26
C. T. ✱ West of mer.	3	32	30
Sum	26	36	56
Middle C. T.	1	18	28
G. M. T. of transit	14	19	40.4
Chro. slow of G. M. T.	1	01	12.4

	h	m	s
R. A. M. ☉ Gr. M. N. April 4,	0	47	58.0
Red. for λ Table III	+		50.3
Sid. time local 0^h	0	48	48.3
L. S. T.=✱'s R. A.	10	04	03.2
Sid. int. from noon	9	15	14.9
Red. Table II	−	1	31.0
L. M. T. of transit	9	13	43.9
Longitude West	5	05	56.5
G. M. T. of transit	14	19	40.4

Ex. 207.—April 5, 1918, at Sydney, N. S. W., Lat. S. 33° 51′ 41″, Long. E. 151° 12′ 23″, observed equal altitudes of the sun's lower limb, using artificial horizon, for C. C. on G. M. T. Mean of a. m. watch times 8^h 32^m $25^s.4$; C—W 2^h 21^m 20^s. Mean of p. m. watch times 8^h 17^m $34^s.2$; C—W 2^h 21^m 21^s. I. C. +2′, both a. m. and p. m. Find the C. C.

	h m s
Long.	= 10 04 49.5 E
Long. Wash.	= 5 08 15.8 W
	15 13 05.3 E of Washington = $15^h.218$ E

Times and Chro. Error.		Value of t.	
	h m s		h m s
A. M. W.	8 32 25.4	P. M. W.	3 17 34.2
C—W	2 21 20	C—W	2 21 21
A. M. C. T.	10 53 45.4	P. M. C. T.	5 38 55.2
P. M. C. T.	5 38 55.2	A. M. C. T.	10 53 45.4
Mid. C. T.	2 16 20.3	$2t$	6 45 09.8
Eq. of eq. alts. = tdt	= + 12.222	t	3 22 34.9
C. T. A. noon	2 16 32.522	t	$3^h.3764$
Eq. of T.	− 3 02.139	t	50° 38′ 43″.5
C. T. M. noon	2 13 30.383		
Long. East	10 04 49.533		
Chro. fast of G. M. T.	18 19.916		

L	S 33° 51′ 41″	tan	+ 9.82672		
d	N 5° 42′ 32″			tan	− 8.99987
dd	N 57″.172	log	− 1.75718	log	− 1.75718
t	$3^h.3764$	log	0.52845	log	0.52845
t	50° 38′ 43″.5	cosec	10.11169	cot	9.91386
	15	colog	8.82391	colog	8.82391
−1st part	$-11^s.1674$	log	− 1.04795		
		2d part	+ 1.0550	log	+ 0.02327
		1st part	+ 11.1674		
		tdt	+ 12.222		

Sun's Declination.		Mean Hourly Diff.	
	° ′ ″		
At Wash. app. noon	N 5 57 00.9		N 57″.096
Corr. for λ E of Wash.	S 14 28.9	λ E of Wash.	$15^h.218$
At local app. noon	N 5 42 32.0	Corr.	S 868″.88

	″
H. D. of dec. April 5,	N 57.02
H. D. of dec. April 4,	N 57.26
Change in 24 hours	− 0.24
Change in 1 hour	− 0.01
Change for λ E of Wash. = $15^h.218$	+ 0.152
H. D. April 5, at Wash. A. N.	N 57.02
dd	= N 57.172
Change for $\frac{15^h.218}{2}$	+ .076
H. D. April 5,	N 57.02
Mean H. D. of dec.	N 57.096

Equation of Time.		Mean Hourly Diff.	
(−) to M. T.	m s		− $0^s.732$
At Wash. app. noon	2 51.00	λ E of Wash.	$15^h.218$
Corr. for λ E of Wash.	+ 11.139	Corr.	+ $11^s.1395$
At local app. noon	3 02.139		

	s
H. D. of Eq. of T. April 5,	0.730
H. D. of Eq. of T. April 4,	0.738
Numerical change 24 hours	0.008
Numerical change for $\frac{15^h.218}{2}$	0.002
H. D. April 5, at Wash. app. noon	0.730
Mean H. D. of Eq. of T.	− 0.732

Ex. 208.—April 9, 1918, at Casilda, Lat. of observation spot N. 21° 45′, Long. $5^h\ 19^m\ 48^s$ W., observed equal altitudes of the sun's lower limb, using artificial horizon, for C. C. on G. M. T. Mean of a. m. chronometer times $1^h\ 05^m\ 03^s$. Mean of p. m. chronometer times $8^h\ 57^m\ 41^s$. I. C. −1′, both a. m. and p. m. Required the C. C.

	h m s
Long. place =	5 19 48
Long. Wash. =	5 08 15.8
+	11 32.2 = Long. W of Wash. = $0^h.1923$ W

Times and Chro. Error.		Value of *t*.		Sun's Declination.		Mean Hourly Diff.
	h m s		h m s		° ′ ″	
A. M. C. T.	1 05 03	P. M. C. T.	8 57 41	At Wash. app. noon	N 7 27 22.0	N 55″.879
P. M. C. T.	8 57 41	A. M. C. T.	1 05 03	Corr. for λ W of Wash.	N 10.74	λ W of Wash. $0^h.1923$
Mid. C. T.	5 01 22	2*t*	7 52 38	At L. A. noon	7 27 32.74	Corr. N 10″.745
Eq. of eq. alts. or *tdt*	− 5.672	*t*	3 56 19			
C. T. A. noon	5 01 16.328	*t*	$3^h.9383$			
Eq. of time	− 1 42.567	*t*	59° 04′ 45″			
C. T. M. noon	4 59 33.761					
Long. West	5 19 48					
Chro. slow of G. M. T.	0 20 14.239					

L	N 21° 45′	tan	+ 9.60093		
d	N 7° 27′ 32″.7			tan	+ 9.11702
dd	55″.878	log	+ 1.74724	log	+ 1.74724
t	$3^h.9383$	log	0.59531	log	0.59531
t	59° 04′ 45″	cosec	10.06657	cot	9.77741
	15	colog	8.82391	colog	8.82391
−1st part $6^s.8227$		log	0.83396		
		2d part	+1.1505	log	+ 0.06089
		1st part	−6.8227		
		tdt	−5.6722		

	″
H. D. of dec. April 9,	N 55.88
H. D. of dec. April 10,	N 55.56
Change in 24 hours	− 0.32
Change in 1 hour	− 0.0133
Change for λ W of Wash.	− 0.002
H. D. Apr. 9, at Wash. app. noon	N 55.88
dd	= N 55.878
Change for $\frac{\lambda}{2}$ W of Wash.	− 0.001
H. D. April 9.	N 55.88
Mean H. D. of dec.	N 55.879

Equation of Time.		Mean Hourly Diff.	
(−) to M. T.	m s		
At Wash. app. noon	1 42.70		$-^s.692$
Corr. for λ W of Wash.	− .133	λ W of Wash.	$0^h.1923$
At L. A. noon	1 42.567		$-0^s.1330$

	s
H. D. of Eq. of T. April 9,	0.692
H. D. of Eq. of T. April 10,	0.681
Numerical change 24 hours	0.011
Numerical change for $\frac{\lambda}{2}$ W of Wash.	0.000
H. D. April 9, at Wash. app. noon	0.692
Mean hourly diff.	0.692

limb, using artificial horizon. I. C. +1′, both p. m. and a. m. Position of observation spot Lat. 20° 43′ N., Long. $5^h\ 12^m\ 00^s$ W. Mean of p. m. chronometer times $9^h\ 30^m\ 25^s.33$. Mean of a. m. chronometer times $8^h\ 11^m\ 14^s.5$. Required chronometer error on G. M. T.

	h m s
$\lambda + 12^h$	= 17 12 00
Long. Wash.	= 5 08 15.8
	12 03 44.2 W of Wash. = $12^h.0623$ W at midnight.

Times and Chro. Error.

		h m s
P. M. C. T.		9 30 25.33
A. M. C. T.		3 11 14.5
Mid. C. T.		6 20 49.915
Eq. of eq. alts. or *tdt*	+	8.291
C. T. A. midnight		6 20 58.206
Eq. of time	+	2 16.906
C. T. M. midnight		6 23 15.112
Long. West		5 12 00
Chro. fast of G. M. T.		1 11 15.112

L	N 20° 43′	tan	+ 9.57772
d	N 13° 34′ 10″.9		
dd	N 48″.203	log	+ 1.68308
t	$8^h.8402$	log	0.94646
	132° 36′ 09″	cosec	10.13308
	15	colog	8.82391
+1st part	$+14^s.5965$	log	+ 1.16425

2d part	− 6.3057
1st part	+ 14.5965
tdt	+ 8.2908

Value of *t*.

	h m s
A. M. C. T.	3 11 14.5
P. M. C. T.	9 30 25.33
2*t*	17 40 49.17
t	8 50 24.58
t	$8^h.8402$
t	132° 36′ 09″

tan	+ 9.38267
log	+ 1.68308
log	0.94646
cot	− 9.96361
colog	8.82391
log	− 0.79973

Sun's Declination.

	° ′ ″
At Wash. app. noon	N 13 24 27.8
Corr. for λ W of Wash. at midnight	N 9 43.1
At local app. midnight	N 13 34 10.9

Mean Hourly Diff.

	N 48″.342
Long. W of Wash. at midnight	$12^h.0623$
Corr.	583″.116

	″
H. D. of dec. April 26,	N 48.48
H. D. of dec. April 27,	N 47.93
Change in 24 hours	− 0.55
Change in 1 hour	− 0.023
Change for $12^h.0623$	− 0.277
H. D. Apr. 26 at Wash. app. noon	N 48.48
dd	= N 48.203
Change for $\frac{12^h.0623}{2}$	= − 0.138
H. D. April 26,	N 48.48
Mean H. D. of dec.	N 48.342

Equation of Time.

+ to M. T.	m s
At Wash. app. noon	2 11.780
Corr.	+ 5.126
At L. A. midnight	2 16.906

Mean Hourly Diff.

	+ $0^s.425$
Long. W of Wash. + 12^h	$12^h.0623$
Corr.	+ $5^s.1263$

	s
H. D. of Eq. of T. April 26,	0.430
H. D. of Eq. of T. April 27,	0.409
Numerical change 24 hours	0.021
Numerical change $\frac{12^h.0623}{2}$	0.005
H. D. April 26 at Wash. app. noon	0.430
Mean hourly diff. of Eq. of T.	+ 0.425

Chauvenet's Tables.—If in formula (226) we let $A = -\frac{t}{15 \sin t}$ and $B = \frac{t}{15 \tan t}$, that formula reduces to

$$t\,dt = A.dd. \tan L + B.dd. \tan d \begin{Bmatrix} \text{a. m. and p. m.} \\ \text{observations.} \end{Bmatrix} \quad (228)$$

and formula (227) becomes

$$t\,dt = -A.dd. \tan L + B.dd. \tan d \begin{Bmatrix} \text{p. m. and a. m.} \\ \text{observations.} \end{Bmatrix} \quad (229)$$

Chauvenet has tabulated log A and log B (to the fourth place only), with their proper signs for noon and midnight transits, the argument being $2t$. Particular attention must be paid to the signs. However, the advantage of using these tables is not apparent.

The equation of time is applied to the C. T. A. N. (or C. T. A. M.) with the sign of application to mean time, because, if it is + to mean time, apparent time is greater than mean time, the apparent sun crosses the meridian first, and the C. T. of mean noon (or midnight) is later than the C. T. of apparent noon (or midnight). If the sign of application is (—) to mean time, apparent time is less than mean time, the apparent sun crosses the meridian after the mean sun, and the C. T. of mean noon (or midnight) is less than the C. T. of apparent noon (or midnight).

To the C. T. of mean noon, East longitude in time is added because the Greenwich mean time of Greenwich mean noon is desired, which will not occur till the number of hours representing the longitude have elapsed since local mean noon, when the longitude is East. To the C. T. of mean noon, West longitude is subtractive because Greenwich noon occurs before noon at places in West longitude by that number of hours.

For similar reasons, longitude is applied to the C. T. of mean midnight; adding when East, subtracting when West.

Attention is called to the fact that the elapsed time, to be exact, should be corrected for the rate of the chronometer and also for the change in the equation of time during the interval; but such refinement is usually neglected, being of no practical importance, except when the chronometer has a large rate.

In case of planets.—When working equal altitudes of planets, formulæ (226) and (227) will apply provided $2t$ may be expressed in units of the hour angle of the body observed instead of in mean time units, $2t$ being the sum of the hour angles of the body itself. Therefore,

let $2t =$ the sum of the two hour angles of the planet;
$2s =$ the elapsed sidereal time between the observations;
$2t_m =$ the elapsed mean time between the observations;
$dr =$ the mean change in the planet's right ascension in one hour of mean time, expressed in decimals of an hour.

Then $2t_m dr$ will be the total change in the planet's right ascension in $2t_m$, or the elapsed time by chronometer, and the elapsed sidereal time will be greater than the sum of the hour angles by this total change; or,

$$2s = 2t + 2t_m dr, \text{ or } s = t + t_m dr. \qquad (230)$$

From (137), $s = t_m + .0027379\ t_m;$

therefore, $$t = t_m + t_m\,(.0027379 - dr). \qquad (231)$$

Again, if dd represents, as in the formulæ (226) and (227), the hourly change of the planet's declination at the instant of transit for one of its own hours, the total change in declination will be

$$2t.dd. = [2t_m + 2t_m\,(.0027379 - dr)]dd.;$$

and, if dd_m represents the change in the planet's declination for one hour of mean time at the instant of the planet's transit, the total change will be $2t_m dd_m$; therefore,

$$[2t_m + 2t_m\,(.0027379 - dr)]\ dd = 2t_m dd_m; \text{ or}$$

$$dd = \frac{dd_m}{1.0027379 - dr}. \qquad (232)$$

270. Littlehales' method of equal altitudes.—Equation 226 may be placed under the form

$$tdt = \frac{-tdd}{15}\left(\frac{\sin L \cos d - \cos L \sin d \cos t}{\cos L \cos d \sin t}\right),$$

but $\sin L \cos d - \cos L \sin d \cos t = \cos h \cos M$, and

$$\frac{\cos h}{\cos L \sin t} = \frac{1}{\sin M};$$

therefore, $$tdt = \frac{-tdd}{15} \sec d \cot M. \qquad (226a)$$

d and dd are taken from the Ephemeris for local apparent noon and marked as explained in Art. 269; then tdt found from (226a) is applied in finding chronometer error as explained in the same article, the form of arrangement of work being modified only in the logarithmic column.

To illustrate the process let us refer to Ex. 207 in which Lat. $= 33° \ 51' \ 41''$ S., $d = 5° \ 42' \ 32''$ N., $dd =$ N. $57''.172$, and $t = 3^h.3764$.

Entering the Azimuth Tables, with 33° 52′ in declination column, 5° 42½′ in latitude column, and $8^h \ 37^m$ or $12^h - t$ (since d is $-$), we find $M = 46° \ 38'$; therefore by substitution in (226a), we find tdt as shown in the columns to right.

$d = 5° \ 42' \ 32''$ N	sec	$+10.00216$
$M = 46° \ 38'$	cot	$+\ 9.97523$
$dd = 57''.172$ N	log	$-\ 1.75718$
$t = 3^h.3764$	log	0.52845
15	colog	8.82391
$-tdt = -12^s.22$	log	$-\ 1.08693$
$\therefore tdt = +12^s.22$		

The sign of application of tdt to the middle chronometer time may be found as above by following the signs of the quantities involved, or by using the following simple precept: "*For values of the position angle less than 90°, tdt should be added when the polar distance is increasing and subtracted when the polar distance is decreasing; and for values of the position angle greater than 90°, the reverse is the case.*"

When the first observation is west of the meridian.—It is evident when equal zenith distances are observed in a latitude L, in this case, that their supplements may be considered as equal zenith distances observed at the antipode in latitude $-L$ on the same meridian; and that in the triangle to be considered, which includes the elevated pole and the antipode, the angle at the body will be $180° - M$ instead of M. Hence, we shall obtain the equation for noon at the antipode or *for* *midnight at the place of observer* by substituting 180° — M for M in (226a);

therefore, $$tdt = \frac{+tdd}{15} \sec d \cot M. \qquad (227a)$$

d and dd are taken from the Ephemeris for local apparent midnight and marked as in Art. 269. The sign of application of tdt to the middle C. T. is found by following the signs of the quantities involved in (227a), or, *by a reverse application of the precept given for a. m. and p. m. observations.* When taking out M from the azimuth tables, it must not be forgotten that the hour angle from the upper meridian in this case is taken as the supplement of half the elapsed time or $12^h - t$. (See Appendix C.)

271. To correct the middle time for a small difference of altitude.

The altitude at the second observation may differ slightly from that at the first observation through a change in refraction which may be learned by noting the barometer and thermometer during both observations; through a change in the index correction; or through interference of clouds or other unexpected causes.

From Art. 237, $$dt = -\frac{\cos h dh}{15 \cos L \cos d \sin t} \begin{cases} dh \text{ in arc,} \\ dt \text{ in time.} \end{cases} \qquad (233)$$

In this formula, dh is negative when dt is positive, since as hour angle increases, altitude decreases and vice versa.

If dh represents the difference between the altitude observed, and the one that should have been observed, dt will be the corresponding difference in hour angles. This being the change during the whole elapsed time, $\frac{1}{2}dt$ will be the correction to be added to the middle chronometer time when the western altitude is the greater; to be subtracted, when the western altitude is the smaller.

Or, if desired, take the difference between two readings of the sextant representing double angles by artificial horizon, and the difference of corresponding times. Find the change in time due to 1′ or 1″ of double altitude and multiply it by the known inequality of altitudes. This result will be the correction to the middle chronometer time, to be added when the western altitude is the greater; otherwise, subtracted.

Ex. 210.—April 29, 1918, in Lat. 34° 26′ 10″ N., Long. 119° 42′ 42″ W., observed equal altitudes of the sun, using artificial horizon. Twice the sextant altitude of the lower limb, both a. m. and p. m., was 62° 30′. Mean of a. m. chro. times $2^h\ 10^m\ 30^s$. Mean of p. m. chro. times $10^h\ 37^m\ 06^s$. I. C. in the forenoon −1′ 20″. Inadvertently its change to +1′ 40″ in the afternoon was not noticed till after the p. m. observations had been taken. Required the correction to the middle chronometer time.

$$\text{Formula: } \tfrac{1}{2}dt = -\frac{\cos h\,dh}{30 \cos L \cos d \sin t} \begin{cases} dh \text{ in arc.} \\ dt \text{ in time.} \end{cases}$$

Change in double altitude 3′ 00″, in single altitude 1′ 30″. Therefore, $dh = -1'\ 30'' = -90''$. G. A. T. of noon = $7^h\ 58^m\ 50^s.8$. Eq. t. = $2^h\ 41^m.5$. G. M. T. = $7^h\ 56^m\ 09^s.3 = 7^h.935$.

Altitude.		Alt. Corr.		Declination.		H. D. of Dec.		Value of *t*.	
	° ′ ″		′ ″		° ′ ″		″		h m s
2 ☉	62 30	S. D.	+15 54		N 14 17 39.5	April 29, H. D.	N 46.91	P. M. C. T.	10 37 06
P. M. I. C.	+ 1 40	p. & R.	− 1 28	Corr.	N 6 11.5	April 30, H. D.	N 46.32	A. M. C. T.	2 10 30
	2 | 62 31 40	Corr.	+14 26		N 14 23 51.0	Change in 24 hrs.	− 0.59	2*t*	8 26 36
	31 15 50					Change for $\frac{\text{G. M. T.}}{2}$	− 0.097	*t*	4 13 18
Corr.	+ 14 26					H. D. April 29,	N 46.91	*t*	63° 19′ 30″
⊖	31 30 16					Mean H. D.	N 46.813		
						G. M. T.	$7^h.935$		
						Corr. for G. M. T.	N 371″.461		

	° ′ ″		
$h =$	31 30 16	cos	0.93075
$dh = -$	90	log	− 1.95424
30		colog	8.52288
$L =$	34 26 10	sec	10.08367
$d =$	14 23 51	sec	10.01386
$t =$	63 19 30	cosec	10.04887
$\frac{1}{2}dt =$	$3^s.5832$	(−) log	− 0.55427

Since the p. m. altitude is greater than the a. m. altitude, the p. m. hour angle is too small, and the correction, $\frac{1}{2}dt$, must be added to the middle chronometer time.

Time and method of observation.—The most favorable position of a heavenly body for observations for equal altitudes is when the body is on the prime vertical; small errors in altitude having the least effect on the resulting hour angle at that time, though the altitude should be sufficiently great to eliminate errors of refraction, at all events to lessen the probabilities of them. For preparing the artificial horizon see Art. 154.

The altitudes of the same limb should be observed at regular intervals of 10′ or 20′ of arc with the artificial horizon; as soon as contact is made at one division, set the sextant at the next, watch for contact, and mark the time at each contact. In forenoon single altitudes the following method may be used: having observed the 2 $\overline{\odot}$ at intervals of 10′ of arc through 50′ of a given degree, run the instrument arm back and observe the 2 $\underline{\odot}$ at intervals of 10′ up to 50′ through the same degree. The mean of these will be the sextant altitude 2 $\ominus$, eliminating correction for S. D. The reverse procedure should be followed for afternoon single altitudes. The corresponding sets of altitudes on each side of the meridian should be taken under like conditions, with the same instrument and adjustments and with the same end of the roof toward the observer. In equal altitudes it is not necessary that the exact altitude at either observation be known, but only the times at which the altitudes East and West of the meridian are equal, since the elapsed time alone, and not the altitude, is required for the computation.

272. Comparison of methods.—In equal altitudes of stars, latitude and declination do not enter, and hence errors in them do not affect the result.

The mean of errors by double altitudes of the sun, at about the same altitudes East and West of the meridian, will be practically the same as the error by equal altitudes. In high latitudes, the double altitude method is preferable on account of the large value of tan L, in the equation of equal altitudes.

LONGITUDE.

273. Longitude has been defined as the difference in the hour angles of the same heavenly body at a given instant at the local and prime meridians; the prime meridian being usually that of Greenwich, and is marked East when the local time is the greater, West when the Greenwich time is the greater.

In Fig. 120, let PG be the meridian of Greenwich; PM the local meridian West of Greenwich; PM' the meridian of a place East of Greenwich; PS the declination or hour circle of a heavenly body; $P\,\Upsilon$ the hour circle passing through the vernal equinox. Then GPS is the Greenwich hour angle of the body S; MPS its hour angle at all places on the meridian PM; $M'PS$ its hour angle at all places on the meridian PM'. GPM is the longitude of the meridian PM and is marked West. GPM' is the longitude of the meridian PM' and is marked East.

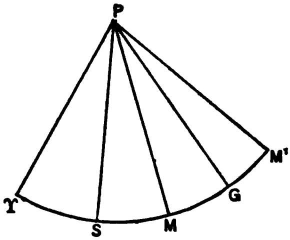


FIG. 120.

$$\left.\begin{aligned} GPM &= GPS - MPS, \\ \text{also } GPM' &= M'PS - GPS. \end{aligned}\right\} \qquad (234)$$

If PS is the true sun's hour circle, GPM or GPM' is the difference of apparent times for the same instant at the Greenwich and local meridians; if it is that of the mean sun, GPM or GPM' is then the corresponding difference of mean times.

Again,

$$\left.\begin{aligned} GPM &= GP\Upsilon - MP\Upsilon, \\ \text{and } GPM' &= M'P\Upsilon - GP\Upsilon. \end{aligned}\right\} \qquad (235)$$

or the longitude of a place is the difference between the Greenwich and local sidereal times at the same instant.

The problem, then, of finding the longitude of a place consists in determining for the same instant of time the difference of the Greenwich and local times, apparent, mean, or sidereal.

When the navigator knows the error and rate of his chronometer, found at a place of known longitude, he can find for the instant of observation at the place the Greenwich mean time corresponding. This Greenwich mean time may be converted into Greenwich apparent time, in case the sun is the body observed, by applying the equation of time; or, into Greenwich sidereal time (Art. 192), if the body observed is the moon, a planet, or a star.

The problem of finding the hour angle of a heavenly body has been considered in Art. 226.

In case the body observed is the sun, the hour angle reckoned positively to the West up to 24 hours is the local apparent time (astronomically considered); the difference between which and the G. A. T. found above will be the longitude, East when the L. A. T. is greater than the G. A. T., otherwise West.

In case the body observed is any other heavenly body, solve the astronomical triangle for t, the hour angle, mark it $+$ when the body is West of the meridian, ($-$) when it is East of the meridian. The algebraic sum of the body's hour angle and right ascension will be the local sidereal time; the difference between which and the G. S. T. found above will be the longitude, East when the L. S. T. is the greater, West when the G. S. T. is the greater (see Art. 179).

Determination of Longitude.

274. Longitude may be determined

(a) Ashore, by electric signals;
(b) Ashore or afloat,
    (1) By equal altitudes,
    (2) By single or double altitudes.

(a) Ashore.

275. By electric signals.—For this method there should be observatories, permanent or portable, in telegraphic communication with each other. Each observatory should be provided with a transit instrument, an electro-chronograph, and requisite telegraphic instruments. An astronomical clock of known error and rate should be at one station, or it may be at any place which is in telegraphic communication with both stations. The electric connections should be such that during the times of observations each successive beat of the astronomical clock will be recorded simultaneously on the chronograph sheet at each station, as well as the instant of transit across each meridian of a given star previously agreed upon; the clock beats being recorded by connection through the pendulum, and the times of transit through a signal key in the hands of the observers. The intervals between two successive beats of the clock, as recorded on the chronograph sheet, being subdivided by scale, the instants of transit of the star across both eastern and western meridians, and hence the elapsed time between transits, may be obtained to fractions of a second from each chronograph.

The difference between the times of transit of the same star across the two meridians, as indicated on each chronograph, is corrected for the rate of the clock in the interval, and the mean of the two values, thus corrected, is taken as the difference in longitude required.

Another method is to have at each station a number of break circuit chronometers, mean or sidereal, instead of the astronomical clock at one station. Sidereal chronometers are preferred as they can be rated by star observations with less computation.

The chronometers at each station, having been carefully rated for local time by transits of stars, are compared by signals sent first one way, then the other way; the times of sending and receiving signals being recorded at both stations. The readings of the chronometers at both stations are reduced to local time, and, if the signals are recorded simultaneously at both stations, the difference of the local times at the instant of comparison will be the difference of longitude. In other words, if T_e is the local time of that instant at the eastern station A and T_w the corresponding local time at the western station B, then $D = T_e - T_w$.

However, the record of signals is not simultaneous, time is lost in completing the circuit and in the action of the armature; this lost time, called the "wave and armature time," may be represented by x. Therefore, if the signal is sent from A, the time recorded at B will not be T_w but will be $T_w + x$, and the difference of times represented by D' will be

$$D' = T_e - (T_w + x).$$

If, however, the signal is sent to A by the observer at B at the local time T_w of the western station, the corresponding time recorded at A will not be T_e but will be $T_e + x$ and the difference of times represented by D'' will be

$$D'' = (T_e + x) - T_w,$$

but $\dfrac{D' + D''}{2} = \dfrac{2T_e + x - 2T_w - x}{2} = T_e - T_w$ which equals D.

It is thus seen that if we take the difference of longitude to be the difference of the local times indicated at the instant of comparison when the signal is sent from the eastern to the western station, the difference of longitude will be too small by a fraction of a second; and, in the same way, it would be too large by the same amount when the signal is sent from the western to the eastern station; the error being eliminated, however, by transmitting signals both ways and taking the mean of the two results for the correct difference of longitude.

(b) Ashore or Afloat.

276. Longitude by equal altitudes.—If equal altitudes of a heavenly body be observed East and West of the meridian, and the times noted by a chronometer, or by a watch compared with a chronometer, the mean of the chronometer times will be the chronometer time of its meridian transit, provided the observer has not changed his position, or the body its declination, in the interval. The known chronometer correction having been applied to this middle chronometer time, the result will be the Greenwich mean time of transit. If the observation is made ashore, one condition is fulfilled; if the body observed is a star, the other condition is fulfilled. The star's right ascension will be the L. S. T. of transit and knowing the G. M. T. of transit, the L. M. T., and then the longitude, may be found.

If the declination of the body has changed in the interval between observations, as may be expected in the case of the sun, the moon, or a planet, the correction to the middle G. M. T. of observation for such a change must be ascertained and properly applied to find the G. M. T. of transit.

Observations for time of the moon and planets being undesirable for reasons stated in Art. 231, consideration of the sun alone comes under this article.

The general method of finding the correction to the middle time due to a change of declination in the interval *t*, or the value of *tdt*, pursued in Art. 269, when working for chronometer error, may be followed when working for longitude, with certain essential changes in the method.

When working for chronometer error, the longitude must be known, and in the case of the sun, the values of the declination, equation of time, and *dd* are easily found for the instant desired, either of local apparent noon or midnight. However, when working equal altitudes for longitude, the nearest known time to the instant of the sun's transit is the G. M. T. of the middle instant between observations.

Take from the Nautical Almanac for this G. M. T. the declination, equation of time, and *dd,* though it must not be forgotten that, strictly speaking, they should be for the instant of transit, an unknown time in this case. In other words, follow the methods of Ex. 208, Art. 269, in correcting the declination and equation of time and finding *dd,* but substituting G. M. T. of the middle instant for longitude.

Then, having found from the equation of equal altitudes (226) or (227), according as the first observation of the sun was East or West of the meridian, the value of *tdt,* and having the equation of time, we would have the following form for the arrangement of the work:

G. M. T. of middle instant =
tdt = ±
G. M. T. of local apparent noon or midnight (according as first observation was E. or W. of meridian). =
Eq. of time (sign of application to M. T.) =
G. A. T. of local apparent noon or midnight (according as first observation was E. or W. of meridian). =

The G. A. T. of noon, if less than 12 hours, is the longitude West; if greater than 12 hours, take it from 24 hours and the remainder is the longitude East.

From the G. A. T. of midnight subtract 12 hours; the remainder, if plus, is the longitude West; if minus, it is the longitude East.

The declination of the sun, the H. D. of declination, and the equation of time, when working for longitude, are taken out from the Ephemeris; for the middle G. M. T., though strictly speaking, they should be taken out for the instant of transit.

In case the value of *tdt* is of such a magnitude as to appreciably affect those quantities, it would be better, after finding the G. M. T. of transit, to take them out for that time and repeat the process. Such refinement, however, will probably be unnecessary, especially when the first observation is East of the meridian.

277. When the position of the observer changes, as it does at sea, in order to attain any approach to accuracy, it is necessary that the elapsed interval should be small and also that the conditions should be favorable for finding time. In low latitudes, especially when the latitude and declination are nearly the same, and the observations are of the sun taken within a few minutes of noon, the vessel nearly stationary or not changing her latitude, the conditions may be said to be favorable for the following simple solution.

(a) **Approximate method for longitude from equal altitudes of the sun.**—Note the time by a chronometer of known error when the sun is at the same altitude, East and West of the meridian. The mean of these chronometer times may be taken without much error as the chronometer time of local apparent noon; apply the chronometer correction, finding the G. M. T. of local apparent noon; reduce this to G. A. T. by applying the equation of time. This G. A. T. will be the longitude West from Greenwich; if the G. A. T. is greater than 12 hours, subtract it from 24 hours, the remainder will be the longitude East from Greenwich.

Ex. 211.—Jan. 11, 1918, on Sacrificios Is., Lat. of observation spot 19° 10′ N., observed equal altitudes of the sun, a. m. and p. m. Mean of a. m. chro. times $8^h\ 20^m\ 23^s$. Mean of p. m. chro. times $9^h\ 40^m\ 29^s$. Chronometer slow of G. M. T. $1^m\ 58^s.39$. Required the longitude.

Times and Longitude.	h m s
A. M. C. T.	3 20 23
Chro. corr.	+ 1 58.39
G. M. T. (a. m.)	3 22 21.39
G. M. T. (p. m.)	9 42 27.39
Mid. G. M. T.	6 32 24.39
tdt	− 4.113
G. M. T. of apparent noon	6 32 20.277
Eq. of T.	− 7 57.900
G. A. T. of apparent noon or Long. West	6 24 22.377
Long. West	96° 05′ 35″.6

Values of *t*.		h m s
P. M. C. T.		9 40 29
Chro. corr.		1 58.39
G.M.T. (p. m.)		9 42 27.39
G.M.T. (a. m.)		3 22 21.39
2*t*		6 20 06
t	=	3 10 03
	=	$3^h.1675$
	=	47° 30′ 45″

L	19° 10′ N	tan	+ 9.54106		
d	21° 50′ 48″ S			tan	− 9.60306
dd	23″.229 N	log	+ 1.36603	log	+ 1.36603
t	$3^h.1675$	log	0.50072	log	0.50072
t	47° 30′ 45″	cosec	10.13228	cot	9.96186
15		colog	8.82391	colog	8.82391
−1st pt.	$+2^s.3120$	log	+ 0.36400		
		2d part	= −1.8013	log	− 0.25558
		1st part	= −2.3120		
		tdt	= −4.1133		

Sun's Declination.	° ′ ″
At G. M. noon	S 21 53 19.4
Corr. for G. M. T. of Mid. instant	N 2 30.97
At Mid. G. M. T.	S 21 50 48.43

Mean Hourly Diff.	
	N 23″.084
G. M. T.	$6^h.54$
Corr.	N 150″.97

	″
H. D. of dec. Jan. 11,	N 22.94
H. D. of dec. Jan. 12,	N 24.00
Change in 24 hours	+ 1.06
Change in 1 hour	+ 0.0442
Change for G. M. T. = $6^h.54$	+ 0.289
H. D. Jan. 11 at G. M. noon	N 22.94
dd	= N 23.229
Change for $\frac{\text{G. M. T.}}{2}$	= + 0.144″
H. D. Jan. 11,	N 22.94
Mean H. D. of dec.	N 23.084

Equation of Time. (−) to M. T.	m s
At G. M. noon	7 51.35
Corr. for G. M. T.	+ 6.55
At Mid. G. M. T.	7 57.90

Mean Hourly Diff.	
	$+1^s.001$
G. M. T.	$6^h.54$
Corr.	$+6^s.546$

	s
H. D. of Eq. of T., Jan. 11,	1.004
H. D. of Eq. of T., Jan. 12,	0.979
Numerical change in 24 hrs.	0.025
Numerical change for $\frac{\text{G. M. T.}}{2}$	0.003
H. D. Jan. 11 at G. M. noon	1.004
Mean H. D. of Eq. of T.	+ 1.001

Such an approximate application of "equal altitudes" is only available in the tropics under conditions named. The method of equal altitudes for longitude has a more extended application when stars are used, as suitable ones can be found in any latitude.

(b) **Method of equal altitudes for longitude when the positions of ship and body change.**—When the body observed is on or near the prime vertical and the change of latitude is small, the error involved through neglecting this change will be small; however, if it is desired to correct for change of position, it may be done very closely in one of the following ways:

(1) The correction may be made approximately by resetting the sextant at the second observation, so that the second altitude will be increased by the number of minutes of arc equal to the number of sea miles in the difference of latitude, when the vessel sails toward the sun; or decreased in the same ratio when she sails away from the sun. The mean of the times of observations will then be without appreciable error the time of transit.

(2) The mean of the times of equal altitudes of a heavenly body corresponds to the time of the maximum altitude, so that if we find the hour angle of the sun at its maximum altitude (Art. 246), that is, the interval of time between maximum altitude and meridian passage, and apply it to the mean of the Greenwich mean times of observation, we will have the Greenwich mean time of local apparent noon. Applying to this the equation of time, we will obtain the G. A. T. of local apparent noon or longitude West. Should this be greater than 12 hours, subtract it from 24 hours; the remainder will be the longitude East.

Remember that *t*, the H. A. of the sun at maximum altitude is easterly when the sun and zenith are separating, westerly when approaching (Art. 246); and, if easterly, that *t* is additive to the mean of chronometer times, if westerly it is subtractive from that mean to give the C. T. of local apparent noon.

In other words, when the ship and sun are approaching, the H. A. at maximum altitude is subtracted from the mean of chronometer times to give the chronometer time of meridian passage; when the ship and sun are separating, the reverse rule holds.

Ex. 212.—On April 22, 1918, latitude by D. R. 26° 00′ N., longitude by D. R. 46° 03′ W., observed from the bridge of a vessel steaming 315° (true) 20 knots per hour equal altitudes of sun's lower limb as follows: In the forenoon $\underline{\odot}$ 75° 45′. W. 11^h 48^m 12^s. C—W 3^h 01^m 28^s. Chronometer slow of G. M. T. 2^m 05^s. I. C.+1′. Height of eye 45 feet. After the lapse of about 20 minutes, the same limb of the sun was observed at the same altitude, W. 12^h 08^m 14^s. C—W 3^h 01^m 27^s. The chronometer error, I. C., and height of eye as before. Required the longitude at noon.

The sun's declination corrected for longitude is N. 12° 03′ 27″, the H. D. N. 50″.69, Eq. of T. corrected for longitude 1^m 25^s.7 (+ to M. T.), and from Table 26, Bowditch, $\Delta_0 h = 7''.13$.

Course.	Distance.	N	W
315°	20′	14′.1	14′.1

L_0 26° 07′ 03″ N
D= 15′.71 W
4

Hourly change of Long. = D= 62^s.84 W expressed as time.

Observer's change in lat. 14′.1 N per hour or 14″.1 N per minute.
Change in sun's dec. 50″.69 N per hour or 0.845 N per minute.
Δc (Art. 246) combined velocity of separation = 13.255 N per minute.

From formula (203), $t = \frac{\Delta c}{2\Delta_0 h} = \frac{13''.255}{14''.26} = 0^m.9295 = 0^m\ 55^s.77.$

As observer's zenith and the body are separating, *t,* the H. A. of maximum altitude is easterly, and as the ship changes longitude to the westward at the rate of 62^s.84 per hour, or 0^s.97 in 0^m.930, the corrected H. A. is 0^m 56^s.74 and the time of maximum altitude is $0^m\ 56^s.74$ before the instant of upper meridian transit.

A. M. Times.		P. M. Times.		Time of Apparent Noon.	
	h m s		h m s		h m s
W.	11 48 12	W.	12 08 14	G. M. T. of noon	3 02 42.24
C—W	3 01 28	C—W	3 01 27	Eq. of T.	+ 1 25.7
C. C.	+ 2 05	C. C.	+ 2 05		
				G. A. T. of noon	3 04 07.94
G. M. T.	2 51 45	P. M. G. M. T.	3 11 46	Long. =	46° 01′ 59″.1 West
		A. M. G. M. T.	2 51 45		
		Mid. G. M. T.	3 01 45.5		
		t East	56.74		
		G. M. T. of apparent noon	3 02 42.24		

Single and Double Altitudes.

278. What has been said about the general subject of single and double altitudes, their advantages, uses, limiting conditions, etc., under the head of chronometer error (Art. 268), applies to the subject of longitude when these methods of observation are used, either afloat or ashore.

The finding of longitude by these methods has been fully explained in the chapter on "Solutions of the Astronomical Triangle" (Arts. 226-232). The question of finding longitude at sea will be further amplified under the head of Sumner lines.

CHAPTER XIX.

SUMNER'S METHOD.—SUMNER LINES OR LINES OF POSITION.

279. A ship approaching the entrance to Chesapeake Bay, with Cape Charles light in sight on the starboard side and Cape Henry light on the port side, at a given moment, may be located at one of two points on a Mercator chart, without bearings having been taken, if the navigator knows the distance from each lighthouse. Say the ship is p miles from Cape Charles and q miles from Cape Henry; with a pair of dividers and a radius of p miles, describe a circle on the chart with Cape Charles light as a center. Being p miles distant from that lighthouse, the ship is somewhere on that circle which is a line of position, passing, as it does, through the position of the ship. If a bearing, sounding, or other determining factor can be gotten, a fix may be obtained.

If a second circle be described, with a radius of q miles, from Cape Henry light as a center, we shall have a second line of position, at some point of which also the ship is located.

Being on both circles at the same instant, the ship must be at one of the two intersecting points. If the ship's position is further restricted by a sounding, by latitude or by longitude, to the vicinity of one point, the other one, as a position, is eliminated.

280. Lines of position and how determined.—As previously defined, a line passing through a position of the ship, whether a position by D. R. or by observation, is a line of position; it may be straight or curved, and it may be determined from celestial bodies as well as terrestrial objects.

To Captain Sumner, an American shipmaster, is due the credit for first defining a ship's position upon a line, which he called a circle of equal altitudes, from the altitude of a heavenly body and its corresponding G. M. T.; and also for determining the ship's position at one of the two intersecting points of two such circles.

281. A heavenly body's geographical position.—Every heavenly body is at a given instant of time in the zenith of some point on the earth's surface; this point is the geographical position of the body; for the sun, it may be called the subsolar point, for any other heavenly body, the subastral point.

The theory being the same for all bodies, the method, as applicable to the sun, will be described.

282. The sun's circle of equal altitudes.—The sun being in the zenith of a given place, one-half of the earth will be illuminated (neglecting refraction), and the other half will be in darkness; the dividing line, called the circle of illumination, will be everywhere 90° from the subsolar point.

To observers anywhere on the circle of illumination, the sun will be in the horizon; at the subsolar point, the sun will be in the zenith, and therefore its altitude will be 90°. If the observer is at any intermediate point between the circle of illumination and the subsolar point, he will have the sun above his horizon and at an altitude less than 90°. If a plane be passed through this intermediate position, parallel to the circle of illumination, its intersection with the earth's surface will cut out a small circle, at every point of which, at the given instant, the sun will have the same altitude. This circle is called a circle of equal altitudes with respect to the sun, and the sun's zenith distance, at the given instant, is the same at all points of the circle.

Observed zenith distance as radius of a circle of position.—Since the distance of the observer's zenith from the heavenly body, in minutes of arc, is the same as the observer's distance in sea miles from the body's geographical position, and in the case of the sun from the subsolar point, when an observer measures the altitude of the sun, the complement of which is its zenith distance, he actually finds his distance in sea miles from a known spot on the earth, and hence locates himself on a circle of position exactly as did the observer, referred to in Art. 279, who found himself on a circle of position around Cape Charles or Cape Henry.

283. Coordinates of the geographical position of a heavenly body.—The geographical position of a heavenly body is located like any terrestrial point by its latitude and longitude; the latitude being the body's declination, the longitude the body's Greenwich hour angle. In the case of the subsolar point, the latitude equals the sun's declination, and the longitude the Greenwich apparent time.

Use of a terrestrial globe in connection with a Sumner circle.—If the subsolar point be located on a terrestrial globe and a circle, whose radius equals the observed zenith distance of the sun, is drawn on the globe, with the subsolar point as a center, the observer will be somewhere on the circumference of this circle; since the subsolar point bears in a given direction from him, his position is in the opposite direction from the subsolar point, so that the sun's azimuth at the time of the observation indicates the part of the circle on which the observer is situated; and his position would be fixed, if, having the above data, he should find his latitude or longitude, or a second circle of equal altitudes, projected from observations of a second heavenly body, or from observations again of the sun after a lapse of sufficient interval of time, the observer's position remaining unchanged.

However, this graphic method cannot be used for the reason that it is impracticable to carry a globe of such dimensions as to admit of accurate results.

284. On a Mercator chart.—The circles of equal altitude will appear on this chart as shown in Plate XV, end of book, being drawn out towards the North and South points for reasons apparent to anyone familiar with the theory of the Mercator projection. These curves are called "Curves of equal altitudes." Fig. 121, right-hand side, shows the curves at intervals of 10°, in which S_1 is the geographical position of the body observed; all these curves belong to the same system which Sumner called a system of illumination. It will be noticed that all these curves cut the parallels of latitude and meridians of longitude at different angles. Near the North and South points, the curves run about East and West with the parallels of latitude, and a large error in longitude makes but a slight error in latitude; near the East and West points, the curves run with the meridians and a large error in latitude makes but a slight error in the resulting longitude; at intermediate points, the curves cut the parallels and meridians at varying angles, so that the error in longitude due to a given error in latitude depends on the body's azimuth.

These facts can be regarded as additional proofs that bodies should be observed for latitude when on or near the meridian, and for longitude when on or near the prime vertical.

Determination of points on the curve.—If an observer has a given altitude, different assumed latitudes, within the limits of the curve, will give him different longitudes, and vice versa. Each latitude will give two points, one for an altitude East, one for an altitude West of the meridian through the observed body. By assuming a sufficient number of coordinates, the whole curve may be plotted.

285. (a) **Double altitude observations.**—Suppose that on an afternoon in April, an observer at sea on the North Atlantic

Ocean, observes the true altitude of the sun's center to be 50°, the G. M. T. of observation being $1^h\ 15^m\ 30^s$.

The sun's corrected declination is 10° N., the equation of time $0^m\ 06^s$, additive to mean time. The G. A. T. is therefore

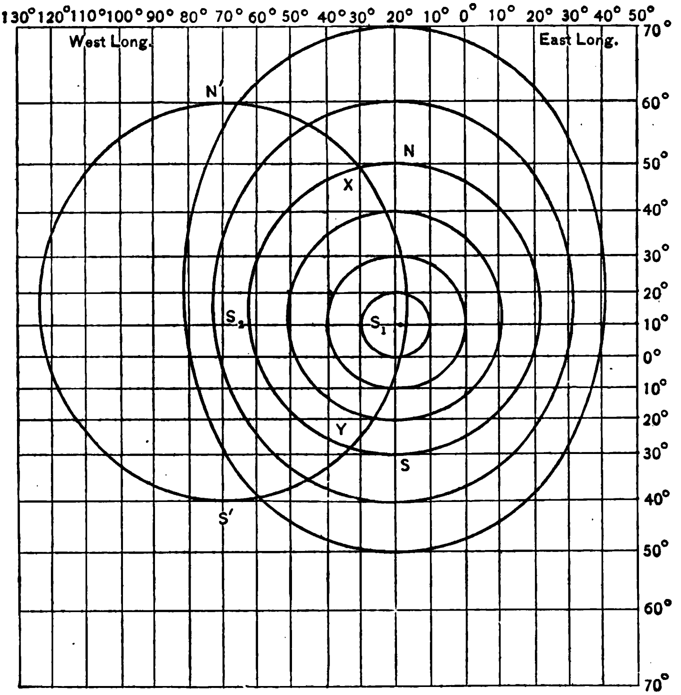


FIG. 121.

$1^h\ 15^m\ 36^s$. Hence the latitude of the subsolar point S_1 is 10° N., and its longitude $1^h\ 15^m\ 36^s$ West, or 18° 54′ W.

From this point S_1 as a center, the curve of equal altitudes with radius of 40° will be the right-hand curve *NXS* on the

Mercator chart (Fig. 121). This curve tells us nothing more than the bare fact that the observer is somewhere on its circumference. However, if the bearing of the sun at the instant of sight is given, the quadrant containing the position is indicated. If, in example, the sun bears southward and eastward, the ship is in the NW. quadrant. Having obtained the curve and bearing of the sun, if either latitude or longitude is given, the ship's position is determined.

Again, suppose that after the lapse of 3^h 24^m 24^s a second observation shows the sun's altitude to be 40°. During this interval, the sun in its diurnal path will have passed to the westward at the rate of 15° of longitude per hour, carrying with it its geographical position and its system of curves of equal altitudes. The G. A. T. becomes 4^h 40^m, or the longitude of the subsolar point S_2 is 70°. The declination is N. 10° 03′ 01″, or the latitude of S_2 is N. 10° 03′ 01″, the zenith distance is 50°, the curve on the Mercator chart is $N'XS'$, and the ship is somewhere on this curve. What we know now is that at the first observation the ship was on the right-hand curve NXS, and at the second observation she was on the left-hand curve $N'XS'$; therefore, if the ship did not change her position in the interval between the observations, she was at one or the other of the two points (X or Y), in which the curves intersected, and the one which was the observer's position depends on the sun's bearings at the times of observation.

(b) **In case the observer changes his position between the observations.**—If the observer, whose position is somewhere on the right-hand curve, can be supposed to make an instantaneous change of position, through a distance of N sea miles directly towards, or directly away from the geographical position S_1, on a great circle passing through S_1, the sun's altitude will be increased or diminished by N minutes of arc; if the course is kept at right angles to the bearing of the sun, he will keep on his original curve of altitude; if the course is at intermediate angles, the altitude will be changed proportionally (the change being expressed in Art. 213 by formula $\Delta h = d \cos (C \sim Z)$), and the observer will be on another circle of equal altitudes, belonging, however, to the same system of circles, that is, the system having S_1 as a center.

In going then from a point on one circle of equal altitudes, which was the ship's position at the first altitude, on a certain course and distance made good, the observer arrives at a point on another circle of equal altitudes, of the same system, however. The altitude at this latter circle corresponds to what would have been observed there at the instant of the first observation on the original circle.

The circle of position can then be found after a run to the place of a second observation, either by reducing the first altitude to what it would have been at the place of the second observation, at the time of the first observation (see Art. 213); or by taking a point in the first curve, representing the ship's approximate position, laying off the course and distance made good in the interval from it to a second point, and then drawing through this second point a curve parallel to the corresponding part of the first curve of altitude. The intersection of this transferred curve with the curve of altitude of the second observation will give the ship's position at the time of the second observation. The general method described in the two sections of this article is known in modern navigation as "Sumner's double altitude method."

Ordinarily, it is impracticable to plot circles of position on a Mercator chart without previous calculations for many apparent reasons, but under certain circumstances this may be done, and from two circles the fix may be found with considerable accuracy.

In certain cases, as may happen in the tropics, the sun may be observed when close to the zenith, say within a degree or so; the subsolar point located by its latitude and longitude (the latitude being the sun's declination and longitude the G. A. T. of observation); and that portion of the circle near the ship's D. R. position drawn with the true zenith distance as a radius.

After the sun's azimuth has altered from 25° to 30°, and under such circumstances it will do so in a very short time, draw a second arc as the result of a second observation. Transfer the first arc for the run between sights, and the intersection of the transferred arc with the arc corresponding to the second observation will give the fix with a fair degree of accuracy, and this without any of the usual calculations.

In using the method of double altitudes, there should be a change of bearing of the sun between observations of at least two points; of course, the nearer the change is to 90°, the more nearly the resulting lines of position run at right angles to each other, and the better the cut.

Rapidity of change of azimuth dependent on *L* and *d*.— The rapidity of change of the sun's azimuth will depend on the values of *L* and *d*, and the time an observer has to wait for that body to undergo a desired change of bearing may be found by inspection of the azimuth tables. The greater the difference between the values of *L* and *d*, the smaller will be the elapsed interval for a given change; for this reason, the interval in winter months will be smaller for observations of the sun taken on the same side of the meridian.

When the latitude and declination are nearly the same, it will be impossible to work the double altitude problem from observations on the same side of the meridian; however, having obtained an a. m. sight, if the meridian observation is lost, it will not take long for the sun after crossing the meridian to alter the first azimuth 90°, so that lines giving excellent cuts may be obtained by combining observations on both sides of the meridian.

286. Simultaneous observations.—The principles of the Sumner double altitude method, as explained in the case of the sun in the preceding article, apply as well to any other heavenly body; but, as a general thing, when one star may be observed, others are available, so that two (or more) may be observed at one time and the ship may be located at one of the two intersecting points of the resulting circles of position. Suppose two stars are observed at the same moment at a given place; that one, whose subastral point is S_1 (Fig. 121), has an altitude of 50° and bears southward and eastward, and that the other, whose subastral point is S_2, has an altitude of 40° and bears southward and westward. The result of the first observation locates the ship on the NW. arc of the circle of position NXS, the result of the second on the NE. arc of the circle $N'XS'$, and, therefore, at their northern intersection X. Such observations are known as simultaneous observations; for these observations, bodies should be so selected that the resulting circles or lines of position will cut at good angles, not less than 30°.

Advantages of simultaneous over double altitude observations.—The former are preferred to the latter as the position of the ship may be obtained at once without an interval of waiting and the errors of the run when there is a change of position; besides, a third line may be obtained and the fix from two lines either verified or disproved, the fix being verified when the three lines have practically the same point of intersection.

It is probable that the navigator may have such a number of first or second magnitude stars to select from that good observations may be obtained in all latitudes.

287. Relation between circles of equal altitude and the astronomical triangle.—Let Fig. 122 represent a projection on the horizon of a point S the geographical position of a heavenly body; PQ, the meridian of that point; PG, the meridian of Greenwich; PZ_1 and PZ_2, the meridians of places on the earth's surface having at the same instant of time the same altitude of the body S. $Z_1Z_2Z_3$ is a Sumner curve or a circle of equal altitudes with respect to the body S; QS, the latitude of the geographical position equals the body's declination; GPS, the Greenwich hour angle of the body, is the

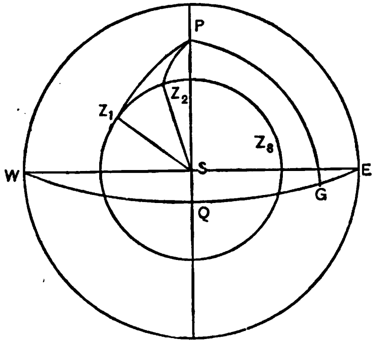


FIG. 122.

longitude of S. The triangles Z_1PS and Z_2PS are projections of astronomical triangles. The angle Z_1PS is the hour angle of the body at a place Z_1 on the circle of equal altitudes; Z_2PS, the hour angle of the same body at the same instant at the place Z_2, also on the same circle.

Since the Greenwich hour angle, the declination, and the altitude are the same at Z_1, Z_2, and other places on the same circle of altitude, the astronomical triangles Z_1PS, Z_2PS, etc., have two sides of one equal to two sides of the others, one side equal to $90° - h$ and the other equal to $90° - d$, but they differ in the values of the third side (PZ_1, PZ_2, etc.), which side is the complement of the latitude, and also in the values of the local hour angles, Z_1PS, Z_2PS, etc. The hour angles of the body at the local meridian, and hence the longitudes, are dependent on the different assumed values of the latitude when solving the astronomical triangle with given values of h and G. M. T. For the sun, the hour angles Z_1PS, Z_2PS, etc., are the local apparent times, or 24 hours—those apparent times, according as the times are less or greater than 12 hours; GPS is the Greenwich apparent time, or 24 hours—that time, according as the apparent time at Greenwich is less or greater than 12 hours; and GPZ_2 and GPZ_1 are the longitudes from Greenwich, respectively, of Z_2 and Z_1.

By assuming latitudes and finding the corresponding longitudes, or by assuming longitudes and finding the corresponding latitudes, any number of points of the curve may be found and the whole circle projected.

288. Rule for assuming coordinates.—The rule for assuming coordinates, based on what has been said as to the varying angles at which the curve of equal altitudes cuts meridians and parallels of latitude (Art. 284), and the demonstrations (see Arts. 237 and 248) as to the best times to observe for latitude or longitude, is as follows: assume latitudes and solve for longitudes when the body's Z_N lies between 45° and 135°, or 225° and 315°; otherwise, assume longitudes and solve for latitudes.

289. Actual sea practice and method of determining the line.—In actual practice at sea, it is never necessary to determine more than a small portion of the circle of equal altitudes, since the observer's position is generally known to be within certain limits, both of latitude and of longitude. This small portion is the only part to be considered; it is called **a line of position, and is at right angles to the heavenly body's true bearing.** If three or more coordinates are assumed, especially if they are far apart or the body's altitude is great, the line may be a curved line. *It is customary,* *however, when working for longitudes, to assume two latitudes differing say by 20′, or when working for latitudes to assume two longitudes differing by two minutes of time or 30′ of arc; in both cases, the dead reckoning position should be between the assumed coordinates.* For such short distances the chord thus obtained is practically coincident with the included arc of the circle. This is known as the "**method of chords**" and for years was the practice of the officers of the U. S. Navy. It is evident that, in the case of a line thus determined, its angle with the meridian may be found by either middle latitude or Mercator sailing, and thence the true azimuth of the body whose bearing is at right angles to it.

Since the circle of equal altitudes is at right angles to the true bearing of the body, a tangent to the circle at a given point, and for short distances either side, may be taken as practically the same as the arc itself. Therefore, to determine a line, assume a latitude and find the corresponding longitude, or assume a longitude and find the corresponding latitude, both assumptions within the limits of the curve, thus determining one point of the circle of equal altitudes.

The true azimuth of the body for the instant of observation having been determined in one of three ways (1) from the azimuth tables (Art. 221) or an azimuth diagram; (2) by observation, the compass bearing being corrected for variation and deviation of the compass; (3) by solution of the astronomical triangle (Arts. 218 and 219); (2) and (3) not being, however, the usual practice at sea; a line is drawn through the determined point at right angles to the body's true bearing. This line is a line of position, and this method of determining it is known as the "**method of tangents.**"

290. To define a line of position.—From what has been said in previous articles, a line may be defined in two ways—when determined by the chord method, it is defined by its two points A_1, A_2, thus:

$$A_1\begin{cases}18^\circ\ 50' & \text{S.}\\ 2^\circ\ 46'\ 24'' & \text{W.}\end{cases}\qquad A_2\begin{cases}19^\circ\ 10' & \text{S.}\\ 2^\circ\ 37'\ 31'' & \text{W.}\end{cases}$$

When determined by the tangent method, it is defined by its one position point A and its direction thus:

Position Pt. A	19° 00′ S. 2° 41′ 58″ W.	Azimuth of body $Z_N = 67° \; 10'$ obtained from azimuth tables, given $L = 19°$ S, $t = 8^h \, 01^m \, 52^s$, $d =$ N 11° 50′.
Line of Position	337° 10′	

From the data of line A_1A_2, by middle latitude sailing, the direction of the line is found to be 337°.2 and the body's true bearing, Z_N, 67°.2. In other words, the line has the same direction, whichever way it is determined.

There is a third method of defining a Sumner line described in the next chapter. It is known as the Method of Saint-Hilaire, and consists in laying off from an assumed geographical position, along the line of direction in which the observed body bore at the time of the sight, the determined distance to the Sumner line.

Coordinates "computed" and "by observation."—In this work, wherever they occur, the terms Lat. and Long. by observation will be taken as applying only to the ship's position or fix; the term computed latitude, as referring to that obtained from a sight by using the D. R. longitude or an assumed longitude; and the term computed longitude, as referring to that obtained by using a D. R. latitude or an assumed latitude.

The method of determining the line and the methods of finding the intersection of two lines will be considered in Arts. 295-310.

291. Uses of a Sumner line.—If the G. M. T. and altitude of the heavenly body are correct, the line determined from the data passes through the position of the ship, and if the line, or the line produced, passes through a lighthouse, point of land, or a danger, the direction of the line gives at once the bearing of that particular object; if desiring to make that light or point of land, the navigator knows the course to steer; to avoid the object, if a danger, it is only necessary to run at right angles to the direction of the line for a safe distance, and then, by changing the course not more than 90° from this last course, the ship will go clear.

If in Fig. 123, A_1A_1 is a line of position passing through

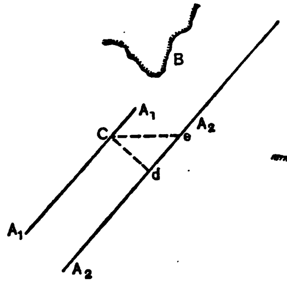


FIG. 123.

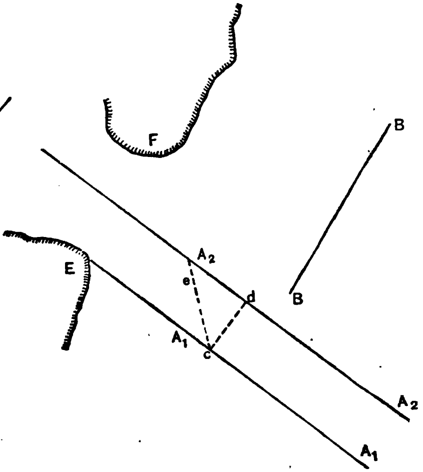


FIG. 124.

a lighthouse or point of land *B*, the course to be steered to make *B* is the direction of the line towards *B*.

If *B* is a danger, run the course and distance represented by *cd*, or, if safe, run a proper distance in the direction *ce*; then the direction A_2A_2 will clear the danger.

If the direction of the line is into the port of destination, the course in will be known; if its direction is towards a point to one side of the entrance, A_1A_1 (Fig. 124), draw a line A_2A_2 on the chart from the entrance *EF*, parallel to the line of position, shape a course *cd* at right angles to A_1A_1, or, if safe, a course in direction *ce* till the vessel arrives on the parallel line A_2A_2; then steer in its direction for the entrance. If the line runs parallel to the coast (*BB*, Fig. 124), the distance off shore will be known.

A fix may be obtained by a verified sounding or by a bearing of an object of known position on the chart; and in this connection, when in the vicinity of dangers, attention is again called to the fact that, *if a line is obtained from an observation of a body on the prime vertical, the longitude will be well determined; if from an observation of a body on the meridian, the latitude will be well determined; even though the other coordinate may be somewhat in error.*

Owing to the fact that a line is always at right angles to the bearing of the body, it is often possible, especially at night when heavenly bodies in all directions are available, to get a line running in any desired direction, so as to show the bearing of land, distance of coast, etc.

If an observation of a heavenly body is taken *when bearing directly abeam*—and the opportunities are many for so observing not only stars but the sun—*the resulting line of position will be in the direction of the course;* and if the line leads clear of danger the navigator may keep his course, if towards danger he may run off 90° for a safe distance, then resume his course, clearing the danger.

Two lines intersecting at angles of not less than 30° (90° preferred) will give good fixes. When possible, it is better to verify this fix by a third line.

During morning or evening twilight, or moonlight, when stars are visible, several may be observed at the same time, and they may be so selected that the corresponding lines will cut at excellent angles, and hence give excellent fixes.

GRAPHIC OR CHART INTERSECTIONS.

292. Finding the noon position on a Mercator chart and the intersection of a line with another moved parallel to itself for the run between observations.—The parallel of the latitude found at noon is nothing more nor less than a line of position obtained when the body is observed on the meridian, and the noon position is the intersection of this

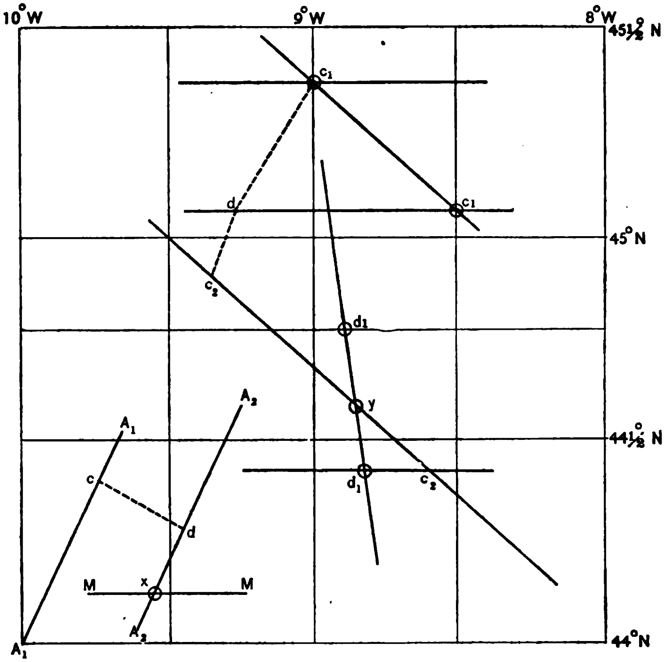


FIG. 125.

parallel with the a. m. line moved for the run between observations

Let Fig. 125 be a section of a Mercator chart with Sumner lines plotted thereon. Suppose the a. m. observation gave a line A_1A_1 (Fig. 125), and that the run to noon was 120° (true) 15 miles (in the direction of and equal to *cd*), and the noon latitude was that of the parallel *MM*; then the intersection of *MM* with A_2A_2, the forenoon line moved up for the run, was the noon position.

Again, in Fig. 125 c_1c_1 is a line of position obtained by working a $\phi''\phi'$ sight of the sun for latitude, having assumed longitudes 8° 30′ W. and 9° W. After sailing 211° true 21.4 miles (c_1d) and 205° 10.8 miles (dc_2), a second line d_1d_1 was found from an observation of the sun worked as a time sight; it is required to find the ship's position at the second observation. Having plotted on the Mercator chart the first line c_1c_1 by its coordinates, it is apparent that the ship is somewhere on the line at the time of the observation, though the exact point is unknown. From any point of this line c_1c_1, lay off the true courses and distances run (in this case in the directions c_1d and dc_2) and through the determined point draw c_2c_2 parallel to c_1c_1; the ship at the time of the second observation is on the line c_2c_2. The second line by observation is plotted by its coordinates and intersects c_2c_2 in y which is the position of the ship at the time of the second observation.

The data for the two lines and for the ship's run of the second case in this article are given below.

First line c_1c_1	λ'_1	8° 30′	W	λ'_2	9° 00′	W
	L'_1	45° 04′ 03″	N	L'_2	45° 22′ 12″	N
Second line d_1d_1	L''_1	44° 26′ 40″	N	L''_2	44° 46′ 40″	N
	λ''_1	8° 47′ 44″	W	λ''_2	8° 52′ 26″	W

Run between lines

211° (true) 21.4 miles.
205° (true) 10.8 miles.

293. Uncertainty in G. M. T.—If there is an uncertainty in the G. M. T., parallels may be drawn on either side of the line, at a distance in longitude equal to the amount of the uncertainty, so that the true position will then be restricted in the case of a single line to a belt instead of a line, and in case of two lines to the area of a small parallelogram.

294. Uncertainty in altitude.—If there is an uncertainty in altitude, parallels to the line may be drawn each side, at a perpendicular distance from it in nautical miles equal to the number of minutes of error in altitude.

For a given uncertainty, to illustrate say 1′ of altitude, **when one body is on the meridian and the other on the prime vertical,** the position may be anywhere in a square, with a maximum uncertainty of 2′ both as to latitude and longitude. Thus if **aa′** and *bb′* (Fig. 126), represent the two lines of position, the observer would be at *O*, provided there was no error; but to allow for a possible error of 1′ of altitude, + or —, lines must be drawn on each side at the perpendicular distance of one sea mile. The observer may be at 1, 2, 3, or 4, or anywhere within the square, making the limits of uncertainty two miles for both latitude and longitude.

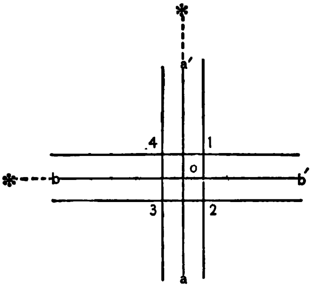


FIG. 126.

For a difference of azimuth of eight points, where neither body is on the meridian or prime vertical, the rectangle will be shifted, and the uncertainty in latitude and longitude will increase till when the lines run NE. and NW. (SW. and SE.), each becomes a maximum; the position may vary in latitude 2.8 sea miles, and in longitude the same amount (Fig. 127).

For a difference of azimuth greater or less than eight points.—When the difference of azimuth of the two lines is greater or less than 90°, an error in altitude, + or (—), will affect the possible position of the ship so as to make the uncertainty in latitude greater and in longitude less, or vice versa, according to the direction in which the parallelogram is elongated. For instance, if the difference of azimuth is small and the mean azimuth is near East or West, as in Fig. 128, where aa′ and *bb′* are the lines of position, it is seen that the possible variation in longitude (1 to 3) is small compared to the variation in latitude (2 to 4). So that, whilst longitude may be determined when the mean azimuth is near E. or W., the latitude may be far out when the difference of azimuths is small. Exactly the reverse is true when the mean azimuth is near N. or S. and the difference of azimuth is small.

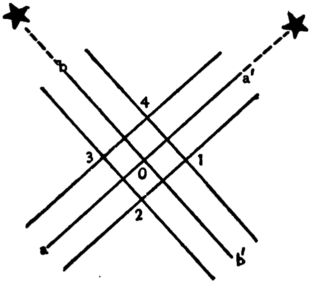


FIG. 127.

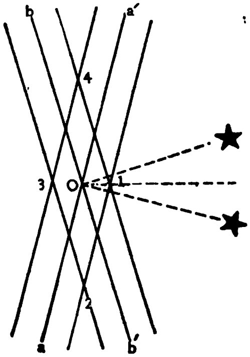


FIG. 128.

Finding the intersection of Sumner lines.

295. The intersection of lines of position may be found:

(1) By plotting, on a Mercator chart of the locality in which the ship may be, the lines determined (a) by the chord method, (b) by the tangent method, or (c) by the Method of Saint-Hilaire.

(2) By computation.

296. (a) **The plotting of lines determined by the chord method.**—If a line of position is determined from celestial observations by assuming two latitudes and finding the corresponding longitudes, or, by assuming two longitudes and finding the corresponding latitudes, the assumed coordinates being about equally distant each side of the dead reckoning position, it is plotted on a Mercator chart by locating the two points thus determined and drawing a straight line between them.

If a second line is plotted in the same way, the observer will be at the intersection of this second line with the first, provided there has been no change of his position in the interval between observations; however, if there has been a change, then the observer's position will be at the intersection of the second line with the first line after having been moved parallel to itself for the run in the interval. The principles involved are shown in Fig. 125.

In case the observation is of a body on the meridian, the line of position becomes a parallel of latitude; if the body is observed on the prime vertical, the line will be a meridian.

297. (b) **The plotting of lines determined by the tangent method.**—In this method, take the D. R. latitude and determine from the given observations, by solution of the astronomical triangle, the corresponding longitude, calling it computed longitude; or take the D. R. longitude and find the corresponding latitude, calling it computed latitude. The point thus determined will be one point of the line and is plotted on the chart. Through this point draw a line at right angles to the bearing of the body for the instant of observation, this bearing being found from the azimuth tables having given *L, d,* and *t,* or from an azimuth diagram. The line drawn will be a line of position. ***Considering the azimuth of the body less than 90°, the direction of the line is easily obtained from the true bearing of the body by reversing either***

*letter of the bearing and taking the complement of the angle. Thus, if the body bore S. 30° E., the corresponding line of position runs N. 60° E. or S. 60° W.**

The second line having been plotted in the same way, the observer's position will be at the intersection of the two lines, as explained in Art. 292.

The plotting of lines by the Method of Saint-Hilaire is explained at page 648, chapter XX.

298. Before explaining the methods of finding the intersection of Sumner lines by computation, it is desirable to give a few definitions.

Definition of longitude factor.—The longitude factor of a line of position, represented by the letter *F*, is the change in longitude due to 1′ change in latitude. In the case of a line determined by the chord method, it is found directly by dividing the difference of the longitudes of the two points by the difference of their corresponding latitudes, or $F = \frac{\lambda_2 \sim \lambda_1}{L_2 \sim L_1} = \frac{\Delta\lambda}{\Delta L}$, where ΔL is a change in latitude due to a change of $\Delta\lambda$ in longitude, and vice versa (Fig. 129).

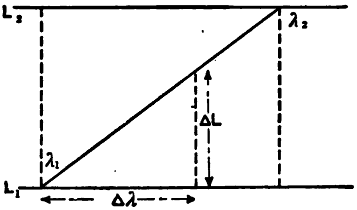


FIG. 129.

Definition of latitude factor.—The latitude factor of a line of position, represented by *f*, is $\frac{1}{F}$, or the change in latitude due to a change of 1′ of longitude. In this method of defining a line, $f = \frac{L_2 \sim L_1}{\lambda_2 \sim \lambda_1} = \frac{\Delta L}{\Delta\lambda}$ (Fig. 129).

When a line is determined by the tangent method, *F* equals $\frac{\Delta\lambda}{\Delta L}$ and *f* equals $\frac{\Delta L}{\Delta\lambda}$, as before; but in this case it is necessary to investigate and ascertain the relation between ΔL and $\Delta\lambda$ and the determining quantities of a tangent line, namely latitude and the body's azimuth.

---

* If the true bearing of the body is given in the form of Z_N, simply add or subtract 90° to obtain the true direction of the line as estimated from 0° at North around to the right.

Values of F and f and where they may be found.—By reference to Art. 237, it is seen that, by differentiation of the general equation of the astronomical triangle, $\frac{dt}{dL} = \sec L \cot Z$, dt being the change in longitude due to a change of dL in latitude. Now, if dL represents one minute of latitude, $dt = \sec L \cot Z$ is the change in longitude due to a change of 1′ of latitude, and, therefore, the longitude factor

$$F = \sec L \cot Z \quad (236).$$

This may be shown graphically from Fig. 130 and Fig. 131, in which the azimuth is considered as less than 90°. In Fig. 130, the body bears either in the NE. or SW. quadrants and the direction of the line of position is NW. or SE. In Fig. 131, the sun bears in the NW. or SE. quadrants and the direction of the line is in the NE. or SW. quadrants. In both cases,

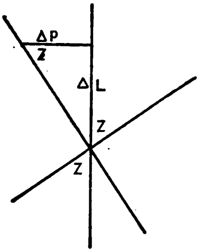


FIG. 130.

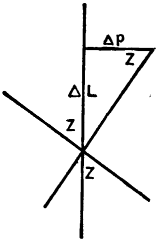


FIG. 131.

$$\left.\begin{array}{lr} & \Delta p = \Delta L \cot Z; \\ \text{but} & \Delta p = \Delta\lambda \cos L; \\ \text{therefore,} & \Delta\lambda \cos L = \Delta L \cot Z, \\ \text{and} & \Delta\lambda = \Delta L \sec L \cot Z; \\ \text{but} & \Delta\lambda = \Delta L \times F; \\ \text{therefore,} & F = \sec L \cot Z. \end{array}\right\} \quad (236)$$

and the quantity sec L cot Z is the longitude factor. It is tabulated in Table I of this book, the arguments being L and

Z. This factor may also be found in Table 47 of Bowditch, in Table C of Lecky, in Inman's Tables, and in Table II of A. C. Johnson's most excellent work on finding latitude and longitude in cloudy weather. Since $f = \frac{1}{F}$, f may be found from the above tables by first finding from them the value of F and taking its reciprocal. This factor is tabulated in Table 48, Bowditch.

It is now understood that if F is known and either ΔL or $\Delta\lambda$ also known, the other may be found from the expression

$$\Delta\lambda = \Delta \mathrm{L} \times \mathbf{F}, \tag{237}$$

and also that $$\Delta \mathrm{L} = \Delta\lambda \times \frac{1}{\mathbf{F}}, \text{ or } \Delta \mathrm{L} = \Delta\lambda \times \mathbf{f}. \tag{238}$$

Rule for naming ΔL and $\Delta\lambda$ when the azimuth of the body is given.—*Regarding the azimuth of the body as less than 90° and as estimated from either the North or South point of the horizon towards East or West, we have the following obvious rule: "If the change in latitude ΔL is of the same name as the first letter of the bearing, the change in longitude $\Delta\lambda$ is of the contrary name to that of the second letter, and vice versa." Thus, if the body bears S. 45 E., if ΔL is S., $\Delta\lambda$ is W.; and if ΔL is N., $\Delta\lambda$ is E.*

INTERSECTION BY COMPUTATION.

299. We are now prepared to find the intersection of lines of position by computation.

The simplest case to be dealt with occurs when one line runs due N. or S., as when a body is observed on the prime vertical, and another line runs due E. or W., as when a body is observed on the meridian. The longitude of the ship is that of the first line, the latitude is the parallel of the second line.

Ex. 213. April 15, 1912, p. m., Long. W., Lat. S., observed simultaneous altitudes of α Leonis (Regulus) on the meridian bearing North and of α Canis Majoris (Sirius) on the prime vertical West of the meridian. Sextant altitude of Regulus 52° 48' 40", of Sirius 42° 18' 30". I. C. +1'. Height of eye 25 feet. W. T. of obs. 8h 32m 10s. C—W 2h 28m 20s. Chron. slow of G. M. T. 3m 07s.3. Required the ship's position.

Mer. Alt. of Regulus.

Alt and Lat.		Corr. to Altitude.		Times.	
	° ' "		' "		h m s
☆'s h_o	52 48 40	I. C.	+ 1 00	W.	8 32 10
Corr.	4 38	D.	4 54	C—W	2 28 20
		R.	— 0 44		
☆'s h_o	52 44 02 N			C.	11 00 30
☆'s z_o	37 15 58 S	Corr.	4 38	C. C.	+ 3 07.3
☆'s d	12 21 54 N	Or (Tab. 46, Bowditch)	—5' 38"	G. M. T. 15th April	11 03 37.3
Lat.	24 54 04 S	I. C.	+1 00	R. A. M. ☉	1 33 09.1
		Corr.	— 4 38	G. S. T.	12 36 46.4

Observation of Sirius.

Altitude.		Corr. to Altitude.		R. A. M. ☉.	
	° ' "		' "		h m s
☆'s h_o	42 18 30	I. C.	+ 1 00	At G. M. N.	1 31 20.1
Corr.	— 4 58	D.	— 4 54	Corr. G. M. T.	1 49.0
		R.	— 1 04		
☆'s h	42 13 32				1 33 09.1
		Corr.	— 4 58		
		Or (Tab. 46)	= —5' 58"		
		I. C.	+1 00		
		Corr.	—4 58		

Star.	R. A.	Dec.
	h m s	° '
Regulus	10 04 03.1	N 12 21.9
Sirius	6 41 33.2	S 16 36.4

	° ' "		
h	42 13 32		
L	24 54 04	sec	10.04237
p	73 23 36	cosec	10.01851
	140 31 12		
s	70 15 36	cos	9.52860
s — h	28 02 04	sin	9.67210
		2	19.26158
	h m s		
(Sirius) t	+ 3 22 24.0	sin	9.63079
" R. A.	6 41 33.2		
L. S. T.	10 03 57.2		
G. S. T.	12 36 46.4		
Long.	2 32 49.2 W		
Arc	38° 12' 18" W		

300. The next case is where the line of position (original or transferred for run), runs at an angle with both parallels and meridians, and is intersected by a line running due E. and W.; this last line being a parallel of latitude from the meridian altitude of a body.

This is a case of simultaneous observations, say of two stars; one on the meridian, the other off the prime vertical at the time of observation. As in example 213 the latitude from the meridian observation, being well determined, is used to work the time sight.

Again, a case under this heading occurs when finding the noon position from an a. m. observation, a run to noon, and latitude from a meridian altitude of the sun. In the latter case, there are two ways of finding the intersection, according as the a. m. line is determined by the chord or tangent method.

301. (1) **The chord method.**—Assume two latitudes, about 10′ each side of the D. R. latitude, work a line of position, obtaining the longitudes corresponding to the two assumed latitudes and hence two points of the line. Divide the difference of the computed longitudes by the difference of the two assumed latitudes. The result is the longitude factor F.

Correct each position of the line for the run to noon, obtaining the corresponding points of the line at noon. The difference between the latitude of one point of this line, after being moved for the run, as an origin, and the latitude by meridian altitude is ΔL but $\Delta\lambda = \Delta L \times F$.

Knowing which way the line of position runs, the sign of application of $\Delta\lambda$ is apparent. The result obtained by applying $\Delta\lambda$ to the longitude of the point taken as an origin will be the noon longitude by observation.

Rule for naming ΔL and $\Delta\lambda$ when the direction of the line of position is given.—*Regarding the direction of the line as an angle less than 90°, and as estimated from either the North or South point of the horizon, towards the East or West* *point, we have the following rule: "If ΔL is of the same name as the first letter of the direction, $\Delta\lambda$ is of the same name as the second letter; and vice versa." Thus, if the line runs N. 30° E., and the change of latitude is to the northward, the change of longitude will be to the eastward. This rule applies to the "chord method," and the rule in Art. 298 to the "tangent method."*

If the longitude by observation is desired at the time of the a. m. sight, having found the a. m. line, it is only necessary to run the noon latitude back to the time of a. m. sight by applying the run in latitude from sight to noon backward, thus getting the true latitude at the time of the a. m. sight.

The difference between this true latitude at the time of a. m. sight and the latitude of one point of the line in its a. m. position as an origin, multiplied by the longitude factor F, gives the correction in longitude, or $\Delta\lambda$, to be applied to the longitude of the same point. The result will be the longitude by observation at the time of the a. m. sight.

If, to this, the run in longitude from the time of sight till noon is applied, the result will be the longitude at noon by observation, which should agree with that obtained as in the previous article.

302. (2) **The tangent method.**—Work up the dead reckoning to the time of a. m. longitude sight. Work the time sight with the D. R. latitude, calling the resulting longitude computed longitude.

With the latitude, declination, and L. A. T. from the sight, find the sun's true azimuth from the azimuth tables or an azimuth diagram.

The azimuth must be considered as less than 90°; so, if that from the tables exceeds 90°, estimated from one pole, use its supplement and reckon it from the opposite pole.

With the D. R. latitude and the sun's azimuth, find from Table I (or Table C in Lecky, Table 47, Bowditch, or from Inman's Tables) the longitude factor F; write the value of F

in the form for work, and near it the direction of the line thus: $F = a. \diagup$, meaning that the variation in longitude for 1′ of latitude is a and the line of position runs NE^d. and SW^d., or $F = a. \diagdown$ in case the line runs NW^d. and SE^d.; *the direction of the line being obtained from the bearing (regarded as less than 90°) by changing either letter of the bearing and taking the complement of the angle.*

To this D. R. latitude and computed longitude apply the run to noon, obtaining at noon a D. R. latitude and a computed longitude. The line at noon remains parallel to its direction at time of sight, and F has, of course, the same value.

With the computed longitude, work the meridian altitude sight and find latitude at noon by observation.

The difference between the latitude at noon by D. R. and by observation is ΔL, or the error in latitude. As before, $\Delta\lambda = \Delta L \times F$.

In the absence of Table I, the correction $\Delta\lambda$ may be found thus: Enter Table 2 of Bowditch's Useful Tables with the complement of the bearing as a course, find ΔL in the latitude column, and take the corresponding departure from the departure columns. This departure, converted into difference of longitude, will be $\Delta\lambda$.

Having found $\Delta\lambda$, its sign of application may be found from rule of Art. 298, if the azimuth of the sun is considered; or, rule of Art. 301, if the direction of the line is considered. Apply $\Delta\lambda$ to the computed longitude for noon, the result will be the longitude at noon by observation.

Should it be desired to find the true longitude at time of sight, run the noon latitude back to the time of the a. m. sight by applying the run in latitude from sight to noon backwards, getting the true latitude at time of sight. The difference between this latitude and the D. R. latitude at sight is ΔL; then $\Delta\lambda = \Delta L \times F$ is the correction in longitude to be applied to the computed longitude at time of sight to give the longitude by observation at that time.

SOLUTION BY THE CHORD METHOD.

Ex. 214.—April 3, 1918, about 7.30 a. m., in Lat. by D. R. 25° 40′ S., Long. East, the sextant altitude of the sun's lower limb was 18° 14′. I. C. —1′. Height of eye 19 feet. W. 7^h 30^m 23^s. C—W 5^h 15^m 03^s. Chro. fast of G. M. T. 5^m $41^s.2$. Work a line of position using latitudes 25° 30′ S. and 25° 50′ S.

Having run to noon 109° (true) 46 miles, the latitude by meridian altitude of the sun was found to be 25° 52′ S. Find the longitude factor, run the line up to noon, and find the longitude by observation at noon.

Times.		Altitudes.		Altitude Corrections.		Declination.	H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″	° ′		+to A. T.	— $^s.7$
W.	7 30 23	⊙	18 14 00	S. D.	+16 01	N 4 54.7	N 1′.0	m s	$0^h.66$
C—W	5 15 08	Corr.	+ 7 58	I. C.	— 1 00	N .7	$0^h.66$	3 39.3	
				Dip	— 4 16			— .5	$—0^s.462$
C.	12 45 26	⊖	18 21 58	p. & R.	— 2 47	N 4 55.4	N 0′.66		
C. C.	— 5 41.2							3 38.8	
				Corr.	+ 7 58				
G. M. T. April 2,	12 39 44.8			Or (Tab. 46)					
or	$12^h.66$				= +8′ 58″				
				I. C.	—1 00				
				Corr.	+7 58				

	° ′ ″				° ′ ″		
h	18 21 58			h	18 21 58		
L_1	25 30 00	sec	10.04451	L_2	25 50 00	sec	10.04573
p	94 55 24	cosec	10.00160	p	94 55 24	cosec	10.00160
	138 47 22				139 07 22		
s	69 23 41	cos	9.54645	s	69 33 41	cos	9.54308
$s-h$	51 01 43	sin	9.89067	$s-h$	51 11 43	sin	9.89169
			2 | 19.48323				2 | 19.48210
	h m s				h m s		
L. A. T.	19 32 11.5	sin	9.74162	L. A. T.	19 32 35	sin	9.74105
Eq of T.	+ 3 38.8			Eq. of T.	+ 3 38.8		
L. M. T.	19 35 50.3			L. M. T.	19 36 13.8		
G. M. T.	12 39 44.8			G. M. T.	12 39 44.8		
Long.	6 56 05.5 E			Long.	6 56 29 E		
Arc	104° 01′ 22″.5 E=λ_1			Arc	104° 07′ 15″ E=λ_2		

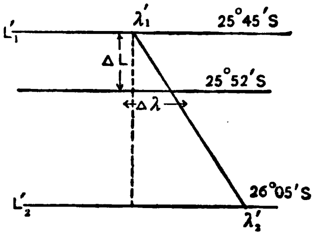
$$F = \frac{\lambda_2 \sim \lambda_1}{L_2 \sim L_1} = \frac{5'\ 52''.5}{20'} = \frac{5'.875}{20'} = .294, \text{ and line runs S}^d \text{ and E}^d.$$

Run to noon 109° (true) 46 miles, $l = 15'$ S, $p = 43'.5$ E, $D = 48'.3$ E.

		° ′		° ′		° ′ ″		° ′ ″
At sight	L_1	25 30 S	L_2	25 50 S	λ_1	104 01 22.5 E	λ_2	104 07 15 E
Run	l	15 S	l	15 S	D	48 18 E	D	48 18 E
At noon	L'_1	25 45 S	L'_2	26 05 S	λ'_1	104 49 40.5 E	λ'_2	104 55 33 E
Lat. by obs.		25 52 S						
ΔL	=	7 S						

Taking origin at $L'_1\ \lambda'_1$
$\Delta\lambda = \Delta L \times F = 7' \times .294 = 2'.058$ E
Since ΔL is S, $\Delta\lambda$ is E

	° ′ ″
Long. of origin	104 49 40.5 E
$\Delta\lambda$	2 08.5 E
Long. by obs. at noon	104 51 44 E



In the solution, both ends of the line are moved for the run to noon; it is necessary, however, to shift but one end.

Ex. 215.—Let example 214 be now worked by the tangent method, the latitude by D. R. at time of sight being 25° 40′ S.

Times.		Altitudes.		Altitude Corrections.	Declination.	H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		° ′			
W.	7 30 23	⊙	18 14 00	Corr. (Tab. 46,	N 4 54.7	N 1′.0	+ to A. T.	− s.7
C—W	5 15 03	Corr.	+ 7 58	Bowditch)	N .7	0h.66	m s	0h.66
				= +8′ 58″			3 39.3	
C.	= 12 45 26	⊖	18 21 58	I. C. = −1 00	N 4 55.4	N 0′.66	− .5	−0s.462
C. C.	− 5 41.2							
				Corr. +7 58			3 38.8	
G. M. T.	12 39 44.8							
or	12h.66							

	° ′ ″		
h	18 21 58		
L	25 40 00	sec	10.04512
p	94 55 24	cosec	10.00160
	138 57 22		
s	69 28 41	cos	9.54477
$s-h$	51 06 43	sin	9.89119
	h m s	2	19.48268
L. A. T.	19 32 23	sin	9.74134
Eq. of T.	+ 3 38.8		
L. M. T.	19 36 01.8		
G. M. T.	12 39 44.8		
Long.	6 56 17 E		
Arc	104° 04′ 15″ E		

From the azimuth tables.

Lat. 25° 40′ S
d 4° 55′.4 N
L. A. T. 7h 32½m (a. m.)
} Z = S 105° 04′ E or N 74° 56′ E (Z_N = 74° 56′)

Table I for { Lat. 25° 40′ S; Z. N 74°.93 E } F = .30↖

Run to Noon.

109° (true) 46 miles, l = 15′ S, p = 43′.5 E, D = 48′.3 E

	° ′ ″		° ′ ″
At sight Lat. D. R.	25 40 00 S	Comp. λ	104 04 15 E
Run to noon, l =	15 00 S	D	= 48 18 E
At noon Lat. D. R.	25 55 00 S	Computed λ	104 52 33 E
At noon Lat. obs.	25 52 S	Δλ	0 54 W
ΔL	= 3 N	Long. by obs. at noon	104 51 39 E

$\Delta\lambda = \Delta L \times F = 3' \times .30 = 0'.9$
$\Delta\lambda = 0'.9 = 54''$

Line of position runs SE^d and NW^d, and ΔL is N ∴ Δλ is W.

When one Observation is of a Body within 45° of the Prime Vertical and the other of a Body within 15° of the Meridian.

303. The mutual correction method.—This method applies to the following: (1) A case of simultaneous observations in which one body is observed near the prime vertical for time and one near the meridian for latitude; (2) a case of double altitudes of the sun with an intervening run—the first altitude observed within 45° of the prime vertical for time, the second altitude observed within 15° of the meridian and worked as a "reduction to the meridian" sight.

Having determined the first line, by either the chord or tangent method, and corrected the coordinates of one point for the run, and having found the value of F_1, it is not unusual to consider the latitude obtained by "reduction to the meridian" (using the computed longitude at the instant of observation in finding H. A.), **as sufficiently exact for all practical purposes,** and for this latitude to find, as in examples 214 and 215, the longitude of fix. Then the noon position is found by applying to the latitude and longitude of fix the run from the time of the second observation till noon.

However, as the body at the second observation is not on the meridian and the resulting line of position is not East and West in direction, and as the sight is worked with a longitude which may be in error, the latitude obtained may be in error.

For more precise results, the "mutual correction method" may be used, correcting the longitude from the time sight by the formula $\Delta\lambda_1 = \Delta L_1 \times F_1$,

in which ΔL_1 is the difference between the latitude of a position point in the first line corrected for the run to second observation and the computed latitude at the second observation;

F_1 is the longitude factor of the first line;
and $\Delta\lambda_1$ is the correction to be applied to the computed longitude at the second observation to give the longitude of fix.

Then the latitude from the "reduction to the meridian" sight should be corrected by the formula $\Delta L_2 = \Delta\lambda_1 \div F_2$;
in which F_2 is the longitude factor of the second line obtained from Table I, knowing the latitude and sun's azimuth at second observation;
and ΔL_2 is the correction to be applied to the computed latitude at the second observation to give the latitude of fix.

This method of "mutual correction" is applicable only where one observation is a time sight and one a sight near the meridian, the latitude from this latter sight being nearly correct.

The following rules are given for the second case under this heading, that of double altitudes of the sun; modifications necessary to make them fit the first case, that of simultaneous observations, will be apparent.

Rules.—Work the time sight by either the chord or tangent method; using the D. R. latitude in the latter case, or assuming latitudes about 10′ each side of the D. R. latitude in the former case.

Find the longitude factor of first line F_1.

To the coordinates of one position point of the first line apply the run to the instant of second observation and obtain a D. R. latitude and a computed longitude; and, with this longitude, work the second sight by the "reduction to the meridian" method, obtaining a computed latitude.

With the computed latitude and azimuth at the second observation, find the longitude factor of the second line F_2.

Take ΔL_1 equal to the difference between the D. R. latitude and the computed latitude at the second observation and find

$\Delta\lambda_1 = \Delta L_1 \times F_1$; apply $\Delta\lambda_1$ to the computed longitude at the second observation and obtain the longitude of fix.

From $\Delta\lambda_1$ find $\Delta L_2 = \Delta\lambda_1 \div F_2$; apply ΔL_2 to the computed latitude at the second observation and obtain the latitude of fix.

Ex. 216.—About 7.45 a. m., January 1, 1918, from an observation of the sun in latitude 16° 21′ 34″ N. by D. R. found the computed longitude to be 63° 15′ 30″ W. True azimuth of sun $= Z_N = 120\frac{2}{3}°$. Ran thence till $11^h\ 40^m$ a. m. 315° (true) 30.7 miles when, by the "reduction to the meridian" method, the latitude was found to be 16° 45′ 04″ N. True azimuth of sun $Z_N = 172°.8$. Ran thence till noon 315° (true) 2.7 miles. Required the noon position.

D. R. between sights.

True Course.	Distance.	Diff. Lat.	Dep.	Diff. Long.
315°	30.7	21.7 N	21.7 W	22′.64 W

	° ′ ″		° ′ ″
Lat. by D. R. at 7.45 a.m.	16 21 34 N	Long. computed at 7.45 a.m.	63 15 30 W
Diff. of Lat. to 11.40 a.m.	21 42 N	Diff. Long. to 11.40 a.m.	22 38.4 W
Lat. by D. R. at 11.40 a.m.	16 43 16 N	Computed Long. at 11.40 a.m.	63 38 08.4 W
L_0	= 16 32 25 N		

		° ′ ″
At 11.40 a. m. computed Lat.		16 45 04 N
At 11.40 a. m. Lat. by D. R.		16 43 16 N
$\Delta L_1 =$	1′.8 N =	1 48 N

To correct the longitude from the a. m. time sight.

Lat. 16°.36 N, $Z_1 = S\ 59\frac{1}{3}°\ E$, $F_1 = .62$

$\Delta\lambda_1 = \Delta L_1 \times F_1 = 1.8 \times .62 = 1'.116$

$\Delta\lambda_1 = 1'.116 = 1'\ 07''$

	° ′ ″
Computed Long. at 11.40 a. m.	63 38 08.4 W
$\Delta\lambda_1$	1 07 E
Long. of fix 11.40 a. m.	63 37 01.4 W

ΔL_1 is northerly, Z_1 is southward and eastward; therefore, $\Delta\lambda_1$ is easterly.

To correct the latitude obtained from the sight near noon.

Lat. 16°.75 N, $Z_2 = S\ 7°.2\ E$, $F_2 = 8.29$

$\Delta L_2 = \Delta\lambda_1 \div F_2 = 1'.116 \div 8.29 = 0'.13$

$\Delta L_2 = 0'.13 = 7''.8$

	° ′ ″
Computed Lat. at 11.40 a. m.	16 45 04 N
ΔL_2	7.8 N
Lat. of fix 11.40 a. m.	16 45 11.8 N

$\Delta\lambda_1$ is easterly, Z_2 southward and eastward; therefore, ΔL_2 is northerly.

To find the noon position.

D. R. 11.40 a. m. to noon.

True Course.	Distance.	Diff. Lat.	Dep.
315°	2.7	1.9 N	1.9 W

	° ′ ″		° ′ ″
Lat. of fix 11.40 a. m.	16 45 11.8 N	Long. of fix 11.40 a. m.	63 37 01.4 W
Diff. of Lat. to noon	1 54 N	Diff. of Long. to noon	2 00 W
Lat. in at noon	16 47 06 N	Long. at noon	63 39 01 W

304. To determine the intersection of two lines running at an angle with both meridians and parallels, when position points having a common latitude are known, one for each line.

Two lines with position points on a common parallel may be considered when we have simultaneous observations of two bodies favorably situated for finding time, the Z_N of each being from 45° to 135° or 225° to 315°; also when a line from a time sight is combined with one from a $\phi''\phi'$ sight previously taken, the computed latitude from the latter, after correction for the run in the interval, being used in the time sight.

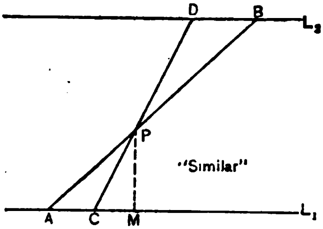


FIG. 132.

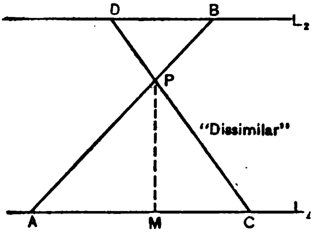


FIG. 133.

Two cases occur under this head: (1) when both lines are in the same or opposite quadrants, being then called *similar;* (2) when the two lines run in adjacent quadrants, being then called *dissimilar.*

The chord method.—Let A and B be two points of a line determined by assuming latitudes L_1 and L_2, and C and D two points of a second line having the same coordinates; the two lines being the results of simultaneous observations of two different bodies; or, of observations taken at different times, whether of the same or different bodies, one line, however, being brought up to the time of the second observation for the run in the interval. Let λ'_1 and λ'_2 be the longitudes of A and B; λ''_1 and λ''_2, of C and D respectively.

Let AB be the first line, F_1 its longitude factor found as in Art. 298. Then, if P is the point of intersection, $\Delta\mathbf{L} = \mathbf{PM}$, and the corresponding difference of longitude from A is $\Delta\lambda_1 = \mathbf{AM}$; therefore, $\Delta\lambda_1 = \Delta\mathbf{L} \times \mathbf{F}_1$.

If CD is the second line, its longitude factor is F_2 and, in the same way as above, $\mathbf{CM} = \Delta\lambda_2$ and $\Delta\lambda_2 = \Delta\mathbf{L} \times \mathbf{F}_2$.

In both figures, AC is the known difference of longitude of both lines for one assumed latitude. In Fig. 132,

$$\mathbf{AC} = \mathbf{AM} - \mathbf{CM} = \mathbf{PM}\ (\mathbf{F}_1 - \mathbf{F}_2),$$
$$\text{or } \mathbf{AM} - \mathbf{CM} = \Delta\mathbf{L}\ (\mathbf{F}_1 - \mathbf{F}_2);$$
$$\text{but } \Delta\lambda_1 = \mathbf{AM} = \Delta\mathbf{L} \times \mathbf{F}_1,$$
$$\text{and } \Delta\lambda_2 = \mathbf{CM} = \Delta\mathbf{L} \times \mathbf{F}_2.$$

In this figure, where the lines are *similar,* both $\Delta\lambda_1$ and $\Delta\lambda_2$ are applied in the same direction, both East or both West, so as to make the resulting longitudes the same.

Then, knowing the name of the correction in longitude for either line and the direction of the line, the name of the correction in latitude is apparent, or is found by the rule of Art. 301, in which the direction of the line, instead of the bearing of the body, is considered.

In Fig. 133,

$$\mathbf{AC} = \mathbf{AM} + \mathbf{CM} = \mathbf{PM}\ (\mathbf{F}_1 + \mathbf{F}_2) = \Delta\mathbf{L}\ (\mathbf{F}_1 + \mathbf{F}_2)$$
$$\text{and } \Delta\lambda_1 = \mathbf{AM} = \Delta\mathbf{L} \times \mathbf{F}_1 \text{ and } \Delta\lambda_2 = \mathbf{CM} = \Delta\mathbf{L} \times \mathbf{F}_2.$$

In this figure, where the lines are "*dissimilar,*" $\Delta\lambda_1$ and $\Delta\lambda_2$ are applied in the opposite directions, westerly to the more easterly longitude and easterly to the more westerly longitude, so as to make both resulting longitudes the same, which must be the case; *otherwise an error has been made.* Then, knowing the name of the correction in longitude for either line, and the direction of the line, the name of the correction for latitude is found. Apply the correction in latitude to the latitude of the parallel used as origin to find the latitude of fix.

Ex. 217.—April 5, 1918, p. m., Lat. by D. R. 20° 38′ S., Long. by D. R. 90° 10′ E., observed simultaneous altitudes of stars α Tauri (Aldebaran) bearing northward and westward and β Leonis (Denebola) bearing northward and eastward. Sextant altitude of α Tauri 26° 00′ 40″, of β Leonis 25° 17′ 00″. I. C.—1′ 00″. Height of eye 19 feet. W. T. of obs. $7^h\ 16^m\ 44^s$. C—W $5^h\ 55^m\ 24^s$. Chro. fast of G. M. T. $1^m\ 38^s$. Required the ship's position, assuming latitudes 20° 30′ S. and 20° 50′ S.

Times.		Data for α Tauri.					Data for β Leonis.				
		Altitude.		Corrections.		✱'s R. A. and Dec.	Altitude.		Corrections.		✱'s R. A. and Dec.
	h m s		° ′ ″		′ ″			° ′ ″		′ ″	
W.	7 16 44	✱'s h_s	26 00 40	I. C.	− 1 00	R.A. $4^h\ 31^m\ 14^s.0$	✱'s h_s	25 17 00	I. C.	− 1 00	R.A. $11^h\ 44^m\ 55^s.8$
C—W	5 55 24	Corr.	− 7 15	Dip	− 4 16	*d.* N 16° 20′.7	Corr.	− 7 19	Dip	− 4 16	*d.* N 15° 01′.5
				Ref.	− 1 59				Ref.	− 2 03	
C.	1 12 08	✱'s h	25 53 25				✱'s h	25 09 41			
C. C.	− 1 38			Corr.	− 7 15				Corr.	− 7 19	
				Or (Tab. 46)					Or (Tab. 46)		
G. M. T. 5th of Apr.	1 10 30				− 6′ 15″					− 6′ 19″	
				I. C.	− 1 00				I. C.	− 1 00	
R. A. M. ⊙	0 51 54.6										
Corr. G. M. T.	11.6			Corr.	− 7 15				Corr.	− 7 19	
G. S. T.	2 02 36.2										

Line from α Tauri.

	° ′ ″		
h	25 53 25		
L_1	20 30 00	sec	10.02841
p	106 20 42	cosec	10.01792
	152 44 07		
s	76 22 04	cos	9.37234
$s-h$	50 28 39	sin	9.88726
	h m s	2	19.30593
t = +	3 33 48.8	sin	9.65296
(α Tauri) R. A.	4 31 14.0		
L. S. T.	8 05 02.8		
G. S. T.	2 02 36.2		
Long.	6 02 26.6 East		
Arc	90° 36′ 39″ East $=\lambda'_1$		

	° ′ ″		
h	25 53 25		
L_2	20 50 00	sec	10.02937
p	106 20 42	cosec	10.01792
	153 04 07		
s	76 32 04	cos	9.36709
$s-h$	50 38 39	sin	9.88830
	h m s	2	19.30268
t = +	3 32 57.3	sin	9.65134
✱'s R. A.	4 31 14.0		
L. S. T.	8 04 11.3		
G. S. T.	2 02 36.2		
Long.	6 01 35.1 East		
Arc	90° 23′ 46″.5 East $=\lambda'_2$		

Longitude factor $= \frac{12'\ 52''.5}{20'} = \frac{12'.875}{20'}$ or $F_1 = 0.644'$

Line from β Leonis.

	° ′ ″				° ′ ″		
h	25 09 41			h	25 09 41		
L	20 30 00	sec	10.02841	L	20 50 00	sec	10.02937
p	105 01 30	cosec	10.01511	p	105 01 30	cosec	10.01511
	150 41 11				151 01 11		
s	75 20 36	cos	9.40317	s	75 30 36	cos	9.39831
$s-h$	50 10 55	sin	9.88541	$s-h$	50 20 55	sin	9.88646
		2	19.33210			2	19.32923
	h m s				h m s		
(β Leonis) t —	3 40 54.3	sin	9.66605	t	— 3 40 07	sin	9.66462
✳'s R. A.	11 44 55.8			✳'s R. A.	11 44 55.8		
L. S. T.	8 04 01.5			L. S. T.	8 04 48.8		
G. S. T.	2 02 36.2			G. S. T.	2 02 36.2		
Long.	6 01 25.3 East			Long.	6 02 12.6 East		
Arc	90° 21′ 19″.5 East $=\lambda''_1$			Arc	90° 33′ 09″ East $=\lambda''_2$		

Longitude factor $= \frac{11'\ 49''.5}{20'} = \frac{11'.825}{20'}$ or $F_2 = 0.591$

The lines are "dissimilar"; therefore, $\Delta L\ (F_1+F_2) = \lambda'_1 \sim \lambda''_1$

		° ′ ″	
λ'_1	$=$	90 36 39	E
λ''_1	$=$	90 21 19.5	E
$\lambda'_1 \sim \lambda''_1$	$=$	15 19.5	E
	$=$	15′.325	

$\Delta L\ (.644+.591) = 15'.325$

$\Delta L = \frac{15'.325}{1.235} = 12'.409 = 12'\ 24''.5$

$\Delta\lambda_1 = \Delta L \times F_1 = 12'.409 \times .644 = 7'.991 = 7'\ 59''.46$ W

$\Delta\lambda_2 = \Delta L \times F_2 = 12'.409 \times .591 = 7'.334 = 7'\ 20''.04$ E

	° ′ ″	
L_1	20 30 00	S
ΔL	12 24.5	S
Lat.	20 42 24.5	S

		° ′ ″	
λ'_1	$=$	90 36 39.0	E
$\Delta\lambda_1$		7 59.46	W
Long.		90 28 39.54	E
or			
λ''_1	$=$	90 21 19.5	E
$\Delta\lambda_2$		7 20.04	E
Long.		90 28 39.54	E

Rules for the chord method using a common latitude.—

(1) *Find the longitude factors* F_1 *for the first line, and* F_2 *for the second line (Art.* 298).

(2) *Divide the difference of the longitudes computed for each line with a given common latitude by the difference or sum of the longitude factors, according as the lines are "similar" or "dissimilar." The result is the correction in latitude,* ΔL.

(3) *The correction in latitude* (ΔL), *multiplied by the longitude factor of each line* (F_1 *and* F_2), *gives the correction in longitude for that line* ($\Delta\lambda_1$ *for first line,* $\Delta\lambda_2$ *for second line). For "similar" lines, apply the corrections in longitude the same way to the computed longitudes so as to make the resulting longitudes the same and obtain the longitude of the fix. If they do not come out the same, a mistake has been made. This fact serves as a check on the work.*

(4) *Knowing the name of the longitude correction and the direction of the line, find by the rule (Art.* 301), *the name of the latitude correction. Apply the correction in latitude to the given common latitude and find the latitude of fix.*

In the above example, the common latitude was taken as 20° 30′ S. The lines are *"dissimilar"* and the correction $\Delta\lambda_1$, the longitude correction to the coordinate of the first line, is marked West because the position point of that line has the most eastern computed longitude; $\Delta\lambda_2$, the longitude correction to the coordinate of the second line, is marked East because the position point of that line has the most western longitude. ΔL is marked S. as per rule in last paragraph of Art. 298.

305. The tangent method.—Fig. 132 and Fig. 133 apply in this case. *Only one latitude, and that by D. R., is used.* The longitude factor of each line is gotten from Table I, as explained in Art. 298, and the direction of each line is obtained as shown in Art. 297. From this point on, the mode of procedure and the rules of the chord method apply.

Ex. 218.—Let example 217 be now worked by the tangent method, with the latitude by D. R. at the time of sight 20° 38′ S.

Times.		Data for α Tauri.			Data for β Leonis.		
		Altitude.	Corr.	✳'s R. A. & Dec.	Altitude.	Corr.	✳'s R. A. & Dec.
	h m s	° ′ ″			° ′ ″		
W.	7 16 44	✳'s h_s 26 00 40	Corr. (Tab. 46, Bowditch)	R.A. 4h 31m 14s.0	✳'s h_s 25 17 00	Corr. (Tab. 46, Bowditch)	R.A. 11h 44m 55s.8
C—W	5 55 24	Corr. — 7 15		*d.* N 16° 20′.7	Corr. — 7 19		*d.* N 15° 01′.5
			−6′ 15″			−6′ 19″	
C.	1 12 08	✳'s *h* 25 53 25	I. C. −1 00		✳'s *h* 25 09 41	I. C. −1 00	
C. C.	− 1 38						
			Corr. −7 15			Corr. −7 19	
G. M. T. 5th of Apr.	1 10 30						
R. A. M. ☉	0 51 54.6						
Corr. G. M. T.	11.6						
G. S. T.	2 02 36.2						

For α Tauri.

	° ′ ″		
h	25 53 25		
L	20 38 00	sec	10.02879
p	106 20 42	cosec	10.01792
	152 52 07		
s	76 26 04	cos	9.37025
s—*h*	50 32 39	sin	9.88768
	h m s	2	19.30464
✳'s *t*	+ 3 33 28.5	sin	9.65232
✳'s R. A.	4 31 14.0		
L. S. T.	8 04 42.5		
G. S. T.	2 02 36.2		
Long.	6 02 06.3 East		
Arc	90° 31′ 34″.5 East=λ'_1		

For β Leonis.

	° ′ ″		
h	25 09 41		
L	20 38 00	sec	10.02879
p	105 01 30	cosec	10.01511
	150 49 11		
s	75 24 36	cos	9.40123
s—*h*	50 14 55	sin	9.88583
	h m s	2	19.33096
✳'s *t*	— 3 40 35.5	sin	9.66548
✳'s R. A.	11 44 55.8		
L. S. T.	8 04 20.3		
G. S. T.	2 02 36.2		
Long.	6 01 44.1 East		
Arc	90° 26′ 01″.5 East=λ''_1		

From Azimuth Tables.

L	20° 38′ S	α Tauri bears true	N 58°.85 W or 301°.15
d	16° 21′ N	Direction of line	N 31°.15 E or 31°.15
t	3h 33½m	From Table I,	F_1=0.64/

From Azimuth Tables.

L	20° 38′ S	β Leonis bears	N 61°.1 E or 61°.1
d	15° 02″ N	Direction of line	N 28°.9 W or 331°.1
t	3h 40½m	From Table I,	F_2=0.59\

Continued on page 630.

This is a case of "*dissimilar*" lines with a common latitude (Fig. 133). Therefore, we have for

$$\text{Lat. } 20^\circ\ 38'\ \text{S} \begin{cases} \lambda'_1 = 90^\circ\ 31'\ 34''.5 \text{ East} \\ \lambda''_1 = 90\ \ 26\ \ 01.\ 5 \text{ East} \end{cases}$$

$$\text{in the common latitude, } \lambda'_1 \sim \lambda''_1 = \quad 5'\ 33'' = 5'.55$$

$$\text{but } \lambda'_1 \sim \lambda''_1 = \Delta L\ (F_1 + F_2)\ ;\ \text{therefore, } 5'.55 = \Delta L\ (0.64 + 0.59)$$

$$\Delta L = \frac{5'.55}{1.23} = 4'.512 = 4'\ 30''.7$$

Now, to find the corrections in longitude:

$$\Delta\lambda_1 = \Delta L \times F_1 = 4'.512 \times .64 = 2'.888 = 2'\ 53''.28$$
$$\Delta\lambda_2 = \Delta L \times F_2 = 4'.512 \times .59 = 2'.662 = 2'\ 39''.72$$

To find the fix:

$$\begin{array}{ll} & \quad ^\circ\quad '\quad '' \\ \lambda'_1 & = 90\ 31\ 34.5 \text{ E} \\ \Delta\lambda_1 & = \quad\ \ 2\ 53.3 \text{ W} \\ \hline \text{Long. of fix} & = 90\ 28\ 41.2 \text{ E} \end{array} \qquad \begin{array}{ll} & \quad ^\circ\quad '\quad '' \\ \Delta\lambda_2 & = 90\ 26\ 01.5 \text{ E} \\ \Delta\lambda''_2 & = \quad\ \ 2\ 39.7 \text{ E} \\ \hline \text{Long. of fix} & = 90\ 28\ 41.2 \text{ E} \end{array} \qquad \begin{array}{ll} & \quad ^\circ\quad '\quad '' \\ \text{Lat. by D. R.} & \ \ 20\ 38\ 00 \text{ S} \\ \Delta L & \qquad\quad 4\ 31 \text{ S} \\ \hline \text{Lat. of fix} & = 20\ 42\ 31 \text{ S} \end{array}$$

The lines being dissimilar in this example, $\Delta\lambda_1$ and $\Delta\lambda_2$ are applied in opposite directions to give the longitude of fix.

The first line running northward and eastward and $\Delta\lambda_1$ being westerly, ΔL is southerly; or, the second line running northward and westward and $\Delta\lambda_2$ being easterly, ΔL is southerly (Art. 301).

306. To determine the intersection of two lines running at an angle with both meridians and parallels, when position points having a common longitude are known, one for each line.

In previous articles we have considered a position point in each line with a common latitude. It may be necessary to consider two lines with position points on a common meridian, as in the case of two simultaneous observations worked for latitude by the $\phi''\phi'$ method; or, in the case of a line from a $\phi''\phi'$ sight combined with one from a time sight, the computed longitude, from the latter, after correction for the run in the interval, being used in the $\phi''\phi'$ sight.

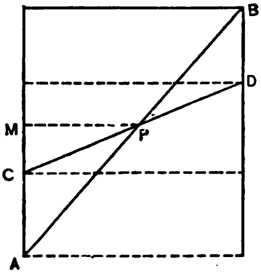


FIG. 134.

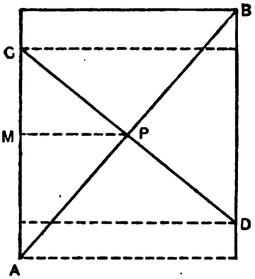


FIG. 135.

The latitude factor.—In this case, instead of using the variation in longitude for 1′ of latitude, or the factor *F*, we use the variation in latitude for 1′ of longitude, or the latitude factor $f\left(\text{which equals } \frac{1}{F}\right)$. This factor is tabulated in Table 48, Bowditch.

The chord method.—Let *AB* and *CD* (Fig. 134) and (Fig. 135) be two lines of position obtained by working sights with the same assumed longitudes, the longitudes of the meridians of *AC* and *BD;* or, let *AB* be a first line moved for the run between observations; *A* and *B* being the coordinates of the first line at the instant of the second observation, the longitudes of A and B are used to work the sight for the second line CD. Let L'_1 and L'_2 be the latitudes of A and B; L''_1 and L''_2, of C and D respectively.

AB being the first line, f_1 (found as in Art. 298) is its latitude factor; then, if P is the point of intersection of AB with the second line, taking the meridian of AC as origin, we have $\Delta\lambda = PM$ and the corresponding ΔL from parallel of A is AM, but $AM = PM \times f_1$; therefore, $\Delta L_1 = \Delta\lambda \times f_1$.

If CD is the second line, its latitude factor is f_2, and, in the same way as above, $CM = PM \times f_2$, or $\Delta L_2 = \Delta\lambda \times f_2$.

In both figures, AC is the known difference of latitude of the position points of the two lines on the common meridian AC.

In Fig. 134, $\mathbf{AC} = \mathbf{AM} - \mathbf{CM} = \mathbf{PM}(\mathbf{f}_1 - \mathbf{f}_2)$, or the difference of latitude $\mathbf{AC} = \Delta\lambda\ (\mathbf{f}_1 - \mathbf{f}_2)$, then $\mathbf{AM} = \Delta\mathbf{L}_1 = \Delta\lambda \times \mathbf{f}_1$ and $\mathbf{CM} = \Delta\mathbf{L}_2 = \Delta\lambda \times \mathbf{f}_2$.

In this figure, where the lines are "*similar,*" both corrections to the latitude, ΔL_1 and ΔL_2, are applied in the same way, either to N. or to S. so as to make the resulting latitudes the same.

Knowing the name of the correction in latitude for either line, the name of the correction in longitude for the same line is apparent, or is obtained by the rule of Art. 301.

In Fig. 135, $\mathbf{AC} = \mathbf{AM} + \mathbf{CM} = \mathbf{PM}(\mathbf{f}_1 + \mathbf{f}_2)$, or the known difference of latitude $\mathbf{AC} = \Delta\lambda(\mathbf{f}_1 + \mathbf{f}_2)$; then $\mathbf{AM} = \Delta\mathbf{L}_1 = \Delta\lambda \times \mathbf{f}_1$ and $\mathbf{CM} = \Delta\mathbf{L}_2 = \Delta\lambda \times \mathbf{f}_2$.

In this figure, where the lines are "*dissimilar,*" the corrections for latitude are applied in the opposite directions, northerly to the more southern latitude and southerly to the more northern latitude, so as to make both resulting latitudes the same. The name of the correction in longitude is found by rule (Art. 301). The correction in longitude, applied with its proper sign to the longitude of the common meridian, will give the longitude of fix.

Rules for chord method, using a common longitude.—

(1) *From the coordinates of the two points of each line, find the latitude factor* f_1 *for the first line, and* f_2 *for the second line.*

(2) *Divide the difference of the computed latitudes of the position points on the common meridian by the difference or sum of the latitude factors, according as the lines are "similar" or "dissimilar." The result is the correction in longitude.*

(3) *The correction in longitude, multiplied by the latitude factor of a line, gives the correction in latitude to be applied to the latitude of that line's position point on the common meridian. For "similar" lines, the latitude corrections are applied the same way, for "dissimilar" lines the opposite way, to the computed latitudes, so as to make the resulting latitude the same and to give the latitude of fix.*

(4) *Knowing the name of the latitude correction, and the direction of the line of position corresponding, find by the rule (Art.* 301) *the name of the longitude correction. Apply this with its proper sign to the longitude of the common meridian; the result will be the longitude of fix.*

Ex. 219.—Jan. 18, 1918, a. m., in Lat. by D. R. 40° 24′ N., Long. by D. R. 70° 34′ W., the sextant altitude of the sun's lower limb was 12° 48′ 30″. I. C. +1′. Height of eye 45 feet. W. T. of obs. $8^h\,30^m\,15^s$. C—W $5^h\,01^m\,55^s$. Chro. fast of G. M. T. $2^m\,10^s$. Having worked a line of position using latitudes 40° 15′ N. and 40° 35′ N., ran 250° (true) 25 miles when the sextant altitude of the sun's lower limb was observed to be 25° 50′; the sun bearing southerly and easterly. I. C. +1′. Height of eye 45 feet. W. T. of obs. $10^h\,29^m\,54^s$. C—W $5^h\,01^m\,56^s$. Chro. fast of G. M. T. $2^m\,10^s$. Correct the coordinates of the first line for the run, and, with the longitudes thus found, work the second sight as a $\phi''\ \phi'$ sight; then find ship's position.

Times.		Altitude.		Altitude Corrections.		Declination.		H. D.		Eq. of T.		H. D.	
	h m s		° ′ ″		′ ″		° ′		N 0′.5	— to M. T.			+0ˢ.8
W.	8 30 15	⊙	12 48 30	S. D.	+16 17		S 20 38.9	G. M. T.	1ʰ.5		m s	G. M. T.	1ʰ.5
C—W	5 01 55	Corr.	+ 6 39	I. C.	+ 1 00	Corr.	N .8				10 24.4		
				Dip	− 6 36			Corr.	N 0′.75	Corr.	+ 1.2	Corr.	+1ˢ.20
C.	1 32 10	$\underline{\odot}$	12 55 09	p. & R.	− 4 02		S 20 38.1						
C. C.	− 2 10										10 25.6		
				Corr.	+ 6 39								
G.M.T. Jan. 18,	1 30 00			or (Tab. 46)									
Eq. of T.	− 10 25.6				+5′ 40″								
				I. C.	+1 00								
G. A. T.	1 19 34.4												
				Corr.	+6 40								

	° ′ ″				° ′ ″		
h	12 55 09			h	12 55 09		
L_1	40 15 00	sec	10.11734	L_2	40 35 00	sec	10.11949
p	110 38 06	cosec	10.02880	p	110 38 06	cosec	10.02880
	163 48 15				164 08 15		
s	81 54 08	cos	9.14880	s	82 04 08	cos	9.13982
h	12 55 09			h	12 55 09		
$s-h$	68 58 59	sin	9.97010	$s-h$	69 08 59	sin	9.97059
	h m s	2	19.26504		h m s	2	19.25870
L. A. T.	20 36 44.1	sin	9.63252	L. A. T.	20 38 19	sin	9.62935
G. A. T.	1 19 34.4			G .A. T.	1 19 34.4		
Long.	4 42 50.3 W			Long.	4 41 15.4 W		
Arc	70° 42′ 34″.5 W $=\lambda_1$			Arc	70° 18′ 51″ W $=\lambda_2$		

Run to Second Observation.	Course.	Distance.	S	W	
	250° (true)	25	8.6	23.5	$L_0 = 40\frac{1}{3}°$ N $D = 30.83$ W

	° ′		° ′ ″		° ′ ″		° ′ ″		° ′ ″
Lat. D. R.	40 24 N	L_1	40 15 00 N	L_2	40 35 00 N	λ_1	70 42 34.5 W	λ_2	70 18 51 W
$\frac{l}{2}$	4.3 S	l	8 36 S	l	8 36 S	D	30 49.8 W	D	30 49.8 W
		L'_1	40 06 24 N	L'_2	40 26 24 N	λ'_1	71 13 24.3 W	λ'_2	70 49 40.8 W
L_0	40 19.7 N			L'_1	40 06 24 N			λ'_1	71 13 24.3 W
				$L'_2 \sim L'_1$	$=20$			$\lambda'_2 \sim \lambda'_1$	$=23\ 43.5$
									$=23'.725$

Latitude factor of 1st line $= f_1 =$ 20′ [illegible] $= .8480$

Now, work the second observation as a ϕ'' ϕ' sight, using the longitudes of the first line brought up for the run ($\lambda''_1 = \lambda'_1$ and $\lambda''_2 = \lambda'_2$).

Times.	Altitudes.	Altitude Corrections.	Declination.	H. D.	Eq. of T.	H. D.
h m s W. 10 29 54 C—W 5 01 56 C. 8 31 50 C. C. — 2 10 G.M.T. Jan. 18, 3 29 40 Eq. of T. 10 27.2 G. A. T. 3 19 12.8 λ'_1 West 4 44 53.6 $t_1 = (-)$ 1 25 40.8 $t_1 = (-)$ 21° 25′ 12″	° ′ ″ $\odot$ 25 50 00 Corr. + 8 49 $\ominus$ 25 58 49	′ ″ S. D. +16 17 I. C. + 1 00 Dip — 6 36 p. & R. — 1 52 Corr. + 8 49 or (Tab. 46, Bowditch) +7′ 50″ I. C. +1 00 Corr. +8 50	° ′ S 20 37.9 Corr. N .8 S 20 37.1	N 0′.5 G. M. T. $1^h.5$ N 0′.75	— to M. T. m s 10 26.0 Corr. + 1.2 10 27.2	$0^s.8$ G. M. T. $1^h.5$ Corr. $+1^s.20$
h m s G. A. T. 3 19 12.8 λ'_2 West 4 43 18.7 $t_2 = (-)$ 1 24 05.9 $t_2 = (-)$ 21° 01′ 28″.5						

	° ′ ″				
h	25 58 49			sin	9.64154
d	20 37 06 S	tan	9.57547	cosec	10.45329
t_1	21 25 12 E	sec	10.03108		
ϕ''	22 00 23 S	tan	9.60655	sin	9.57370
ϕ'	62 12 55 N			cos	9.66853
Lat.₁	40 12 32 N $= L''_1$				

	° ′ ″				
h	25 58 49			sin	9.64154
d	20 37 06 S	tan	9.57547	cosec	10.45329
t_2	21 01 28 E	sec	10.02992		
ϕ''	21 57 12 S	tan	9.60539	sin	9.57270
ϕ'	62 17 05 N			cos	9.66753
Lat.₂	40 19 53 N $= L''_2$				

° ′ ″
L''_2 40 19 53 N
L''_1 40 12 32 N
$L''_2 \sim L''_1 =$ 7 21
$=$ 7′.35

° ′ ″
λ'_2 = 70 49 40.8 W
λ'_1 = 71 13 24.3 W
$\lambda'_2 \sim \lambda'_1$ = 23 43.5
= 23′.725

Latitude factor of 2d line

$$= f_2 = \frac{7'.35}{23'.725} = .3098$$

Continued on page 636.

This is a case of similar lines with a common longitude (Fig. 184). Taking the western meridian as the origin, we have

$$\left.\begin{array}{r}\text{With longitude of}\\ \text{Western meridian}\end{array}\right\}=\lambda'_1=71^\circ\ 13'\ 24''.3\ \text{W}\left\{\begin{array}{l}L''_1=40^\circ\ 12'\ 32''\ \text{N}\\ L'_1\ =40\ \ 06\ \ 24\ \ \text{N}\end{array}\right.$$

$$\text{On the common meridian, Diff. of Lat.}=L''_1\sim L'_1=\quad 6\ 08$$
$$=6'.133$$

$$\text{but } L''_1\sim L'_1=\Delta\lambda\ (f_1\sim f_2)\text{ ; therefore, } 6'.133=\Delta\lambda\ (.8430-.3098)$$

$$\Delta\lambda=\frac{6'.133}{.5332}=11'.5022=11'\ 30''.1$$

To find the corrections in latitude:

$$\Delta L_1=\Delta\lambda\times f_1=11'.502\times .8430=9'.6962=9'\ 41''.77$$
$$\Delta L_2=\Delta\lambda\times f_2=11'.502\times .3098=3'.563\ =3'\ 33''.78$$

To find the fix:

	° ′ ″		° ′ ″		° ′ ″
L'_1	=40 06 24 N	L''_1	=40 12 32 N	λ'_1	=71 13 24.3 W
ΔL_1	= 9 41.8 N	ΔL_2	= 3 33.8 N	$\Delta\lambda$	= 11 30.1 E
Lat. of fix	40 16 05.8 N	Lat. of fix	40 16 05.8 N	Long. of fix	71 01 54.2 W

The lines being similar in this example, ΔL_1 and ΔL_2 are both applied the same way, and that to northward, to give the same result, the latitude of fix.

The first line running northward and eastward, and ΔL_1 being northerly, $\Delta\lambda$ is easterly (Art. 301).

307. The tangent method.—The above example (219) may be worked by the tangent method thus: Let the first sight be worked with the D. R. latitude 40° 24′ N., giving a computed longitude; find from the azimuth tables the sun's true bearing Z_1 at the first observation, and from Table 48, Bowditch, the value f_1.

Let this D. R. latitude and the resulting computed longitude be corrected for the run of the ship between observations, thus obtaining a position point L', λ'.

Let the $\phi'' \; \phi'$ sight be worked with this corrected longitude λ', the resulting computed latitude being L''. Find from the azimuth tables the sun's true bearing Z_2 at the second observation and from the Table, the value f_2; the azimuths in both cases being taken less than 90°.

In example 219, the lines run in the same quadrant and are "similar"; therefore,

$$\Delta\lambda = \frac{\text{Diff. of latitudes}}{\text{Diff. of Lat. factors}} = \frac{L'' \sim L'}{f_2 \sim f_1},$$

$$\left.\begin{array}{l} \Delta L_1 = \Delta\lambda \times f_1 \text{ and } L = \text{Lat. of fix} = L' + \Delta L_1, \\ \Delta L_2 = \Delta\lambda \times f_2 \text{ and } L = \text{Lat. of fix} = L'' - \Delta L_2. \end{array}\right\} \text{check}$$

Then mark $\Delta\lambda$ with its proper sign and apply it to λ'; the result λ is the longitude of fix.

CHAPTER XX.

THE NEW NAVIGATION.

308. The method of treating Sumner lines to be described in the following pages is known as the method of Marcq Saint-Hilaire, and is considered in various text-books under the head of "The New Navigation." However, there is nothing new about it—the French have used the method for years.

Referring to Arts. 281-284 and Fig. 121, it is seen that for a given instant an observed heavenly body is in the zenith of some place on the earth's surface, which is the pole of a system of circles of equal altitude called also parallels of altitude. For a given dead reckoning position, this body has a parallel of altitude, and the altitude corresponding may be found by computation.

If from the ship's position at the given instant, the measured altitude of the same heavenly body differs from the computed altitude, the true position of the ship is not on the parallel of altitude passing through the dead reckoning position but on another one of the same system; distant from it on the great circle passing through the dead reckoning position of the ship and the body, the number of sea miles equal to the number of minutes of arc in the difference between the measured and computed altitudes, and in a direction towards or from the observed heavenly body, according as the measured altitude is greater or less than that by computation; therefore, to determine one point of the circle of equal altitudes, which circle is the locus of the ship's possible positions at the given instant, it is only necessary to lay off, in the proper direction from the ship's position by D. R., a great circle distance equal to the above-mentioned difference of altitude called the "altitude difference." The point thus determined is one "position point" and the circle of altitude through it is the required "line of position."

Let Fig. 136 represent a projection of a heavenly body *S* on the plane of the horizon of its geographical position (Art. 281), showing *A*, the D. R. position of a ship from which an altitude of the body *S* has been observed, then *AS* is the great circle direction of the body *S* from *A* and *HA* is the computed true altitude of *S* for that place and instant of observation. If the measured true altitude is also equal to *HA*, the ship's real position is on a circle of altitude whose radius is *SA*; however, if the measured true altitude is not *HA* but *HB* (represented in the figure as $> HA$), then *AB* is the "altitude difference," *bBcd* is the circle of altitude passing through the place of observation, and *B* is one point of this circle which has certain attributes that make it very prominent in the methods of the so-called "New Navigation."

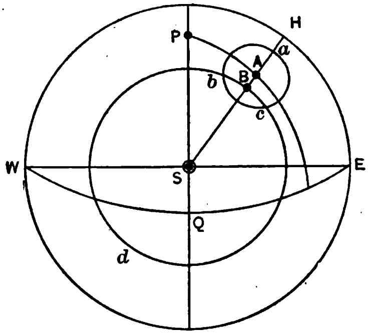


FIG. 136.

This point is always nearer to the real position of the ship than the D. R. position, except when coincident with it, as may be seen by reference to Fig. 136, in which *abc*, a circle described about *A* as a center with a radius equal to the possible error and called "a circle of error," includes the ship's position which, being also on the circle of altitude *bBcd*, must be on the arc *bBc*. Since *ABS* is perpendicular to this arc at its middle point *B* it is evident that *B* is nearer than *A* to the ship's real position, and as *B* occupies the mean of the probable positions, it is less likely to be in error than other points of the arc.

The circle of altitude might be drawn on a globe if the latter should have sufficient dimensions (Art. 283); or it might be practicable to draw it on a Mercator chart in the case of a body of small declination observed at a high altitude, in other words, in case of many observations made in the tropics, by first locating the body's geographical position and then drawing a circle from that point as a center with a radius equal to the body's observed true zenith distance (Arts. 281-286). The point *B* would then be determined by construction at the intersection of this circle and the great circle passing through the D. R. position of the observer and the body's geographical position.

The computed point.—As the use of a globe is impracticable, and since circles of altitude may be represented by circles on a Mercator chart only under special circumstances, it is ordinarily necessary to find by computation the co-ordinates of the point *B* through which the circle of altitude passes, therefore, in the practice of the "New Navigation," this point becomes the first desideratum and will be referred to hereafter as the "computed point." Having determined the "altitude difference," this point may be found with sufficient accuracy in practical navigation by laying off this difference on a loxodrome through the D. R. position instead of along the great circle bearing, the error produced by this substitution, owing to the small size of the "altitude difference," being inappreciable, even under the most unfavorable conditions.

Line of position.—Since the D. R. position so limits that portion of the circle of altitude on which the observer may be, it is necessary in practice to consider only a small arc and this will not differ materially within certain limits from a straight line drawn through the computed point at right angles to the body's bearing regarded as a loxodrome, except when the body observed is very near the zenith, the limits of coincidence depending on the value of the altitude difference as well as the altitude of the observed body.

Formulæ.—The altitude and azimuth may be computed from the formulæ (159), the form for arrangement of work in finding h and Z being as shown in the solution of examples 157 and 158, or these elements may be found, without computation within the usual meaning of that term, from the spherical traverse tables of Aquino, which are printed in Hydrographic Office publication No. 200.

Or the altitude may be computed from the formula

$$\sin h = \cos (L \sim d) - 2 \cos L \cos d \sin^2 \tfrac{1}{2}t \qquad (239)$$

and the true azimuth Z may then be taken from the azimuth tables (Art. 221), from an azimuth diagram, or found from a simultaneous compass bearing corrected for variation and deviation, provided the conditions are such as to admit of an accurate bearing being taken.

For a body observed on the meridian formula (239) reduces to $\sin h = \cos (L \sim d)$ or $z = L \sim d$, which is the usual formula for finding the latitude from a meridian altitude.

Since tables of haversines and log haversines have been more generally supplied (Table 45, Bowditch), the following formulæ obtained from (209) and (210) by substituting $(L \sim d)$ for z_0, have become the most widely used for computing the zenith distance:

$$\left.\begin{array}{ll} & \text{haver } z = \text{haver } (L \sim d) + \text{haver } \theta, \\ \text{where} & z = 90° - h \text{ and } \theta \text{ is defined by} \\ & \text{haver } \theta = \cos L \cos d \text{ haver } t. \end{array}\right\} \qquad (240)$$

Besides their existence in Bowditch, Hydrographic Office publication No. 200 contains a special collection of tables to facilitate the computation of the zenith distance by the above formulæ.

A Sumner line determined by the Marcq Saint-Hilaire method is of course the same as would be determined by any other approved method of solution. It may be combined to determine a fix with a terrestrial bearing or with a line found from a sight worked by some one of the direct methods.

Conditions of observation.—Owing to errors of refraction at low altitudes, and to the small limits within which the circle of altitude and its tangent at the computed point are coincident at very high altitudes as well as the practical difficulties of observation under such circumstances, it is desirable that heavenly bodies be observed, if possible, at altitudes not less than 10° nor greater than 86°.

Advantages of the Marcq Saint-Hilaire method.—*The great advantage of this method of obtaining a line lies in the fact that since the formulæ make it available practically without limitations as to azimuth, altitude, or hour angle, it furnishes one method equally applicable to all conditions, whether these conditions would otherwise require the formulæ of a time-sight, a $\phi''\phi'$ sight, or that of a body observed near the meridian.* Except when finding latitude by meridian altitude, by reduction to the meridian, or by Polaris, or when finding longitude by the time-sight (tangent) method, the process of solution above described is simpler than any other method for either latitude or longitude.

Rule for the determination of a single line.—With the latitude and longitude of the given D. R. position, compute the true altitude of a heavenly body for the instant at which the observed true altitude is known, or will be known, and find from the azimuth tables the body's true azimuth Z for the same instant. Subtract the computed true altitude from the observed true altitude, calling the remainder the "altitude difference" and designating it by the letter *a*. Then run by dead reckoning, or lay down on the chart from the assumed D. R. position, the distance of *a* sea miles on a Mercator course equal to the observed body's azimuth, if the observed altitude is the greater; on a course equal to the azimuth +180°, if the observed altitude is less than that by computation. The point thus determined is the computed point, and a line (which may be drawn on the chart) through this point at right angles to the body's bearing line will be the line of position. In Fig. 137, *AB* is the "altitude difference," *B* the computed point, and *BB'* the line of position from an observation of the body S_1 made at the D. R. position *A*.

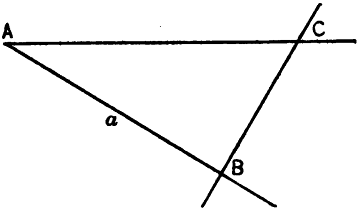


FIG. 136a.

Any point on a line of position together with the line's direction determines the line, and it is sometimes more convenient to use the point at which the assumed latitude cuts the Saint-Hilaire line, thus finding at once the longitude corresponding to assumed latitude or λ_c without finding the "computed point."

In Fig. 136a the point *A* is the assumed position; line *BC* is the line of position; *B*, the computed point; and *C*, the point on the line corresponding to the assumed latitude.

Let the coordinates of *A* be $L=40°$ N., $\lambda=60°\ 30'\ 00''$ W., the altitude difference $a=5$ miles toward the heavenly body whose bearing is 120° and hence the angle $BAC=30°$.

From the traverse tables, using 30° as the course and 5 miles as the difference of latitude we find the value of *AC* in miles from the distance column to be 5.77. Then transforming the departure, 5.77 miles, into difference of longitude for latitude 40°, we have $AC = 7'.53 = 7'\ 32''$ E. Hence the position of point C is, $L = 40°$ N., $\lambda_c = 60°\ 22'\ 28''$ W.

This point is particularly important in finding the elapsed interval from time of A. M. sight to noon, as explained in Chapter XXI.

Double altitudes.—If having determined the computed point B of one line BB' (Fig. 137), the run of the ship is laid off from B to C, at which point another altitude of the same, or of a different body, bearing in the direction CS_2 is both observed and computed, the hour angle from the meridian of C being used in the computation, then a second line of position DD' may be obtained by laying off the "altitude difference" at the second observation as before explained, the point C being the D. R. position and D the computed point of this second line. The intersection of DD' with CC' drawn through C parallel to the first line (CC' being the first line transferred for the run of the ship during the interval between observations) will be the fix F. These lines and the intervening run may be laid down on a Mercator chart (Fig. 137) and the fix found by construction (see Art. 292).

Anticipating the work.—Should the navigator, assuming the position A *for a later instant,* compute the true h and Z of a heavenly body to be observed at that future time and place, he may immediately plot that position on the chart and lay down the body's bearing line; then there will be nothing to do but to lay off the "altitude difference" and draw in the line of position when the actual altitude has been observed at the predetermined instant of G. M. T. The fact that the preliminary computation may be made, perhaps hours in advance of the actual measurement of the altitude, makes the Saint-Hilaire method most useful to any navigator desiring to anticipate his work. If for any reason the obser-

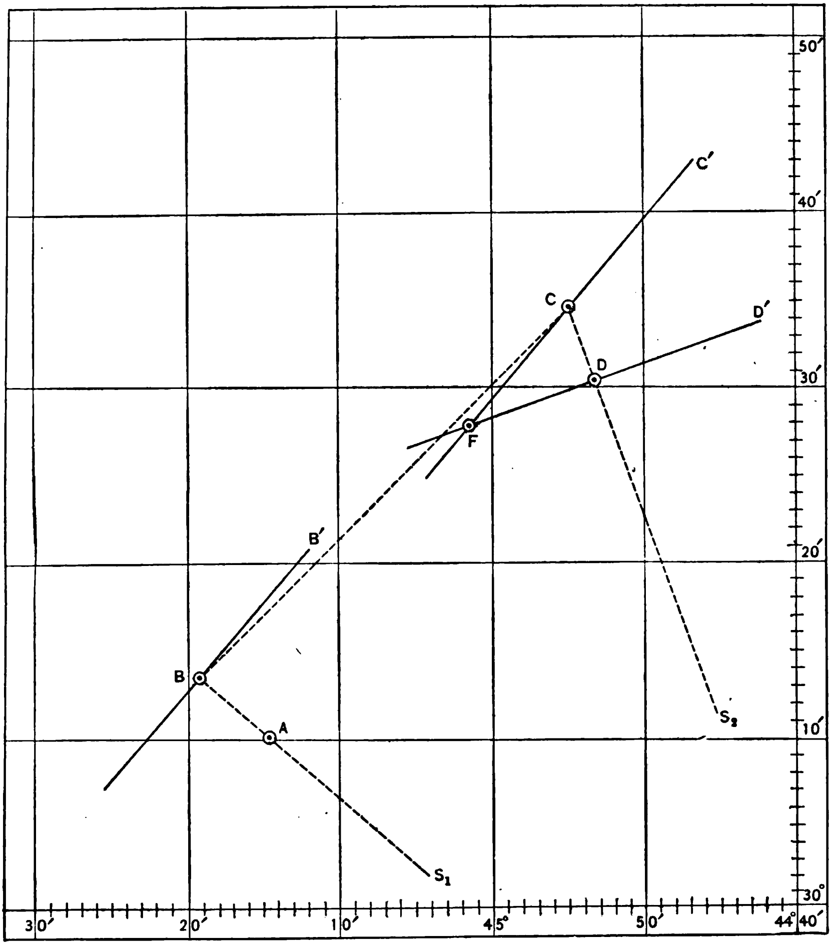


FIG. 137.

vation should not be made at the exact G. M. T. used in the computation, the line plotted as above should be shifted so as to allow for the error (Art. 293), remembering that if the G. M. T. of observation is greater than that of computation the line must be shifted to the westward, otherwise to the eastward. The same result may be accomplished by laying off the "altitude difference" from a new position point found by so altering the longitude of the assumed position (to the westward or eastward as indicated above) that the hour angle corresponding to the G. M. T. of observation may be the same as that used in the solution, thus making the computation for the altitude still hold good (see Ex. 222 and Fig. 145). For any probable difference between the G. M. T. of observation and that of computation the changes in the elements used for the observed body would be so slight as to produce only an inappreciable error in the results. So long as the longitude is changed as above to make the hour angle the same for the G. M. T., both of observation and computation, no other adjustment will be necessary, as for instance for any change in the assumed position due to fleet maneuvers. A line from a previous observation, however, brought up to the instant of the second observation, must be transferred for the exact run in the interval; then the intersection of the two lines will give the "fix" at the instant of second observation.

Intersection by Computation-Double Altitudes.

Referring to Fig. 137, let

AB be the first altitude difference $= a_1$,
CD be the second altitude difference $= a_2$,
Z_1 be the azimuth of the body at the first observation,
Z_2 be the azimuth of the body at the second observation.

Then the position of A being that by dead reckoning at the first observation, the position of *B* is obtained by running a distance a_1 in the direction of the first azimuth, or the opposite direction, as required by the conditions.

The position of *C* (the D. R. position used in the solution of the second observation) is obtained from the position *B* and the dead reckoning between observations. If desired the run a_1 and the run *BC* may be combined in one traverse, thus permitting *C* to be found directly from *A*.

The position of *F*, or fix, is obtained by running a distance *CF* in a direction parallel to that of the first line (see Ex. 220 and Fig. 137). As the line runs in two opposite directions from *C*, it is only necessary to know that the general direction of *CF* is that of *CD*; the direction of *CF* cannot differ as much as 90° from that of *CD*, and hence that direction from the point *C* is considered which fulfills this requirement.

The distance $CF = CD \operatorname{cosec} CFD = a_2 \operatorname{cosec} (Z_1 \sim Z_2)$ and is easily found by computation, or by using the traverse tables, entering the tables with $(Z_1 \sim Z_2)$ as a course and taking out the distance *CF* in the distance column directly opposite the value a_2 found in the departure column (see Art. 125).

The angle *CFD*, which equals the difference of the azimuths of the body, or bodies, at the two observations, or $Z_1 \sim Z_2$, is always acute if the same body is observed in both observations, and is never greater than 90° in case the observations are of two different bodies.

Intersection by Computation-Simultaneous Observations.

When the position of the ship does not change between the sights, and in the case of simultaneous observations, the run *BC* of Fig. 137 is zero; therefore, in such cases, as shown in

Fig. 138, when the "fix" is to be determined by computation, use the D. R. position *A* in the solution of one observation, apply the altitude difference a_1 to the position *A* and find the computed point *B* of the first line *BB′*; then use this computed point *B* in the solution of the other observation, apply the altitude difference a_2 to the same point *B* and find the computed point *D* of the second line *DD′*. The intersection *F* of the two lines will be the "fix" (see Ex. 221).

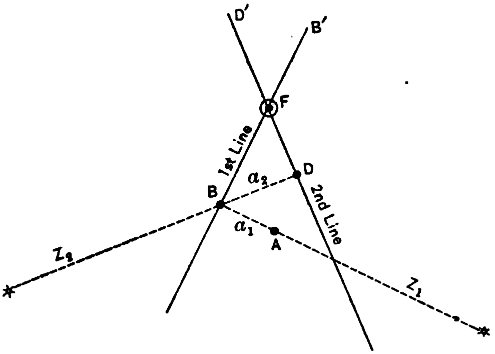


FIG. 138.

Intersection by Construction.

When the ship's position changes between observations, the "fix" by construction should be found by first determining the lines and then plotting them on the chart as indicated in Fig. 137.

Should the ship's position not change between sights, or in the case of simultaneous observations, the computed point *B* of the first line may be used in the solution of the second sight and the "fix" by construction found as indicated in Fig. 138. However, it is customary to work both sights by using the co-ordinates of one and the same point *A*, laying off from that one position both altitude differences, each in its proper direction, for the determination of the computed points; then the intersection of the position lines, when drawn,

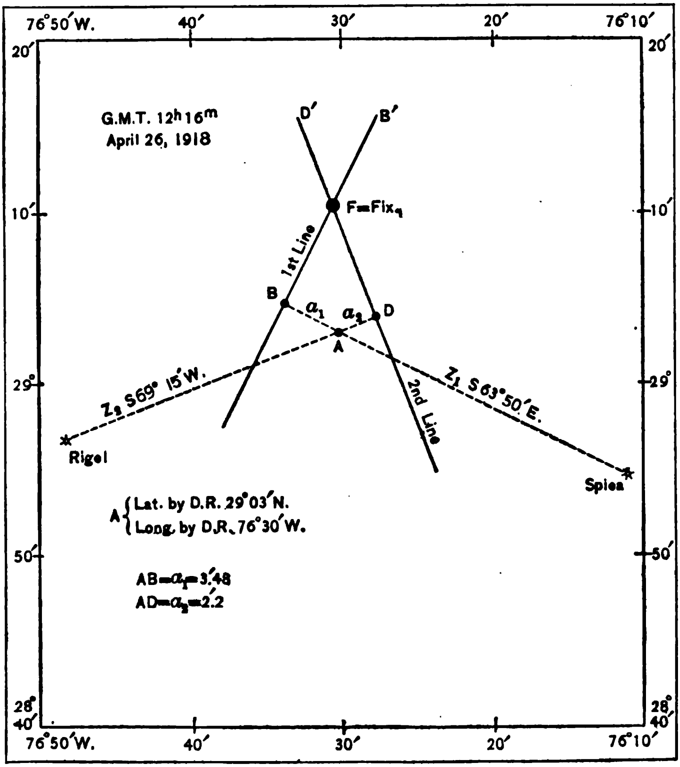


FIG. 139.

will be the "fix" *F* (see Fig. 139). In this connection attention is called to the method of laying off courses and distances on the Mercator and polyconic charts (see Art. 31).

309. Special cases.—The following cases are specially referred to in order that the student may learn how to combine, perhaps with advantage under certain circumstances, the direct methods of Chapters XVII and XIX with the indirect method of Marcq Saint-Hilaire:

(1) When one of the two observed bodies is on the meridian (Figs. 140 and 141). Let A be the D. R. position; B the computed point of the line BB' (or that line transferred for run to the instant of the second observation). Then whether the observation of the body on the meridian is solved by the Saint-Hilaire method, using the computed point B of the line BB' and the altitude difference a_2, or by the direct method (Art. 240), the result will be same; in the one case $BD = a_2$ and in the other it is the difference of latitude between B and D. In either case $BF = BD \operatorname{cosec} BFD = BD \operatorname{cosec} (Z_1 \sim Z_2) = BD \operatorname{cosec} Z_1$, but it is unnecessary to find BF as the latitude is well determined and the longitude alone in doubt. Find the Departure $DF = a_2 \cot Z_1$, then the difference of longitude which applied to the longitude of B will give the longitude of fix.

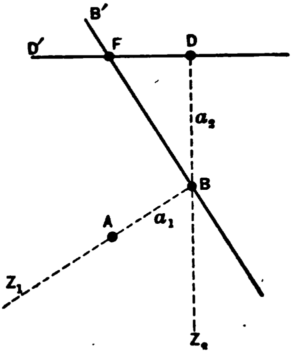


FIG. 140.

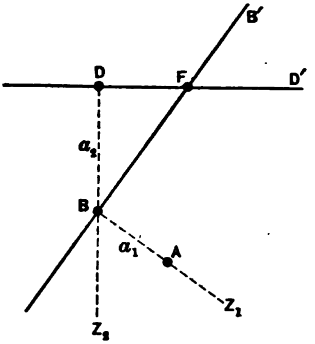


FIG. 141.

In this particular case, however, it must not be forgotten that the direct method of Arts. 299 and 300, as illustrated in Ex. 213, is equally as simple.

(2) When one of two observations is to be solved as a time-sight. Some navigators are averse to giving up the time-sight (tangent) method when conditions justify its use, at the same time preferring that of Saint-Hilaire to the $\phi''\phi'$ method. This procedure may find application in simultaneous observations of stars or in forenoon observations of the sun; the intersection may be found graphically on the chart or by computation as indicated below and in Fig. 142.

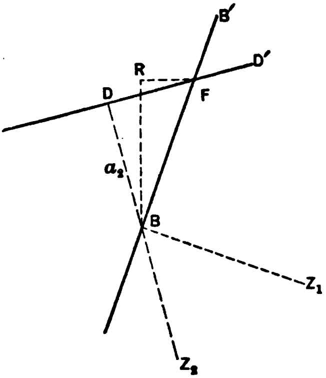


FIG. 142.

Let BB' (Fig. 142) be one line from a time-sight of a body (not on the P. V.), whose azimuth is Z_1, transferred for run to the instant of observation of another body whose azimuth is Z_2; or a line from a time-sight simultaneous with the observation of the body whose azimuth is Z_2. The point B having been used in the solution of the other observation by the Saint-Hilaire method, D is the computed point of the line DD' and F the fix. $BFD = Z_1 \sim Z_2$; $FBR = 90° \sim Z_1$; $BR = l$ and $RF = p$ between the positions of B and F. Entering the traverse tables with $Z_1 \sim Z_2$ as a course, look for a_2 in the dep. column and find BF in the distance column; with the direction of BF, that is $90° \sim Z_1$, as a course and BF as a distance, take out the corresponding l and p; then find values of L_0 and D and, from the coordinates of B, the latitude and longitude of "fix" F.

(3) If the time sight is of a body observed on the prime vertical, then, as indicated in Figs. 143 and 144, the line of position *BB′* will run due north and south, the longitude of "fix" will be well determined and it will be necessary to determine only its latitude. This may be found from the latitude of *B* and the difference of latitude between *B* and *F*. In this case *BF* is this difference of latitude and $BF = a_2 \operatorname{cosec} (Z_1 \sim Z_2) = a_2 \operatorname{cosec} (90^\circ \sim Z_2) = a_2 \sec Z_2$.

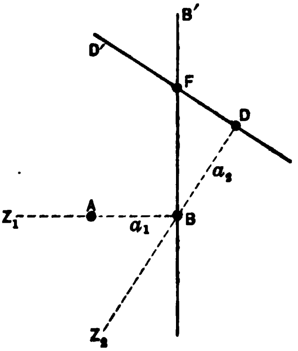


FIG. 143.

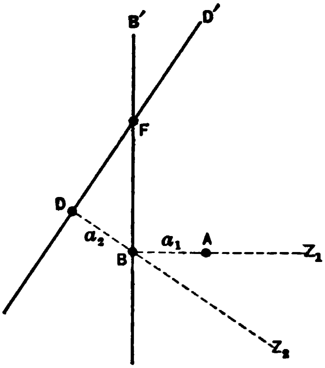


FIG. 144.

However, attention is called to the fact that by the direct methods the longitude would be well determined from the observation on the prime vertical and that an excellent "fix" would then result from using this longitude in the solution of the $\phi''\phi'$ sight.

(4) If the Marcq Saint-Hilaire method is applied to the first sight worked and the latitude of the computed point of this first line is used in working a time-sight (tangent method), the intersection, if not found by construction, may be found as explained in Art. 305.

The ship's most probable position.—From each one of several simultaneous observations, a line of position may be obtained and from n observations n lines will result; in case there have been no errors of observations, or otherwise, these lines should pass through one and the same point. However, there are always errors which may be due to the imperfection of the instrument itself or its adjustment, to error of the tabulated dip or refraction, to incorrect time, or the personal equation of the observer, etc., and, in consequence, generally speaking, there will be more than one point of intersection, and there may be for n lines as many as $\frac{n(n-1)}{2}$ points. It is evident that the ship is not at all points; the "theory of the probability of errors" shows that the most probable position is that point from which if perpendiculars are drawn to the lines of position, the sum of their squares shall be a minimum. The navigator, in his effort to check a "fix" from two lines by means of a third line of position, will often find that the three lines make a plane triangle; and, in such cases, though the most probable position may easily be found by construction, the practical navigator, regarding this procedure as more a matter of theory than of practical value, will assume the ship's position at the center of said triangle, especially if it is small and equilateral.

Use of Table 44, Bowditch.—This table has opposite t in the p. m. column the log sin $\frac{1}{2}t$ in the sine column; so if using formula (239), look for t expressed in time in the p. m. column and from the sine column, directly abreast, take out the log sin $\frac{1}{2}t$ which, multiplied by 2, will be the log $\sin^2 \frac{1}{2}t$. This method is illustrated in Ex. 221, and the method of considering the half-angle in degrees is illustrated in Ex. 220.

Ex. 220.—On January 15, 1918, about 8.30 a. m., in Lat. by D. R. 30° 10′ N., Long. by D. R. 45° 15′ W., the sextant altitude of the sun's lower limb bearing southward and eastward was 17° 40′ 50″. I. C. +0′ 30″. Height of eye 29 feet. W. $8^h\ 30^m\ 10^s$. C—W $3^h\ 02^m\ 40^s$. Chronometer slow of G. M. T. $7^m\ 10^s$. From this position ran NE. (true) 30 miles when the sextant altitude of the sun's lower limb bearing southward and eastward was 35° 32′ 20″. I. C. −0′ 20″. Height of eye 29 feet. W. $10^h\ 50^m\ 09^s$. C—W $3^h\ 02^m\ 41^s$. Chronometer slow of G. M T. $7^m\ 10^s$. Required the ship's position at the second observation, using the method of Marcq Saint-Hilaire.

Solution for first line.

Times.		Altitude.		Altitude Corrections.		Declination.		H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″		° ′		(−) to M. T.	
W.	8 30 10	$\odot$	17 40 50	S. D.	16 17	G. M. N. Jan. 15	S 21 13.3	N 0′.5	m s	$+0^s.9$
C—W	3 02 40	Corr.	+ 8 37	I. C.	+ 0 30			G. M. T. $-\frac{1}{3}^h$	9 22.8	$-\ \frac{1}{3}^h$
				Dip	− 5 17	Corr.	.2		Corr. − .3	
C.	11 32 50	$\ominus$	17 49 27	p. & R.	− 2 53			Corr. S 0′.16		$-0^s.30$
C. C.	+ 7 10						S. 21 13.5		9 22.5	
				Corr.	+ 8 37					
G. M. T. 14th Jan.	23 40 00			Or (Tab. 46)	+8′ 09″					
or 15th Jan.	− 0 20 00			I. C.	+ 30					
				Corr.	+8 39					
G. M. T.	23 40 00	2							log 0.30103	
Long. W	3 01 00	t		$=8^h\ 29^m\ 37^s.5$ a. m.		sin $\frac{t}{2}$		9.64642	2 sin $\frac{t}{2}$ 9.29284	
				° ′ ″						
L. M. T.	20 39 00	L		=30 10 00 N					cos 9.93680	
Eq. of T.	− 9 22.5	d		=21 13 30 S					cos 9.96950	
		$L \sim d$		=51 23 30		Nat cos		0.62399		
L. A. T.	20 29 37.5							0.31635	log 9.50017	
t	= − 3 30 22.5									
Civil T.	$8^h\ 29^m\ 37^s.5$ a. m.	h		=17 55 02		Nat sin		0.30764		

Ex. 220.—Continued.

By the azimuth tables { bearing of sun $Z_N = 128° 54'$ or S 51° 06′ E
{ direction of 1st line 218° 54′ or S 38° 54′ W

	° ′ ″
Observed true altitude of sun	17 49 27
True altitude of sun for D. R. position	17 55 02
Distance of *B* from *A* on reverse bearing of sun or 1st altitude difference $= a_1$ =5′.58=	5 35

Reverse bearing of sun 308° 54′ = 308°.9

Therefore by the traverse tables,

Course	Distance	Diff. Lat. N	Dep. W	
308°.9	5′.58	3.50	4.35	$D = 5'.03$ W

		° ′ ″		° ′ ″
Position Pt. by D. R.	Lat.	=30 10 N	Long.	45 15 W
	l	= 3 30 N	*D*	= 5 02 W
Position Pt. of 1st line	Lat.	=30 13 30 N	Long.	=45 20 02 W

Transfer of first line to position of second observation. Dead reckoning between observations.

True course.	Distance.	Diff. Lat. N	Dep. E	
45°	30	21′.21	21.21	$D = 24'.58$

		° ′ ″		° ′ ″
Position Pt. left	Lat.	30 13 30 N	Long.	45 20 02 W
	l	21 13 N	*D*	= 24 35 E
Position Pt. arrived at	Lat.	30 34 43 N	Long.	44 55 27 W

Ex. 220.—Continued.

Solution for second line.

Times.		Altitude.		Altitude Corrections.		Declination.	Eq. of T.	
	h m s		° ′ ″		° ′ ″	° ′	(−) to M. T.	m s
W.	10 50 09	⊙	35 32 20	S. D.	16 17	S 21 12.4		9 24.6
C—W	3 02 41	Corr.	+ 9 26	I. C.	− 0 20			
				Dip	− 5 17			
C.	1 52 50	⊖	35 41 46	p. & R.	− 1 14			
C. C.	+ 7 10			Corr.	+ 9 26			
G. M. T., Jan. 15	2 00 00			Or (Tab. 46)	+9′ 48″			
Long. W	2 59 41.8			I. C.	−0 20			
L. M. T.	23 00 18.2			Corr.	+9 28			
Eq. T.	− 9 24.6	2					log	0.30103
L. A. T.	22 50 53.6	t	10^h 50^m 53^s.6 a. m.		sin $\frac{t}{2}$	9.17666	2 sin $\frac{t}{2}$	8.35332
t	= − 1 09 06.4		° ′ ″					
Civil T.	10^h 50^m 53^s.6 a. m.	L	30 34 43 N				cos	9.93497
		d	21 12 24 S				cos	9.96955
		$L-d$	51 47 07		Nat cos	0.61861		
						0.03621	log	8.55887
		h	35 37 10		Nat sin	0.58240		

By the azimuth tables { Bearing of sun Z_N=160° 06′ or S 19° 54′ E Z_1 = S 51 06 E
Direction of second line 250° 06′ or S 70° 06′ W Z_2 = S 19 54 E

Observed true altitude of sun 35 41 46 $Z_1 \sim Z_2$ = 31 12 cosec 0.28565
True altitude of sun for D. R. position 35 37 10 a_2 = 4′.60 log 0.66276

CD (Fig. 187) or second altitude difference a_2 . =4′ 36″=4′.60 CF = 8′.88 log 0.94841

Direction of CF=218°.9

The position of fix F is 218°.9, distant 8.88 miles from C.

Course	Distance	Diff. Lat. S	Dep. W	D=6′.48 W
218°.9	8.88	6.91	5.58	

		° ′ ″		° ′ ″
Position Pt. C	Lat.	30 34 43 N	Long.	44 55 27 W
	l	6 55 S	D =	6 29 W
Fix F	Lat.	30 27 48 N	Long.	45 01 56 W

Ex. 221.—April 26, 1918, p. m., Lat. by D. R. 29° 03′ N., Long. by D. R. 76° 30′ W., observed simultaneous altitudes of the stars *α* Virginis (Spica) bearing southward and eastward, and *β* Orionis (Rigel) bearing southward and westward. Sextant altitude of Spica 21° 02′ 00″; of Rigel 18° 05′ 30″. I. C.+1′. Height of eye 45 feet. W. T. of obs. 7h 12m 10s. C—W 4h 58m 38s. Chro. slow of G. M. T. 5m 12s. Required the ship's position by the method of Marcq Saint-Hilaire, using the azimuth tables, and finding the intersection of the lines by computation. (See Fig. 138.)

Solution for first line (✳ Spica).

Times.		Altitude.		Altitude Corrections.		R. A. M. ☉.		Star's R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s	✳'s R. A.	13h 20m 55s.5
W.	7 12 10	✳'s h_s	21 02 00	I. C.	+ 1 00	At G. M. N.	2 14 42.2	✳'s *d*	S 10° 44′.3
C—W	4 58 38	Corr.	− 8 06	Dip	− 6 36	Corr. G. M. T.	2 00.9		
C. C.	+ 5 12			R	− 2 30				
		✳'s *h*	20 53 54			R. A. M. ☉	2 16 43.1		
G. M. T. April 26	12 16 00			Corr.	− 8 06				
				Or (Tab. 46)	− 9′ 07″				
Long. W.	5 06 00			I. C.	+ 1 00				
L. M. T.	7 10 00			Corr.	− 8 07				
R. A. M. ☉	2 16 43.1								
L. S. T.	9 26 43.1	2						log	0.30103
✳'s R. A.	13 20 55.5	*t*	3h 54m 12s.4			sin $\frac{t}{2}$	9.68932	2 sin $\frac{t}{2}$	9.37864
✳'s *t*	(−) 3 54 12.4	*L*	29° 03′ 00″ N					cos	9.94161
		d	10 44 18 S					cos	9.99232
		L ~ *d*	39 47 18			Nat cos	0.76842		
							0.41077	log	9.61360
		h	20 57 22			Nat sin	0.35765		

By azimuth tables { (Spica) Z_1=116° 10′ or S 63° 50′ E
Direction of 1st line, 26 10

Observed true altitude of ✳ Spica,	20° 53′ 54″
Computed true alt. of ✳ Spica for D. R. position,	20 57 22
a_1 (in direction of Z_1+180°)=3′.47=	3 28

	Course.	Distance.	Diff. Lat. N.	Dep. W.	
By the traverse tables	296°.2	3′.47	1′.53	3′.11	*D*=3′.56 W

Position Pt. by D. R., *A*.	Lat.	29° 03′ 00″ N	Long.	76° 30′ 00″ W
	l =	1 32 N	*D* =	3 34 W
Computed Position Pt. of 1st line, *B*.	Lat.	29 04 32 N	Long.	76 33 34 W

Ex. 221.—Continued. Solution for second line (✳ Rigel).

Times.		Altitude.		Altitude Corrections.		R. A. M. ☉.		Star's R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		
G. M. T. April 26	12 16 00	✳'s h_s	18 05 30	I. C.	+ 1 00	At G. M. N.	2 14 42.2	✳'s R. A.	$5^h\ 10^m\ 36^s.6$
Long. W.	5 06 14.8	Corr.	− 8 33	Dip	− 6 36	Corr. G. M. T.	2 00.9	✳'s d	S 8° 17′.8
				R	− 2 57				
L. M. T.	7 09 45.7	✳'s h	17 56 57			R. A. M. ☉	2 16 43.1		
R. A. M. ☉	2 16 43.1			Corr.	− 8 33				
				Or (Tab. 46)	− 9′ 34″				
L. S. T.	9 26 28.8			I. C.	+ 1 00				
✳'s R. A.	5 10 36.6								
✳'s t	+ 4 15 52.2				− 8 34				

2				log	0.30103
t	$4^h\ 15^m\ 52^s.2$	sin $\frac{t}{2}$	9.72402	2 sin $\frac{t}{2}$	9.44804
L	29° 04′ 32″ N			cos	9.94150
d	8 17 48 S			cos	9.99543
$L \sim d$	37 22 20	Nat cos	0.79471		
			0.48529	log	9.68600
h	18 01 28	Nat sin	0.30942		

By azimuth tables { (Rigel) Z_2 = 249° 15′ or S 69° 15′ W
Direction of 2d line, 339 15

Observed true altitude of ✳ Rigel, 17° 56′ 57″
Computed true altitude of ✳ Rigel for D. R. position, 18 01 28

BD (Fig. 138) or second altitude difference a_2 = 4′.517 = 4 31

Z_1 = S 63° 50′ E
Z_2 = S 69 15 W

$Z_1 \sim Z_2$ = 46 55 cosec 10.13646
a_2 = 4′.517 log 0.65485

BF (Fig. 138) = d = 6′.185 log 0.79131
Direction of *BF* = *C* = 26°.2

The position of fix *F* bears N 26°.2 E and is distant 6.185 miles from *B*.

Course.	Distance.	Diff. Lat. N	Dep. E.	
26°.2	6′.185	5′.55	2′.78	D = 3′.12 E

Position Pt. of 1st line, *B*.	Lat.	29° 04′ 32″ N	Long.	76° 33′ 34″ W
	l =	5 32 N	D =	3 07 E
Fix *F* (Fig. 138).	Lat.	29 10 04 N	Long.	76 30 27 W

Ex. 220(*a*).—April 3, 1918, about 7.30 a. m., in Lat. by D. R. 25° 40′ S., Long. by D. R. 104° 06′ East, the sextant altitude of the sun's lower limb was 18° 08′. I. C.—1′. Height of eye 19 feet. W. $7^h 30^m 23^s$. C—W $5^h 15^m 03^s$. Chro. fast of G. M. T. $5^m 41^s.2$. Find a position point and a line of position by the method of Marcq Saint-Hilaire, using the haversine formulæ (240).

Times.		Altitudes.		Altitude Corrections.		Declination.	H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″	° ′		— to M. T. m s	
W.	7 30 23	⊙	18 08 00	S. D.	+16 01	N 4 54.7	N 1′.0	3 39.3	— s.7
C—W	5 15 03	Corr.	+ 7 56	I. C.	— 1 00	N .7	0h.66	— .5	0h.66
				Dip	— 4 16				
C.	12 45 26	⊖	18 15 56	p. & R.	— 2 49	N 4 55.4	N 0′.66	3 38.8	—0s.46
C. C.	— 5 41.2			Corr.	+ 7 56				
G. M. T. April 2,	12 39 44.8			Or (Tab. 46)					
Long. E.	6 56 20				+ 8′ 56″				
				I. C.	— 1 00				
L. M. T.	19 36 04.8			Corr.	+ 7 56				
Eq. of T.	— 3 38.8								
L. A. T.	19 32 26								
t	— 4 27 34								

By tables.

⊙'s Z = S 105° 04′ E

∴ ⊙'s Z_N = 74°.9

$t =$	— $4^h 27^m 34^s$	L. haver	9.48254		
$L =$	25° 40′ 00″ S	L. cos	9.95488		
$d =$	4 55 24 N	L. cos	9.99840		
$\theta =$	....	L. haver	9.43582	N. haver	.27279
$L \sim d =$	30 35 24	....		N. haver	.06958
$z =$	71 37 25	....		N. haver	.34237
$h =$	18 22 35				
$a =$	6′ 39″ = 6′.65 on bearing Z_N + 180°				

By traverse tables.

C_N	Distance.	Diff. Lat. S.	Dep. W.
254°.9	6′.65	1′.73	6′.42

Lat. by D. R.	25° 40′ 00″ S.	Long.	104° 05′ 00″ E.
l	1 44 S.	*D*	7 08 W.
Lat. of Pt.	25 41 44 S.	Long. of Pt.	103 57 52 E.
L_0	25⅝° S.		

Ans. Position Point {Lat. 25° 41′ 44″ S. / Long. 103 57 52 E.} Direction of line 164°.9

Ex. 220(*a*).—Statement on preceding page. This example is solved below by the use of the Aquino spherical traverse tables (Hydrographic Office publication No. 200) for purposes of comparison. The lines obtained should be, of course, identical, although the "Computed Points" differ, and this practical identity is shown at the bottom of the page by comparing the difference of longitude of the two computed points with the difference of longitude obtained by multiplying the difference of latitude of the two points by the longitude factor *F* of the line.

Times.		Altitudes.		Altitude Corrections.		Declination.	H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″	° ′		— to M. T.	
W.	7 30 23	⊙	18 08 00	Tab. 46	+ 8 56	N 4 54.7	N 1′.0	m s	— s.7
C—W	5 15 03	Corr.	+ 7 56	I. C.	— 1 00	N .7	0h.66	3 39.3	0h.66
								.5	
C.	12 45 26	⊖	18 15 56	Corr.	+ 7 56	N 4 55.4	N 0′.66		— s.46
C. C.	— 5 41.2	or	18° 15′.9					3 38.8	

G. M. T. April 2,	12 39 44.8		° ′	Interpolation.		° ′	
Eq. of T.	— 3 38.8	Arc	170 58.5 E	2.50	Arc	170 58.5 E	
		λD. R.	104 05.0 E	10.4	t_A	66 59.1 E	
G. H. A.	12 36 06						
or	— 11 23 54	$t_{D. R.}$	66 53.5 E	26.0	λ_A	103 59.4 E	Position *A*.
Arc	170° 58′.5			.08			
		$a=$	66 30	2.08	L_A	25 34 S	
		h_0	18 15.9		b	12 26	
		h_c	18 19.0		C	38 00	
		Diff.=	3.1 away.		$Z=$	75 01	

By traverse tables.

C_N	Dist.	Diff. Lat. S.	Dep. W.
255°	3.1	′.8	3.0
$L_0=25°.6$		$D=3'.3$ W.	$F=.3$

	° ′		° ′	
L_A	25 34.0 S	λ_A	103 59.4 E	
l	.8 S	D	3.3 W	
Lat. of point	25 34.8 S	Long. of point	103 56.1 E	Aquino Line.
Lat. of point	25 41.7 S	Long. of point	103 57.9 E	Cos—Haver Line.
l	6.9 S			
F	.3			
D	2.07 E	$D=$	1.8 E	

Ex. 221(*a*).—Jan. 26, 1918, about 6.20 p. m., in Lat. by D. R. 13° 30′ N., Long. by D. R. 137° 14′ 40″ E., the sextant altitude of the star Sirius (*a* Canis Majoris) was 24° 26′ 30″. Star east of the meridian. I. C. + 1′. Height of eye 45 feet. W. T. of observation $6^h\ 22^m\ 30^s$. C—W $2^h\ 57^m\ 57^s$. Chronometer fast of G. M. T. $3^m\ 56^s.5$. Find a position point and a line of position by the method of Marcq Saint-Hilaire (using haversine formulæ).

Times.		✱'s Altitude.		Corr. to ✱'s Alt.		R. A. M. ⊙.		✱'s R. A. and Dec.	
	h m s		° ′ ″		′ ″		h m s		h m s
W.	6 22 30	✱'s h_s	24 26 30	I. C.	+ 1 00	At G. noon Jan. 25	20 15 55.8	✱'s R. A.	6 41 34.4
C—W	2 57 57	Corr.	− 7 43	Dip	− 6 36				
				Ref.	− 2 07	Corr. G. M. T.	3 29.7		° ′
C.	9 20 27	✱'s *h*	24 18 47					✱'s Dec.	S 16 36.4
C. C. (−)	3 56.5			Corr.	− 7 43	R. A. M. ⊙	20 19 25.5		
G. M. T. Jan. 25	21 16 30.5			Or (Tab. 46)	− 8′ 44″				
R. A. M. ⊙	20 19 25.5			I. C.	+ 1 00				
G. S. T.	17 35 56.0			Corr.	− 7 44				
Long. E.	9 08 58.7								
L. S. T.	2 44 54.7								
✱'s R. A.	6 41 34.4								
✱'s *t*	− 3 56 39.7								
By tables.									
✱'s *Z* = N 115°.5 E									
∴ ✱'s Z_N = 115°.5									

$t = -\ 3^h\ 56^m\ 39^s.7$ L. haver 9.38689

$L = 13°\ 30'.00''$ N L. cos 9.98783

$d = 16\ 36\ 24$ S L. cos 9.98150

θ .... L. haver 9.35622 N. haver .22710

$L \sim d = 30\ 06\ 24$ .... N. haver .06745

$z = 65\ 44\ 20$ .... N. haver .29455

$h = 24\ 15\ 40$

$a = 8\ 07 =$ 3′.12 on bearing Z_N

By traverse tables.

C_N	Distance.	Diff. Lat. S.	Dep. E.
115°.5	8′.12	1′.34	2′.82

Lat. by D. R.	13° 30′ 00″ N.	Long. by D. R.	137° 14′ 40″ E.
l	1 20 S.	$D = 2'.90$ E =	2 54 E.
Lat. of Pt.	13 28 40. N.	Long. of Pt.	137 17 34 E.
L_e	13½° N.		

Ans. Position Point { Lat. 13° 28′ 40″ N. Long. 137 17 34 E. } Direction of line 25°.5

Ex. 222.—April 7, 1918, a. m., by an observation of the sun taken at the G. M. T. $0^h\ 40^m\ 54^s$ April 8, 1918, a ship was found to be on a line of position *GM* determined by a position point *G* in Lat. 39° 27′ N. and Long. 69° 18′ W., and by the sun's true azimuth $Z_N = 104°\ 58'$. Expecting the ship to maintain for several hours her course 320° (true) and speed 10 knots per hour, the navigator decided to anticipate as far as possible the work for a line from an observation of the sun to be taken three hours later, or at the G. M. T. $3^h\ 40^m\ 54^s$. Assuming that the D. R. position at that time would be Lat. 39° 50′ N. and Long. 69° 43′ W., he found for that time and place the sun's computed true altitude to be 54° 11′ 45″ and its true azimuth or Z_2 to be $Z_N = 153°\ 49'$. The above D. R. position and bearing were immediately laid down on the chart (see *A*, Fig. 145). The navigator failed to get his sight at the exact G. M. T. used in the computation, but 16 seconds later, or at the G. M. T. $3^h\ 41^m\ 10^s$, he observed the sextant altitude of $\underline{\odot}$ 54° 00′ 50″; I. C. + 3′; height of eye 26 feet. It is required to find at the time of the second observation a position point and line by the Marcq Saint-Hilaire method and the "fix" by construction.

Solution.—The sun's true azimuth at second sight from tables, or Z_2, is $Z_N = 153°\ 49'$, the computed true altitude of the sun's center is 54° 11′ 45″, the observed true altitude is 54° 14′ 13″, and hence a_2 is 2′ 28″ to be laid off in the direction of Z_2. Since the G. M. T. of observation was 16 seconds later than that used in the computation, the longitude of the D. R. position of the second line must be so changed as to give for the G. M. T. of observation the same hour angle as that used in the computation; in this case the longitude must be increased by 16 seconds of time, that is by 4 minutes of arc.

Therefore, from *A′*, which is *4′ of longitude directly to* westward of *A* (see Fig. 145), lay off $a_2 = 2'\ 28''$, in this case, toward the sun and *B* will be a position point of the second line *BB′*, and, as *AM′* is the first line brought up for

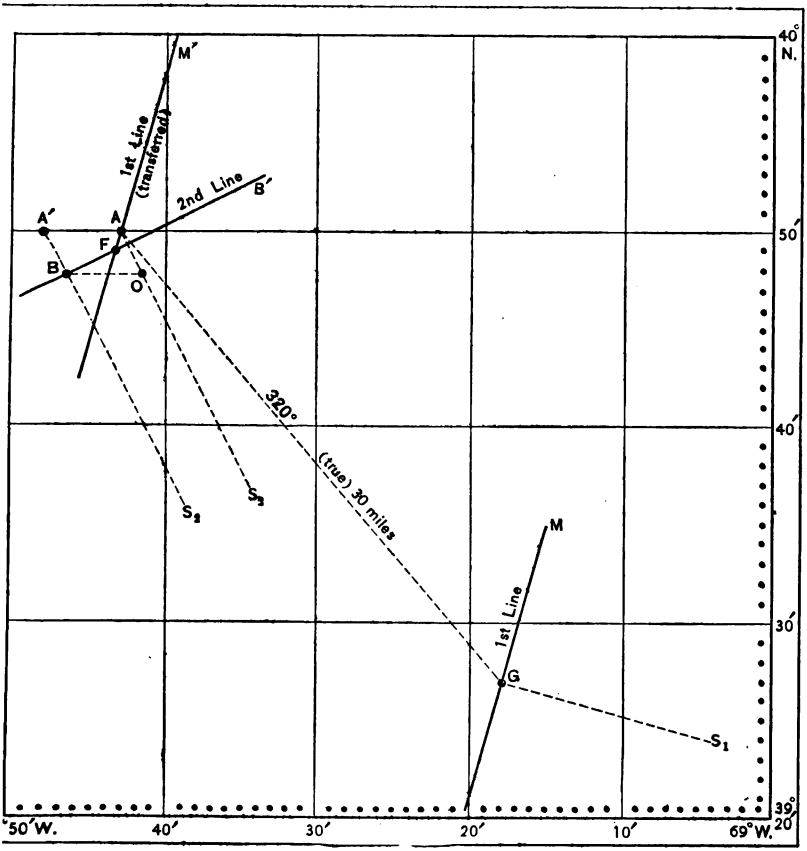


FIG. 145.

the run, *F*, in Lat. 39° 49′ N., and Long. 69° 43′ W., by construction, will be the "fix." It is apparent that a_2 could have been laid off from *A* to *O*, and the line then shifted to *B* in order to allow for the error of time (Arts. 31 and 293).

Littlehales' graphic solution.—The methods of the "New Navigation" have been applied by Mr. G. W. Littlehales of the Hydrographic Office, Navy Department, to the graphic solution of the astronomical triangle by means of a stereographic projection on the plane of the observer's meridian. Knowing the latitude and longitude by D. R., and having found the observed body's hour angle and declination for the instant of observation, the navigator may find graphically the values of *Z* and *h* for the given D. R. position, and, from these and the measured true altitude of the same body the computed point and line of position, as previously explained.

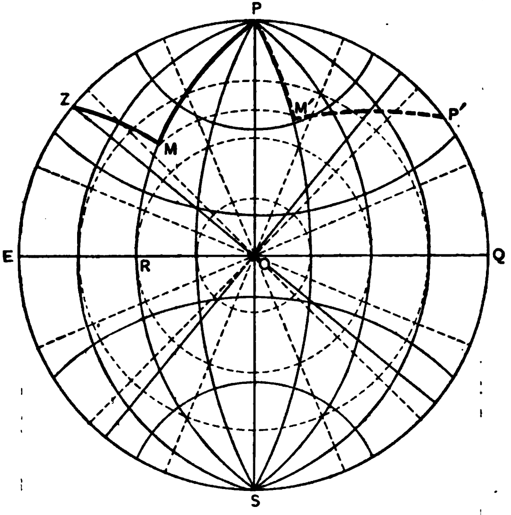


FIG. 146.

The theory embodied may be briefly explained as follows: let Fig. 146 be the projection referred to above on a reduced scale; *P*, the north pole; *Z*, the observer's zenith; *M*, the observed body located by its hour angle *ZPM* and its declination *RM*; then *PZM* is the astronomical triangle in which the known parts are *PZ* the co-*L*, *PM* the co-*d*, and *ZPM* the hour angle *t*. In this method *PZ* is considered as $< 90°$ or $> 90°$ according as *L* is north or south; *PM* is $< 90°$ or $> 90°$ according as *d* is north or south. The required parts of the triangle are *PZM* or *Z* and *ZM* the co-*h*, and they may be easily determined if referred to a system of co-ordinates which, like the equinoctial system, admits of permanent graduations. This is accomplished by revolving the astronomical triangle about the central point *O* of the projection with the side *PZ* kept in coincidence with the bounding meridian till *Z* falls where *P* originally was and *P* and *M* are revolved respectively into the positions *P′* and *M′* so that the unknown parts *PZM* and *ZM*, respectively equal to *P′PM′* and *PM′*, may be measured from the graduations of the projection; *PZM* is reckoned from the left hand bounding meridian and *P′PM* from the right hand bounding meridian.

It is apparent that *M* has described an arc of a circle whose radius is *OM* and which subtends an angle equal to *PZ* or co-*L*, hence to obviate actual revolution of the triangle, a series of equally spaced concentric circumferences and a series of equally spaced radial lines are drawn to facilitate identification, the former numbered from the center outward, the latter numbered so as to indicate the number of minutes of arc estimated from *OS* and around to the right. After having plotted the body by its hour angle and declination, it is only necessary to note the number of the circumference and the number of the radial line passing through the position *M*; add to this latter number the distance *ZP* or co-*L* expressed in minutes of arc; the point where the radial whose number is the latter sum intersects the noted circumference will be the point *M′* whose hour angle and declination, read from the graduations of the projection, will be respectively the required *Z* and *h* of the body *M*.

A stereographic projection has been constructed for a sphere 12 feet in diameter which is on such a scale as to admit of sufficient accuracy for practical navigation and to admit of convenient spacing of the various meridians, parallels, circumferences, and radials.

Each quadrant is subdivided into 92 overlapping sections, making 368 in all; the plates representing them form a book of convenient size, each plate bearing the same number as the corresponding section of a small projection called the index-plate.

The point *M* is roughly plotted on the index-plate and the circumference, radial line, and square are each noted by its number; *M'* is then roughly plotted and the square in which it falls is also noted.

Knowing the numbers of the squares and turning to them in the book of plates, the positions of *M* and *M'* are successively plotted, and the values of *Z* and *h* are taken from the second square.

For plates and further information see Mr. Littlehales' book " Altitude, Azimuth, and Geographic Position."

Solution by nomography.—Lt. Radler de Aquino, Brazilian Navy, has suggested a method of finding *h* and *Z* by using a nomogram constructed by Dr. Pesci; this method, with certain modifications introduced by the author, will be found explained in Appendix D.

Another solution of this nature is published by the Hydrographic Office as chart No. 2776, entitled " Altitude, Azimuth, Hour Angle." The mathematical deduction of the principles of the design appears in the Proceedings of the U. S. Naval Institute for November, 1917.

Solution by Tables.*—Navigators who find logarithmic work laborious may find a position point and a line of position by the Marcq Saint-Hilaire method by using Hydrographic Office publication No. 200 from which *h* and *Z* can be taken for a position of assumed latitude and longitude, the computed point being then found by laying off the altitude difference from this assumed position in the direction of *Z* or 180° + *Z*, as conditions may require. Ex. 220(a) has been solved on p. 660, by the use of these tables.

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* See " Altitude and Azimuth Tables," by Lt. Radler de Aquino, Brazilian Navy, and " Altitude or Position Line Tables," by Frederick Ball, R. N., both books published by J. D. Potter, London.

CHAPTER XXI.

DAY'S WORK AT SEA.

310. In the chapter on the sailings attention was called to the fact that the general subject of a day's work was reserved till after the student had studied and understood the methods of finding latitude and longitude by the observation of celestial bodies.

These methods having been considered, that subject will now be taken up.

In the course of his routine work, a navigator, besides determining his latitude at or near noon and obtaining lines of position from observations of the sun, both a. m. and p. m., would get positions by cross lines of stars or planets, when conditions proved favorable, as in evening or morning twilight or when moonlight renders the horizon sufficiently distinct. Polaris, being available in the northern hemisphere, should be observed when conditions are favorable, and latitude desirable.

The reckoning is estimated from the point of departure (Art. 123), or from the noon position at sea till noon of the following day, or till arrival at the port of destination, if the voyage ends before noon.

Owing to the facility of getting the latitude at noon by observations and the fact that longitude can be determined by observation within a few hours before noon and brought up to that time without appreciable error, it is convenient to compare the run by dead reckoning and by observation from noon to noon, and to regard the difference between the noon positions by dead reckoning and by observation as due to current, *though, as a matter of fact, it may be due to other causes, such as bad steering, faulty logging, and inaccurate values of the variation and deviation of the compass.*

The navigator must report to the commanding officer at noon each day:

(1) **Latitude and longitude by D. R. at noon.**

(2) **Latitude and longitude by observation at noon.**

(3) **Course and distance made good.**

(4) **Set and drift of current.**

(5) **The deviation of the compass** (on the course at time of a. m. sight perhaps).

(6) **Course and distance to destination.**

To attain these results, the following rules are laid down for a minimum of work, but the results are not rigidly accurate unless the ship is making a true North or South course during the whole interval from noon to noon. The interval of time from noon to noon on a ship steaming on any other than a North or South course is not 24 hours and consequently the current in latitude as found by the following rules together with results depending on its value will be slightly in error:

(1) Find the D. R. positions at time of a. m. sight for longitude and at noon by working the traverse from the previous noon or point of departure.

(2) Find an a. m. line of position by the method of Saint-Hilaire or by either the chord or tangent method, and the deviation of the compass when the sun is favorably situated for finding time. Plot this line on a Mercator chart and find graphically its intersection with another line, if possible.

(3) Find the latitude at noon by observation from a meridian altitude of the sun, or by reduction to the meridian; or bring up to noon a latitude obtained from the intersection of a $\phi'' \ \phi'$ line of position and a line of position for longitude, or from the intersection of two Saint-Hilaire lines taken at different times.

(4) Take the difference between the latitude by observation and latitude by D. R. at noon, mark it North or South as the former is to the northward or southward of the latter. This discrepancy may or may not be due to current, though usually so considered in the computation. Its value being for approximately 24 hours, or from the time of departure, a proportional part for one hour, and hence for the interval between the forenoon sight and noon, may be obtained.

(5) Run the noon latitude back to the time of sight, correcting backwards for both the run from sight to noon and the proportion of current in latitude for that time. The result will be the true latitude at time of sight.

Find the longitude by observation at time of sight by finding the position point of the line corresponding to the true latitude at time of sight.

(6) The difference between the longitude by observation and by D. R. at the time of a. m. sight is a discrepancy which may or may not be due to current, though usually so considered in the computation. Its value being from noon of the previous day, or time of departure, to the time of a. m. observation, a proportional part for one hour, and hence for the interval to noon, may be found. It is marked E. or W., according as the longitude by observation is to the eastward or westward of that by D. R.

(7) Run the longitude by observation at time of sight up to noon by applying the run in longitude from time of sight to noon, and also the current in longitude for the same time, each with its proper sign. The result will be the longitude by observation at noon.

(8) The course and distance from the noon position of the previous day, or point of departure, to the noon position by observation arrived at, will be the course and distance made good.

(9) The course and distance from the noon position arrived at by D. R. to that by observation will be the set and drift of the current, so-called (Art. 130).

(10) The course and distance from the noon position by observation arrived at to point of destination by middle latitude or Mercator sailing, will be the course and distance by that sailing to point of destination. Reference is made to chapter VI for manner of working dead reckoning and to chapters XVI and XVII for working of sights.

The following problem will illustrate the points involved:

Ex. 223.—On January 2, 1918, at noon, a ship's position by observation was latitude 7° 05′ 42″ N., longitude 148° 19′ W. Sailed thence until about 8 a. m. next day the following courses and distances; wind, variation, and deviation as indicated.

Wind.	Course (p. c.).	Var.	Dev.	Leeway.	Distance.
Sly and Wly	301°	+8°	−8°	8°	22.8 Miles.
do	285	+8	−4	6	31.6 "
do	276	+8	−5	8	34.5 "
Nly and Wly	256	+8	−3	3	17.9 "
do	233	+8	−1	8	16.1 "
do	212	+8	+1	0	14.9 "
do	220	+8	+1	0	12.6 "

At about 8 a. m. observed an altitude of the sun's lower limb, 23° 42′. I. C. (plus) 1′ 20″. Height of eye 45 feet. Watch 8^h 08^m 45^s. C—W 10^h 03^m 15^s. Chronometer fast of G. M. T. 7^m $21^s.5$. Sun's center bore (p. s. c.) 115°, ship's head 291°, variation + 8°. Work a line of position, using latitudes 7° N. and 7° 20′ N. Work an altitude-azimuth with latitude 7° 20′ N. and find deviation. (The azimuth may be taken from tables.)

Ran thence to noon 291° (p. s. c.), 39 miles, when observed meridian altitude of sun's lower limb, bearing South, 59° 26′ 10″. I. C. (plus) 1′ 20″. Height of eye 45 feet.

1. Find latitude and longitude by D. R. at 8 a. m.
2. Work a line for longitude, and find deviation.
3. Find latitude and longitude by D. R. at noon; true latitude at noon, and current in latitude; true latitude at a. m. sight.
4. Find true longitude at 8 a. m., current in longitude, and true longitude at noon.
5. Find C_N and d made good, and set and drift of current.
6. Find C_N and d to Guam by Mercator sailing using trigonometrical formulæ.

A DAY'S WORK, SOLUTION OF EXAMPLE 228.

Dead Reckoning from Noon till Time of a. m. Sight.

Wind.	Course p. c.	Var.	Dev.	Lee-way.	True Course.	Distance.	Diff. of Lat.		Departure.	
							N	S	E	W
Sly & Wly	301°	+8°	−3°	3°	309°	22.8	14.4			17.7
do	285	+8	−4	6	295	31.6	13.4			28.6
do	276	+8	−5	3	282	34.5	7.2			33.8
Nly & Wly	256	+8	−3	3	258	17.9		3.7		17.5
do	233	+8	−1	3	237	16.1		8.8		13.5
do	212	+8	+1	0	221	14.9		11.2		9.8
do	220	+8	+1	0	229	12.6		8.3		9.5
							35 32 l = 3 N	32	p =	130.4 W

D = 131′.4 W

	° ′ ″
Lat. left	7 05 42 N
Diff. of Lat.	3 00 N
At a. m. sight, Lat. by D. R.	7 08 42 N

	° ′ ″
Long. left	148 19 00 W
Diff. of Long.	2 11 24 W
At a. m. sight, Long. by D. R.	150 30 24 W

A DAY'S WORK, SOLUTION OF EXAMPLE 223.—Continued.

A. M. Line for Longitude and Compass Deviation.

Times.	Altitude.	Altitude Corrections.	Declination.	H. D.	Eq. of T.	H. D.
h m s W. 8 08 45 C—W 10 03 15 C. T. 6 12 00 C. C. − 7 21.5 G. M. T. Jan. 3 6 04 38.5 = 6h.08	° ′ ″ ☉ 23 42 00 Corr. + 8 58 ⊖ 23 50 58	′ ″ S. D. +16 18 I. C. + 1 20 Dip − 6 36 p. & R. − 2 04 Corr. + 8 58 Or (Tab. 46) + 7′ 38″ I. C. + 1 20 Corr. + 8 58	° ′ At 6h, S 22 51.3 p 112° 51′ 18″	N 0′.2 G. M. T. h.08 Corr. N 0′.016	(−) to M. T. m s 4 29.5 Corr. + .1 4 29.6	+1s.2 G. M. T. h.08 Corr. +0s.096

	° ′ ″				° ′ ″				° ′ ″		
h	23 50 58			h	23 50 58			h	23 50 58	sec	10.03876
→ L	7 00 00	sec	10.00325	→ L	7 20 00	sec	10.00357	L	7 20 00	sec	10.00357
p	112 51 18	cosec	10.03551	p	112 51 18	cosec	10.03551	p	112 51 18		
$2s$	143 42 16			$2s$	144 02 16			$2s$	144 02 16		
s	71 51 08	cos	9.49342	s	72 01 08	cos	9.48954	s	72 01 08	cos	9.48954
h	23 50 58			h	23 50 58			p	112 51 18		
$s-h$	48 00 10	sin	9.87109	$s-h$	48 10 10	sin	9.87223	$s \sim p$	40 50 10	cos	9.87885
	h m s		~~2 | 19.40327~~		h m s		~~2 | 19.40085~~				2 | 19.41072
L. A. T.	19 58 22.0	~~sin~~	9.70164	L. A. T.	19 59 06.7	~~sin~~	9.70042	$\frac{Z}{2}$	= 59° 30′ 30″	cos	9.70536
Eq. of T.	+ 4 29.6			Eq. of T.	+ 4 29.6			True Z	N 119° E	True Z_N	119°
										Z_N (p.c.)	115
L. M. T.	20 02 51.6			L. M. T.	20 03 36.3					C. E.	= +4
G. M. T.	6 04 38.5			G. M. T.	6 04 38.5					Var.	= +8
Long.	10 01 46.9 W			Long.	10 01 02.3 W					Dev.	= −4
Arc	150° 26′ 43″.5 W			Arc	150° 15′ 33″ W						

Longitude factor $F = 0.56'$ and $\Delta\lambda = 0.56 \times \Delta L$.
The true azimuth may be found from the azimuth tables, using L, d, and t as arguments.

Dead Reckoning from time of a. m. sight till noon.

Course p. c.	Var.	Dev.	True Course.	Distance.	Diff. of Lat.		Departure.	
					N	S	E	W
291°	+ 8°	− 4°	295°	39	16.5			35.3

	° ′ ″		° ′ ″
At time of a. m. sight, Lat. by D. R.	7 08 42 N	Long. by D. R.	150 30 24 W
To noon, Diff. of Lat.	16 30 N	Diff. of Long.	35 36 W
At noon, Lat. by D. R.	7 25 12 N	Long. by D. R.	151 06 00 W

Latitude by meridian altitude of the sun.

Altitude.		Altitude Corrections.		Sun's Declination.		H. D. of Dec.	
	° ′ ″		′ ″				
☉	59 26 10 S	S. D.	+ 16 18	At 10ʰ,	S 22° 50′.3	G. M. T.	N 0′.2
Corr.	10 31	I. C.	+ 1 20				0ʰ.15
		Dip	− 6 36			Corr.	N 0′.03
⊖	59 36 41 S	p. & R.	− 0 31				
z	30 23 19 N						
d	22 50 18 S	Corr.	+ 10 31				h m s
		Or (Tab. 46)		G. A. T. of local noon			= 10 04 24
Lat. obs. at noon	7 33 01 N		+ 9′ 11″	Eq. t			+ 4 34
		I. C.	+ 1 20	G. M. T. of local app. noon			10 08 58=10ʰ.15
		Corr.	+10 31				

	° ′ ″
At noon, Lat. by obs.	7 33 01 N
At noon, Lat. by D. R.	7 25 12 N
Current in Lat. 24 hrs.	7 49 N
Current in Lat. 4 hrs.	1 18.2 N

	° ′ ″
At noon, Lat. by obs.	7 33 01 N
Diff. Lat. sight to noon	16 30 N
Current in Lat. sight to noon	1 18.2 N
True Lat. at time of a. m. sight	7 15 12.8 N

With Lat. 7° 20′ N and Long. 150° 15′ 33″ W } as origin, $\Delta L = 4'.7866$ S. and $\Delta\lambda = 0.56 \times 4'.7866 = 2'.680 = 2'\ 40''.8$ W.

A DAY'S WORK, SOLUTION OF EXAMPLE 223.—Continued.

	° ′ ″		° ′ ″
Computed Long. for Lat. 7° 20′ N	150 15 33 W	True Long. a. m. sight	150 18 13.8 W
Value of Δλ	2 40.8 W	Diff. of Long. to noon	35 36 W
		Current in Long., sight to noon	2 26 E
True Long. at a. m. sight	150 18 13.8 W	True Long. at noon	150 51 23.8 W
Long. by D. R. at a. m. sight	150 30 24 W		
Current in Long. 20 hrs.	12 10,2 E		
Current in Long. 4 hrs.	2 26 E		

Course and distance made good.

		° ′ ″		° ′ ″
Lat. by obs. at noon		7 33 01 N	True Long. at noon	150 51 23.8 W
Lat. left		7 05 42 N	Long. left	148 19 00 W
Diff. of Lat.	=27′.316 N=	27 19 N	Diff. of Long. 152′.4 W =	2 32 23.8 W
L_0	=	7 19 21.5 N	p =	151′.17 W

C_N (made good) 280°.2. Distance 153.6 miles.

Set and drift of current.

	° ′ ″		° ′ ″
Lat. by obs. at noon	7 33 01 N	True Long. at noon	150 51 23.8 W
Lat. by D. R. at noon	7 25 12 N	Long. by D. R. at noon	151 06 00 W
Diff. of Lat. (current)	= 7 49 N	Diff. of Long. (current) 14′.6 E =	14 36.2 E
	= 7′.816 N	$p=14'.475$ E	
L_0	=7° 29′ 06″ N		

Set 62°. Drift 16.5 miles.

Course and Distance to Destination (GUAM). Mercator sailing.

	° ′ ″				° ′ ″				
L_2	18 25 48 N	M_2	807.8	λ_2	144 39 30 E	$D=3869'.1$	log 3.58761		
L_1	7 33 01 N	M_1	451.2	λ_1	150 51 23.8 W	$m=356'.6$	log 2.55218		
$l=$	5 52 47 N	m	356.6	D	64 29 06.2 W	C. N. 84° 44′ W	tan 11.03543	sec	11.03727
=	352′.8 N			=	3869′.1	$l=352.8$		log	2.54753
						$d=3844.2$ miles.		log	3.58480

Course $C_N=275°$ 16′ }
Distance 3844.2 miles } To Guam.

Had the tangent method been used to work the above time sight, we should have used the latitude by D. R. at the time of a. m. sight, 7° 08′ 42″ N., calling the resulting longitude computed longitude.

With the latitude by D. R., the declination, and the L. A. T. from the sight, the sun's true azimuth, regarded as less than 90°, would have been taken from the azimuth tables; and, with this azimuth and the latitude, the value of F found in Table I. The value of F and the direction of the line would have been written in the form for work thus: $F = a$ ✗ .

Having found the true latitude at the time of a. m. sight, the difference between it and the latitude by D. R. at that time would have given ΔL, and, as before, we should have had $\Delta\lambda = \Delta L \times F$.

Applying $\Delta\lambda$ to the computed longitude at the time of sight, we should have had the true longitude at the time of sight.

The procedure from this point would have been the same as in the chord method fully illustrated in Ex. 223.

Marcq Saint-Hilaire line.—Had the Marcq Saint-Hilaire method been used in working the a. m. sight, we should have used the D. R. position at the time of sight, Lat. 7° 08′ 42″ N., Long. 150° 30′ 24″ W., in finding the computed point of the position line, the arrangement of work in finding a position point of a line as shown on pages 654 and 655 being substituted for that on page 672. Then with the azimuth and the latitude of the computed point, we should have found the longitude factor F from Table I and the direction of the line.

The difference between the true latitude at time of sight and that of the computed point would have been ΔL and, as before, we should have had $\Delta\lambda = \Delta L \times F$. Having applied $\Delta\lambda$ to the longitude of the computed point we should have had the true longitude at the time of sight.

The procedure from this point on would have been the same as in the chord method illustrated in Ex. 223.

To Find the Interval from Time of A. M. Sight to Local Apparent Noon.

An important item in the "Day's Work" is the proper setting of the ship's clocks to show local apparent time at local apparent noon each day made necessary by the ship's change of longitude since the preceding noon. This is always done in advance so that the navigator may know what his watch time of noon will be and one method of procedure is explained in Article 253. A simpler method, particularly for use with the Saint-Hilaire line, is presented below, based on the following assumptions which, while not rigidly accurate, are sufficiently so for the purposes of this problem in ordinary cases:

(a) That the longitude corresponding to D. R. latitude, λ_c, (see Article 308) at time of a. m. sight is the true longitude of the ship at that instant. This assumption will be absolutely correct if the D. R. latitude is the same as the true latitude, or if the body observed at a. m. sight was exactly on the prime vertical. The more these conditions are departed from, the less accurate will be the assumption.

(b) That the ship's course and speed from time of a. m. sight to noon are known.

(c) That the current in longitude as determined from the true and D. R. longitudes at time of a. m. sight continues constant to noon.

If a ship does not change her longitude between the time of a. m. sight and local apparent noon, she will be carried by the rotation of the earth on its axis toward the plane of the sun's hour circle at the rate of 15 degrees or 1/24th of a complete rotation per hour and the time that elapses between the time of a. m. sight and of having the sun on the ship's meridian will be, of course, equal to the sun's negative hour angle at time of a. m. sight. This condition obtains when the ship is stationary or is making good a true North or South course. When, however, the ship does change her longitude due to her course and speed, or to current, or to both, she will move toward or away from the sun; that is, she will approach the plane of the sun's hour circle at a greater or less rate than the 15 degrees per hour due to the earth's rotation.

If, then, we divide the negative hour angle of the sun at time of a. m. sight by the rate at which the ship approaches the sun's hour circle the result will be the number of hours that will elapse between time of a. m. sight and noon.

Hence the following rules:

(1) From the longitude corresponding to D. R. latitude, λ_c, and the G. A. T. of a. m. sight find the L. A. T. and thence the negative hour angle of the sun at time of a. m. sight and express it in minutes of arc.

(2) Find the algebraic sum, expressed in minutes of arc, of the rates at which the ship approaches the plane of the sun's hour circle due to the rotation of the earth on its axis (always 900′ toward the East), the change of longitude per hour (East or West) due to the ship's course and speed, the change in longitude per hour (East or West) due to current, all expressed in minutes of arc.

(3) Divide the negative hour angle or distance to be covered obtained in (1) by the rate of approach obtained in (2) and the result will be the interval to noon expressed in hours and decimals of an hour.

Having the watch time at time of a. m. sight and the interval from that time to noon, it is evident that the sum of the two will give the watch time of local apparent noon. The watch should read, $0^h\ 00^m\ 00^s$ at noon, hence it must be set ahead or back the number of minutes that the computed watch time of local apparent noon differs from $0^h\ 00^m\ 00^s$, bearing in mind that this can be done only to the nearest minute as the second hand of the watch cannot be set.

Ex. 223(a).—The watch time of a sight of the sun taken at about 8 a. m. was $7^h\ 59^m\ 43^s$, the corresponding G. A. T. was

$0^h\ 38^m\ 50^s.2$, D. R. position at time of sight Lat. 38° 58′ 54″ N., Long. 63° 23′ 48″ W., and longitude corresponding to D. R. latitude, λ_c 63° 13′ 00″ W. Ship's true course from time of a. m. sight to noon is 81°, speed 15 knots. Find the interval from time of a. m. sight to noon.

Course	Dist. per hr.	p	D
81°	15	14.82	19′.07 E per hour

(The middle latitude between the D. R. position at time of a. m. sight and the noon D. R. position is used for the above transformation of departure into difference of longitude and it may be mentally estimated with sufficient accuracy for this purpose. 39° is used in this case.)

	h m s
G. A. T.	0 38 50.2
λ_o	4 12 52 W
L. A. T.	20 25 58.2
H. A.	− 3 34 01.8
Arc	= 3210′.45
	° ′ ″
$\lambda_{D.R.}$	63 23 48 W
λ_c	63 13 00 W
D	10 48 E

in 20 hours. Current in Long. for 1 hr. = 0′.54 E

$$\text{Interval to noon} = \frac{3210'.45}{900' + 19'.07 + 0'.54} = 3^h.4911 = 3^h\ 29^m\ 28^s$$

	h m s
W. T. at a. m. sight	7 59 43
Interval to noon	3 29 28
W. T. of L. A. noon	11 29 11

∴ If the watch is set ahead 31 minutes, it will be 11 seconds fast at L. A. noon.

Procedure for a Day's Work, Including Finding of Interval to Noon and Setting of Watch. Saint-Hilaire Line.

(1) Find the D. R. position at time of a. m. sight by working the traverse from previous noon or point of departure.

(2) Observe the sun's altitude when as near the prime vertical as conditions permit. With the data of this sight find the a. m. line of position by the method of Saint-Hilaire, picking out the true azimuth of the sun from the azimuth tables for the L. A. T., declination and latitude of the a. m. sight, and the longitude factor F from Table 47, Bowditch. Always indicate the direction of the line by the symbol ↗ or ↘ after the value of F.

(3) Observe the sun's bearing by standard compass and find the true azimuth from the azimuth tables for this instant.

(Some time may be saved if an assistant observes the sun's bearing p. s. c. at the time of the a. m. sight.) The difference between the true and compass bearings of the sun gives the compass error from which the deviation may be found by applying the known variation.

(4) Find the longitude corresponding to the D. R. latitude at time of a. m. sight, λ_e, and the current per hour in longitude.

(5) Find the interval from the time of a. m. sight to noon and adding this interval to the watch time of sight find how much the watch must be set ahead or back to make it read $0^h\ 00^m\ 00^s$ at noon. Set watch to nearest minute and note how much it will then be in error at L. A. N.

(6) Using interval to noon and the known course and speed of the ship find the D. R. position at L. A. N.

(7) Find the true noon latitude by meridian altitude or reduction to meridian sight. (Constants for both these sights should be prepared in advance.)

(8) To the longitude corresponding to D. R. latitude at time of a. m. sight add algebraically the run in longitude of the ship from a. m. sight to noon and the current in longitude from a. m. sight to noon, thus finding the longitude corresponding to noon D. R. latitude. Then find the true noon longitude by adding algebraically to the longitude corresponding to noon D. R. latitude the product of the difference between true noon latitude and D. R. noon latitude multiplied by the longitude factor of the a. m. line, paying close attention to signs.

(9) Find the course and distance made good from the noon position of the previous day or point of departure to the true noon position arrived at.

(10) Find the set and drift of current which is the course and distance from noon D. R. position to true noon position. This distance divided by the elapsed time from noon to noon or from time of taking departure to noon gives the drift of the current in miles per hour which is the usual form of expressing drift.

(11) Course and distance to destination may be found by any suitable method.

Many of these operations may be performed graphically on a chart or plotting sheet, but the principles involved are in no way changed by using the graphic method of solution.

Ex. 223(b).—The U. S. S. *Florida* making passage from Hampton Roads to Brest is, at noon, April 25, 1918, in Lat. 38° 12′ N., Long. 69° 43′ W. Thence she steams on course 81° true, speed 15 knots, until L. A. noon the next day, April 26, 1918.

It is cloudy until about 8.30 a. m. by watch on April 26 when the navigator observes the sun as follows:

Watch $8^h\ 30^m\ 19^s$, C—W $4^h\ 54^m\ 13^s$, chro. fast $18^m\ 13^s$, sextant altitude of sun's lower limb 42° 10′ 30″, I. C. + 1′ 00″, height of eye 40 feet. Shortly afterward, when the watch read $8^h\ 32^m\ 49^s$, the bearing of the sun's center by standard compass was 125°, var. 16° 40′ W. At L. A. N., April 26, the meridian altitude of the sun's lower limb was observed to be 63° 58′ 15″, I. C. and height of eye as before.

(1) Find the D. R. position at time of a. m. sight.

(2) Work a line of position by the method of Saint-Hilaire, finding altitude difference, bearing of line and longitude factor.

(3) Find the deviation of the standard compass.

(4) Find the longitude corresponding to the D. R. latitude at time of a. m. sight and the current per hour in longitude.

(5) Find the interval from time of a. m. sight to noon, the amount to set the watch so that it will be correct to the nearest minute at noon and how much it will then be in error.

(6) Find the D. R. position at L. A. N.

(7) Find the true noon latitude.

(8) Find the true noon longitude.

(9) Find course and distance made good since preceding noon.

(10) Find set and drift of current.

A DAY'S WORK, SOLUTION OF EXAMPLE 223 (b).
Saint-Hilaire Line.

Dead Reckoning from Noon till Time of a. m. Sight.

Course, True.	Distance, 20½ hrs.	l	p	D
81°	307.5 miles	48.1	303.7	388′.6 E

	° ′ ″		° ′ ″
Lat. left	38 12 00 N	Long. left	69 43 00 W
Diff. of Lat.	48 06 N	Diff. of Long.	6 28 36 E
At a. m. sight, Lat. by D. R.	39 00 06 N	At a. m. sight, Long. by D. R.	63 14 24 W

L_0=38°.6 N

A. M. Line of Position.

Times.		Altitude.		Altitude Corrections.		Declination.	H. D.	Eq. of T.	H. D.
	h m s		° ′ ″		′ ″	° ′		+ to M. T. m s	
W.	8 30 19	⊙	42 10 30	Tab. 46	+ 8 47	N 13 20.3	N 0′.8		+ .4
C—W	4 54 55	Corr.	+ 9 47	I. C.	+ 1 00	N .9	t 1ʰ.1	2 9.6	t 1.1
								+ .4	
C.	1 25 14	⊖	42 20 17	Corr.	+ 9 47	N 13 21.2	N ′.88		+ .44
C. C.	− 18 13							2 10.	

	h m s	
G. M. T.	1 07 01	Apr. 26
Eq. of T.	+ 2 10	
G. A. T.	1 09 11	
Long.	4 12 57.6 W	
L. A. T.	20 56 13.4	

From Tab. 47.
$F=.45\times$

L. A. T.	20ʰ 56ᵐ 13ˢ.4	L. hav	9.18276		
	° ′ ″				
d	13 21 12 N	L. cos	9.98809		
L	39 00 06 N	L. cos	9.89049		
		L. hav	9.06134	N. hav	.11517
$L \sim d$	25 38 54			N. hav	.04927
z	47 50 45			N. hav	.16444
Computed h_0	42 09 15				
Observed h	42 20 17				
a	11 02=11 miles toward sun.				

By Azimuth Tables.

Bearing of sun	109° 25′
Direction of line	19° 25′

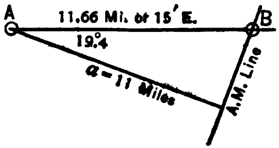
Deviation of Standard Compass.

	h m s
W. T. of a. m. sight	8 30 19
W. T. of sun's compass bearing	8 32 49
Difference in time	{ 2 30 { $2^m.5$

	° ′
Sun's true bearing at time of a. m. sight	109 25
Change in azimuth for $2^m.5$ from tables	+ 32
True bearing at time of compass bearing	109 57
Observed compass bearing of sun	125 00
Compass error	− 15 03
Variation from chart	− 16 40
Deviation of compass on head. 81° true	+ 1 37

Current in Longitude and Longitude Corresponding to Latitude at Time of A. M. Sight.

D. R. position at time of a. m. sight. Point A.



Point B is the intersection of the a. m. line of position with the D. R. latitude at time of a. m. sight.

From traverse tables with course 109° 25′ − 90° = 19°.4 find distance AB = 11.66 miles.

With middle latitude 39° transform AB = 11.66 miles into difference of longitude = 15′ E.

Current in longitude for 20.5 hours = 15′ E.
Current in longitude for 1 hour = ′.732 E.

	° ′ ″
At time of a. m. sight, D. R. Long.	63 14 24 W
Current in Long. for 20.5 hours	15 00 E
Long. corresponding to a. m., D. R. Lat. (λ_c)	62 59 24 W

Setting of Watch to be Correct at L. A. Noon.

	h m s
G. A. T. at a. m. sight	1 09 11
λc	4 11 57.6
L. A. T. for position *B*	20 57 13.4
H. A.	− 3 02 46.6
Arc	45° 41′ 39″
	=2741′.65

Run in Long. for one hour.

Course.	Dist.	*l*	*p*	*D*
81°	15	2.3	14.82	19′.098 E (L_0=39°.1)

Interval to noon $= \dfrac{2741'.65}{900' + 19'.098 + '.732} = 2^h.9806 = 2^h\ 58^m\ 50^s$

	h	m	s
W. T. at time of a. m. sight	8	30	19
Interval a. m. sight to noon	2	58	50
W. T. at L. A. noon	11	29	09

∴ Set the watch ahead 31 minutes and it will then be 9 seconds fast at L. A. noon.

D. R. Position at L. A. Noon.

Run from time of a. m. sight to noon.

$2^h.9806 \times 15 = 44.7$ miles.

Course.	Dist.	*l*	*p*	*D*
81°	44.7	7.	44.15	56′.9 E

	° ′ ″		° ′ ″
A. M. D. R. Lat.	39 00 06 N	Long.	63 14 24 W
Diff. of Lat.	7 00 N	*D*	56 54 E
Noon D. R. Lat.	39 07 06 N	Long.	62 17 30 W

L_0=39°.1

Meridian Altitude.

	° ′ ″
$\odot$	63 58 15
Corr.	+ 10 20
$\ominus$	64 08 35
z	25 51 25 N
d	13 23 42 N
True noon Lat.	39 15 07 N

Altitude Corrections.		Declination.	H. D.
	′ ″	° ′	
Tab. 46	+ 9 20	N 13 23.6	N 0′.8
I. C.	+ 1 00	N .1	λ .15
Corr.	+10 20	N 13 23.7	N .12

True Noon Longitude.

Current in Long. a. m. sight to noon 2.9806 × .732 = 2′.2.

	°	′	″	
Noon D. R. Lat.	39	07	06	N
Noon true Lat.	39	15	07	N
$l=$		8	01	N
$F = .45 \times D$ =		3	36	E

	°	′	″	
Long. corresponding to D. R. Lat. at a. m. sight	62	59	24	W
Run in Long. from a. m. sight to L. A. N.		56	54	E
Current in Long. from a. m. sight to L. A. N.		2	12	E
Long. corresponding to D. R. Lat. at noon	62	00	18	W
D		3	36	E
True noon longitude	61	56	42	W

Course and Distance Made Good.

		°	′	″			°	′	″	
True position at noon April 25	Lat.	38	12	00	N	Long.	69	48	00	W
True position at noon April 26	Lat.	39	15	07	N	Long.	61	56	42	W
Course 80°.2	l	1	03	07	N	D	7	46	18	E
	=		63′.1		N	=	466′.3			E
Distance 369.2 miles	$L_0=$		38°.72			$p=$	363.8 miles			E

Set and Drift of Current.

		°	′	″			°	′	″	
True position at noon April 26	Lat.	39	15	07	N	Long.	61	56	42	W
D. R. position at noon April 26	Lat.	39	07	06	N	Long.	62	17	30	W
Set 63⅝°	l		8	01	N	D		20	48	E
Drift 18 miles	= 8 miles				N	=	20′.8			E
per hour ¾ mile	$L_0=$ 39°.2					$p=$	16.11 miles			E

CHAPTER XXII.

TIDAL WAVES, TIDAL CURRENTS, AND FINDING TIME OF HIGH WATER.

311. Closely related to the subject of the moon's meridian transit is the subject of the tides, which, though a very broad one for a work of this scope, may be presented, even in its elementary form, with advantage to the student; by applying general rules, he may approximate to the time of high water for those places not tabulated in the tide tables.

312. Definitions.—The phenomena of tides, as usually observed in tide-water regions, are a periodic rise and fall and a recurrent flood and ebb of the water; the word tide or tide-wave, properly refers to the vertical movement only, the horizontal movement being characterized as tidal current. The maximum height to which the tide rises is called high water, the lowest level to which it falls low water; that moment at either high or low water when no vertical movement takes place is called stand, and the difference in height between low and high water is called range.

Flood is the inflow of tide water from the general direction of the ocean, ebb its recession towards the sea; the set of a current is the direction towards which it is flowing, drift the distance through which it flows in a given time, rate its velocity per hour, and slack the term applied to the period between tidal currents when there is no horizontal motion.

313. Causes of the tides.—The tides are caused by the difference of the attractions exerted by the moon, and, in a less degree by the sun, upon the earth and waters of the earth.

By the law of gravitation, the attractive forces of the sun and moon decrease as the square of the distance increases, and hence exert a greater force on the nearer surface and a less force on the farther surface, than on intermediate parts; the resultant effect being a tendency to recede from the center in the parts not only just under the attracting body, but in the parts diametrically opposite.

For purposes of illustration.—The earth may be considered as surrounded by a uniform envelope of frictionless water, and, as illustrated in Fig. 147, let M be the moon whose mean attractive force on the solid part of the earth may be assumed as acting at the center E; therefore, the moon exerts a greater force on the waters at A, just beneath it, than on the earth at E; a greater force on the earth at E, than on the water at A' diametrically opposite. The water at any other position, as at L, though attracted by the moon less strongly than that at A, will have its gravity toward the center diminished, and a tendency to go toward A, due to that component of the force along LM, which acts in the direction of the tangent at L; while the water at L' will have a tendency to go toward A'. The waters of the entire envelope, being free to yield to a similar tendency, would, if the water were either very deep or devoid of inertia, assume a spheroidal shape with the longer axis toward the moon, and thus two tidal waves, called lunar

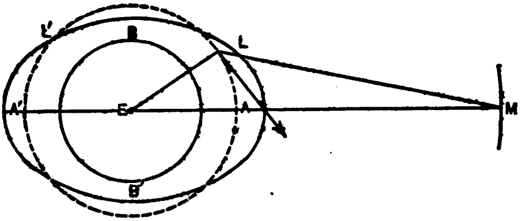


FIG. 147.

waves, will be formed at the points A and A'. These will be points of high water, and midway between these elevations will be depressions of the water level, called low water, as at B and B'.

Number of tides per day.—Ordinarily there are two principal alternations of high and low water at a given place in a lunar day; but, owing to the moon's declination, the two high waters or the two low waters generally differ in height from each other. That is, there is a daily or diurnal inequality in the heights of the high and low waters.

This daily inequality is due to the inclination of the plane of the moon's orbit to that of the equator, and to the rotation of the earth on its axis.

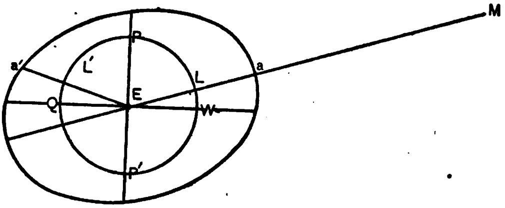


FIG. 148.

In Fig. 148, let PP' be the earth's axis, P the North pole, QW the plane of the equator; let M be the moon whose declination is North, and equals the angle MEW; let L be a place on the earth's surface having the moon in its zenith. The tidal wave at L is the superior wave, its height may be represented by La, but at a place L' in the same latitude, and distant 180° in longitude, the height of the tide will be represented by $L'a'$; owing to the revolution of the earth on its axis, these two places will change situations with respect to the moon in about 12 hours, and the height of the tide at L will then be equal to what it was at L' 12 hours before. This will be known as the inferior wave at L.

If the moon remained stationary in her orbit, high waters would occur at intervals of 12 sidereal hours; but owing to its advance to the eastward in its orbit, thereby arriving at the same branch of the same meridian later each day by a mean amount of 50 minutes, the inferior wave, or tide of the lower culmination, will follow the superior wave, or tide of the upper culmination, by the average time of 12 hours and 25 minutes.

314. Effect of the sun.—The attraction of the sun causes in the same way two solar waves at diametrically opposite points, which reinforce or diminish the lunar waves according to the relative positions of the sun and the moon in their respective orbits.

Owing to the sun's great distance, the inequality of its attractions on the earth and waters of the earth is small, and the mean force of the moon in causing tides is about 2½ times as great as the sun.

When the sun and moon are in conjunction or opposition, they act together in producing the tidal wave, and the maximum high and minimum low water of the month called spring tides result, with maximum tidal range; unusually high tides would result should the sun and moon happen to be respectively, at perihelion and perigee at the time of new or full moon, and especially so if moon and sun are in the celestial equator.

At the first and third quarters of the moon, the sun and moon act at right angles to each other, and the effect of the solar wave is to diminish the height of the lunar wave; the lowest high waters and the highest low waters of the month, called neap tides, result with a minimum tidal range.

Priming and lagging.—When the moon is in the first and third quarters, the solar wave is to the westward of the lunar wave, and there is an acceleration in the time of high water called priming of the tides. When the moon is in the second and fourth quarters, the solar wave is to the eastward of the lunar wave, and there is a retardation in the time of high water, called lagging of the tides.

315. Luni-tidal interval.—The theoretical assumptions in the preceding articles are, of course, not justified by facts; the earth is not entirely covered with water, and the depth of the oceans is not sufficient for permitting its surface to assume an equilibrium form in so short a time as a tidal period. As a matter of fact, the tides in the various oceans are, in the main, independent of one another. Moreover, the observed tide has generally been propagated some distance in comparatively shallow water. For these reasons high tide is not coincident in time with the time of the moon's meridian transit, and the interval of time between the moon's meridian transit and the following high water is not the same for each day of the month. These intervals are known as luni-tidal intervals.

The mean of these intervals on days of new and full moon is called the vulgar or common establishment of a port. It is frequently spoken of as the time of high water on full and change days, being found in the tidal data of charts as H. W. F. & C.

The mean of all the luni-tidal high-water intervals observed throughout at least a lunar month, is called the corrected establishment of the port, and, when known, should be used, in preference to the common establishment, in finding the time of high water. It will be found tabulated for many ports in Appendix IV, Bowditch.

316. Age of tide.—The greatest effect of the sun and moon in producing the tidal wave occurs at new and full moon, and the interval of time from the instant of new or full moon to the highest subsequent tide at any place is known as the retard or age of the tide. This varies with the locality, being one day on the Atlantic Coast of North America, and as much as 2½ days on the Coast of England.

General laws.—Though the subject of tidal waves is complicated by the fact that the sun, moon, and earth do not occupy the same relative position more than once in a period of about 18 years, and by the further fact that every tide is largely affected by local conditions, such as depth of water, configuration of the coast, and even by interference of different parts of the same wave; still the following elementary laws may be laid down as general for the moon's effect.

(1) Two high tides will occur daily at a given place.

(2) When the declination of the moon is 0°, the two daily tides at a given place will be nearly equal; and the equatorial tides will, as a rule, be much larger than those in high latitudes.

(3) When the moon's declination is not 0°, the two tides of the day at all places will be unequal, the maximum daily inequality usually occurring soon after extreme declination is reached. If the higher high water follows the upper transit of the moon by a certain interval when extreme north declination is reached, it will follow the lower transit by this same interval when the declination is south. A similar remark is true concerning the low waters.

(4) The time of high water occurs after the moon's upper transit a number of hours equal to the establishment of the port. The time of the following low water 6 hours and 13 minutes after high water, and the time of the next high water at a mean interval of 12 hours and 25 minutes after the first high water.

Tidal currents.—A distinction must be drawn between tidal waves and tidal currents, the former referring to the vertical oscillations of the water, the latter to the horizontal inflow and outflow caused by the interferences offered the tidal waves by local formations and the frictional resistances of the bottom and sides of shoal, narrow and contracting channels, etc.

Whilst it is of importance to know the times of high water when about to enter or leave a harbor, it is of more practical importance in the navigation of a vessel to be able to anticipate a probable set and drift of a current and to allow for the same.

It must not be forgotten that the changes of tidal currents seldom correspond with high and low waters, perhaps never except on open coasts or in wide and shallow basins, certainly not in large bodies of water having a relatively contracted entrance to the sea, as in the cases of Delaware and Chesapeake Bays.

Furthermore, a current in certain localities may flow in the offing one to three hours after it has turned along the shore; such peculiarities may often be found described in the sailing directions of those regions and should be studied by the navigator.

In the tide tables issued by the U. S. C. & G. Survey will be found current diagrams for Georges Bank, Boston Harbor, Nantucket and Vineyard Sound, New York entrance and East River, Delaware and Chesapeake Bays, and current tables, restricted, however, to points on the Atlantic and Pacific Coasts of the United States. In the diagrams, the set and rate of the current are given for three hours before and after high water. In certain localities tables are provided for the purpose of predicting the current from the times of the high and low waters.

Full or daily predictions of the times of slack water are given for five stations on the Atlantic Coast and four on the Pacific Coast. Brief tables of differences are provided whereby the times of slack water at a considerable number of subordinate stations may be obtained from the daily current predictions.

An examination of these, when a vessel may be in the localities therein considered, will often point out the most favorable conditions under which the current should be encountered.

When lying in a port of which the tidal information is incomplete, and under circumstances that will admit of observations, a navigator should make every effort to gather all possible information about the local currents. For the method of making tidal observations and a description of the instruments used, etc., the student is referred to any standard work on Marine Surveying.

317. Times of high and low water.—The quickest, most accurate, and hence most satisfactory method of finding the times of high and low water is by taking this information from tide tables which are furnished navigating officers of the navy. General tide tables published by various foreign governments may be purchased in almost any seaport; and the U. S. C. & G. Survey publishes annually in advance tables containing, in addition to the current matter referred to in Art. 316, predictions as to the times and heights of every high and low water in the following year at certain principal ports of the world regarded as standard ports for tidal purposes. For these ports, the times of tides are arranged in the order of the occurrence of tides in one line, the corresponding heights above the plane of reference (which for the Coast Survey Charts is that of mean low water) in a second line, a comparison of the heights indicating which are high and which are low waters. These predictions are extended to over 1000 other places by applying to the data of the proper standard port, the tidal differences and ratios corresponding for the places.

High water by computation.—When tide tables are not available, the times of high and low water may be found by applying the principles of rule 4 (Art. 316).

(1) Find the local mean time of the moon's upper transit at the place.

(2) Add to this the high water or low water luni-tidal interval from Appendix IV, Bowditch, according as the time of high water or low water is desired. The result will be the required time.

The H. W. luni-tidal interval, as tabulated in Bowditch, is the corrected establishment of the port; it may be taken from the chart of the locality; or the common establishment found on the chart, as H. W. F. & C., may be used without appreciable error.

The times given in the Nautical Almanac are for the astronomical date.

When the establishment is added to the local time of local transit, the result will be in astronomical time; the corresponding civil time may be a day later, so if the time of high water is desired for a given civil date, and it is found that the sum of the establishment plus the local time of local transit will be greater than 12 hours, take out the time of transit for the preceding date, since in this case the astronomical date is one day less than the civil date, and, when the time is converted into civil time, the civil date of the tide in question will result.

Ex. 224.—Find the times of high and low waters occurring on January 22, 1918, a. m., at Portland, Me. Latitude 43° 39′ 28″ N., longitude 70° 15′ 18″ W.

In this example, the sum of the time of moon's transit and the luni-tidal interval is greater than 12 hours; therefore, take out the time of transit for January 21.

	h m		
G. M. T. of Greenwich transit Jan 21,	7 42	H. D.	$2^{m}.33$
Corr. for Long. $4^{h}.68$ W	+ 11	Long. W	$4^{h}.68$
		Corr.	$+\ 10^{m}.90$
		Or (Tab. 11, Bowditch)	$=+11^{m}$
L. M. T. of local transit Jan. 21,	7 53		
H. W. luni-tidal Int. Appx. IV, Bowditch	11 06		
L. M. T. of high water Jan. 21,	18 59		
or Jan. 22,	6 59 a. m.		
L. M. T. of local transit Jan. 21,	7 53		
L. W. luni-tidal Int. Appx. IV, Bowditch	4 51		
L. M. T. of low water Jan. 21,	12 44		
or Jan. 22,	0 44 a. m.		

Ex. 225.—Find the time of morning high water and afternoon low water on April 14, 1918, at Port Adelaide. Latitude 34° 50′ 25″ S., longitude 138° 26′ 58″ E.

		d h m		
G. M. T. of Gr. upper transit	Apr.	13 2 09	Mean H. D.	$2^{m}.5$
G. M. T. of Gr. upper transit	Apr.	14 3 09	Long. E	$9^{h}.23$
	2	$\boxed{27\ \ 5\ 18}$		
G. M. T. of Gr. lower transit	Apr.	13 14 39	Corr.	$-23^{m}.07$
Corr. for Long. $9^{h}\ 23^{m}$ E	—	23	Or (Tab. 11, Bowditch)	$=-23^{m}$
L. M. T. of local lower transit	Apr.	13 14 16		
H. W. Lun. Int. Appx. IV, Bowditch		4 04		
L. M. T. of H. W.	Apr.	13 18 20		
	or Apr.	14 6 20 a. m.		
L. M. T. of local lower transit	Apr.	13 14 16		
L. W. Lun. Int. Appx. IV, Bowditch		10 22		
L. M. T. of L. W.	Apr.	14 00 38		
	Apr.	14 12 38 p. m.		

Ex 226.—Find the time of high water occurring next afternoon on April 12, 1918, at Hong Kong. Latitude 22° 16′ 23″ N., longitude 114° 10′ 02″ E.

	h m		
G. M. T. of Greenwich transit, April 12,	1 10	H. D.	$2^{m}.41$
Corr. for Long. $7^{h}.61$ East,	− 18	Long. E	$7^{h}.61$
		Corr.	$-18^{m}.34$
L. M. T. of local transit, April 12,	0 52	Or (Tab. 11, Bowditch)	$= -18^{m}$
H. W. luni-tidal Int. Appx. IV, Bowditch	9 20		
L. M. T. of high water, April 12,	10 12		
or April 12,	10 12 p. m.		

CHAPTER XXIII.

IDENTIFICATION OF HEAVENLY BODIES.

318. A navigator is fortunately not dependent on observations of the sun either in locating the position of his ship or in determining the error of his compass. Planets and fixed stars, when visible and favorably situated, are available for that purpose. Owing to the large number of stars of the first two magnitudes of differing right ascensions, it is probable that several may be found favorably situated for cross lines at all hours during twilight, or when the horizon may be made sufficiently distinct by moonlight. In these days of fast ocean steamships, stellar observations are essential and an observer with some practice and a clear horizon should get good results from sights for position; such sights should be avoided, however, when the horizon is uncertain. When working for compass error, it is only necessary to see and to know the star, and to obtain its compass bearing, it being immaterial whether the horizon is clear or clouded. The method of observation as well as the methods of working stellar sights have been fully explained.

319. Distinction between planets and fixed stars.—The planets change their positions in the heavens not only with reference to each other but to the fixed stars; they have a perceptible disc and shine with a steady light; fixed stars do not change their positions relative to other fixed stars, and they appear in the most powerful glasses simply as luminous points shining with a twinkling light.

320. Distinction between planets.—The only planets that need be considered by the navigator are Jupiter, Venus, Mars, and Saturn. Both Jupiter and Venus are larger and brighter than Sirius; when only one is visible, it may easily be taken for the other, but a comparison of the estimated right ascension of the visible planet with the tabulated right ascensions of Jupiter and Venus will decide which it is. When both are visible, (1) the one to the eastward will be the one of greater right ascension as indicated by the tabulated right ascensions of the Nautical Almanac; (2) the motion of Venus in right ascension is more rapid than that of Jupiter and in consequence its change of position among the fixed stars is more noticeable; (3) as Venus is an inferior planet with a maximum elongation of about 47°, it is easily seen that as morning or evening star, it cannot be visible before sunrise or after sunset more than three hours and eight minutes, whereas Jupiter may be visible at any hour of night depending on its elongation which, as with all superior planets, varies from 0° to 180°.

Mars may be recognized by looking up its right ascension and declination; it is larger than a fixed star, and shines with a reddish color, which has caused it to be known as the "Ruddy Planet."

Saturn, owing to its great distance, changes its relative position among the stars very slowly, and by the naked eye may be taken for a fixed star. Estimating its right ascension, or the use of good night glasses, will distinguish it from fixed stars. The three planets first mentioned are more frequently used in practical navigation.

321. Grouping and classification of stars.—From remote ages stars have been grouped in constellations, those of each constellation, as a rule, being arranged in order of brightness and distinguished by having Greek or Roman letters prefixed to the name of the constellation, or by numerals when both alphabets have been exhausted, the brightest star of the group being represented by the letter *α*. Specific names are usually given to the most conspicuous stars.

Stars are found in nautical almanacs, arranged according to their right ascensions and classified by magnitudes or brightness, the lowest magnitude assigned to stars just visible to the naked eye being the sixth. Assigning to sixth magnitude stars an average brightness of unity, and regarding the stars of one magnitude about 2½ times as bright as those of the lower magnitude, the average brightness of first magnitude stars should be 100. Of course, there are marked deviations from this rule, the most notable exception being Sirius, which is perhaps 500 times as bright as a star of the sixth magnitude.

322. Navigational stars. — The twenty brightest stars: *α* Canis Majoris (Sirius), *α* Argus (Canopus), *α* Aurigæ (Capella), *α* Bootis (Arcturus), α^2 Centauri, *α* Lyræ (Vega), *β* Orionis (Rigel), *α* Eridani (Achernar), *α* Canis Minoris (Procyon), *β* Centauri, *α* Aquilæ (Altair), *α* Crucis, *α* Orionis (Betelgeux), *α* Tauri (Aldebaran), *α* Virginis (Spica), *α* Scorpii (Antares), *β* Geminorum (Pollux), *α* Piscis Australis (Fomalhaut), *α* Leonis (Regulus), *α* Cygni (Deneb), and perhaps a dozen more may be classed as navigational stars, and every navigator should be able to recognize these and to select the ones most favorably situated for his purposes. To do so, it is useless to make a study of the constellations based on the fanciful grouping of stars by the ancients; it is only necessary to know (1) one conspicuous constellation in the northern heavens about which to group stars of North declination; (2) one in the region of the equinoctial leading to a knowledge of others in the same region, to some one of which, stars of either North or South declination up to certain limits may be referred; (3) one in the southern hemisphere that may assist in locating the stars adjacent to the South celestial pole.

323. Constellations of reference.—The constellations recommended for obvious reasons in carrying out the above plan are (1) Ursa Major or "The Dipper"; (2) Orion; (3) the Southern Cross. The student having made himself familiar with the visible stars of these constellations, and having learned certain bright stars near them, should trace out others in one of three ways:

(1) by bearings and angular distances;

(2) by prolonging a line (straight or curved) passing through two known stars till at a certain approximate distance it may pass through a required star;

(3) by the geometrical figures, which in many cases, three or more bright stars form with each other.

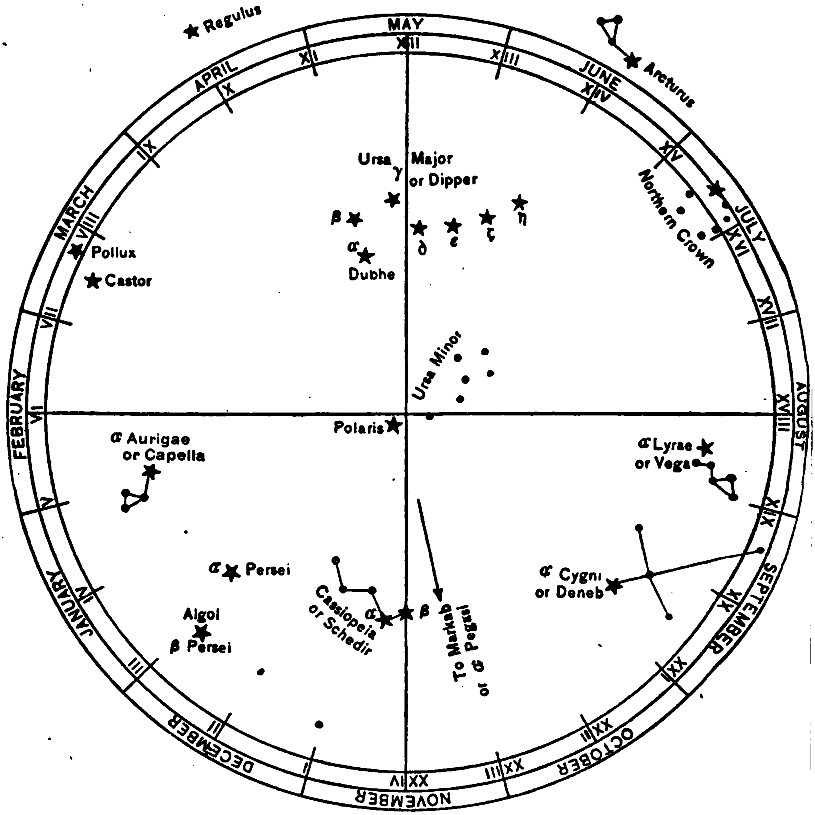
The first method is unsatisfactory as the bearing of one star from another is a great circle bearing and should be noted when the known star is at its upper culmination and as near the zenith as possible—conditions seldom governing. An inspection of star maps on the Mercator projection would only confuse the student as the bearings there shown are not great circle bearings.

The second and third methods in connection with Plates VI to IX will perhaps be found the best and most expeditious methods for indentifying stars when the surrounding heavens are visible.

324. Description of Plate VI.—The plate shows the principal stars in the northern hemisphere whose declination exceeds 30°. The Roman numerals on the margin show the meridians of right ascension at intervals of one hour. As the right ascension of the meridian is the L. S. T., if the observer faces the North and holds the plate so that the numeral which represents the L. S. T. at the time of observation is uppermost, the stars in the upper part of the plate will be shown in the same relative positions as they appear in the heavens.

If the observer faces the North and holds the plate so that

PLATE VI.



THE PRINCIPAL STARS AROUND THE NORTH CELESTIAL POLE OF A DECLINATION GREATER THAN 30° N.

the name of a month found in the margin is uppermost, the plate will show the visible heavens around the pole as they appear about 8.30 p. m. in that particular month; the number of stars in the lower part of plate cut off by the horizon depending on the latitude of the observer.

Ursa Major, commonly called the "Dipper" from its shape, one of the brightest and most conspicuous of the northern constellations, consists of seven principal stars. Beginning with the edge of the bowl they are (α) Dubhe, (β) Merak, (γ) Megrez, (δ) Phecda, (ϵ) Alioth, (ζ) Mizar, (η) Benetnasch. The first two (α and β) are the brightest and, pointing to the pole star (Polaris), are known as the pointers. Polaris is the principal star of Ursa Minor which appears also in the shape of a smaller dipper, Polaris being in the extremity of the handle.

Cassiopeia.—About the same distance from Polaris as the "Dipper," but on the opposite side, is Cassiopeia's chair, whose five principal stars appear in the form of the letter M or W, according to the position of the constellation in its diurnal path.

β Cassiopeia.—A line from γ Ursæ Majoris through Polaris, produced about 30°, leads to β Cassiopeia.

α Cassiopeia.—A line from δ Ursæ Majoris through Polaris leads to α Cassiopeia called Schedir, the farthest one of the chair from the pole star.

Square of Pegasus.—A line from the pointers through Polaris, produced beyond Cassiopeia, leads first to β Pegasi (Scheat), then to α Pegasi (Markab), two stars in a noticeable figure resembling a square; the other two being γ Pegasi (Algenib) and α Andromedæ (Alpheratz), the latter nearer the pole (Plates VI and VIII).

α Lyræ or Vega.—A line from γ passing between δ and ϵ Ursæ Majoris leads to Vega, a very bright star of a decided blue tint, which is attended by five other stars, making, with Vega, two triangles.

α Cygni or Deneb.—A line from γ through δ Ursæ Majoris extended passes between Vega and Deneb. Also a line from Algenib through Scheat (Plate VIII), continued to nearly twice its distance, leads to Deneb.

α Aquilæ or Altair.—A line from Polaris midway between Vega and Deneb leads to Altair, which is further distinguished by having an attendant star each side of it and by proximity to the Dolphin which shows five stars, four of which form a small diamond (Plate VIII). Altair, Vega, and Deneb form a triangle nearly right angled at Vega (Plate VIII).

α Aurigæ or Capella.—A line from γ Ursæ Majoris passing between the pointers (α and β Ursæ Majoris) leads to Capella, a very bright star of a yellow tinge, attended by a small triangle of three stars to the southward of it called "the kids."

A line from the middle star of Orion's belt through Orion's head and β Tauri leads to Capella (Plate VII).

Capella, Algol, and Aldebaran form an equilateral triangle (Plate VII).

α Bootis or Arcturus.—A line from Dubhe passing between γ and δ Ursæ Majoris leads to Arcturus, and the handle of the dipper curves toward it.

Arcturus is a very bright star with a reddish tint, is attended by a small triangle of three stars, is as far from the pointers on one side as Capella is on the opposite side; it forms bold triangles with Spica and Regulus, also with Spica and Antares, both triangles nearly right angled at Spica (Plate VIII).

α and β Geminorum—Castor and Pollux.—A line from δ Ursæ Majoris passing between the pointers leads to Castor and Pollux, which are about as much one side of the Dipper as the Northern Crown is the other side. A line from the middle star of Orion's belt (Plate VII) through Betelgeux leads to Castor, which shines with a greenish light. Betelgeux, Procyon, and Pollux (Plate VII) form a triangle, right angled at Procyon.

α Leonis or Regulus.—A line from δ Ursæ Majoris passing between β and γ Ursæ Majoris leads to Regulus. This is a bright white star and, being in the handle of the so-called sickle or reaping hook, is a very prominent one. It forms a triangle with Spica and Arcturus, right angled at Spica (Plate VIII).

325. Description of Plates VII and VIII.—The principal stars of a declination less than 45°, North and South, are shown in these plates, those whose right ascensions are between 0 and XII hours in Plate VII, those of a right ascension greater than XII hours in Plate VIII.

If about 8.30 p. m. in a particular month, these plates be so held overhead that the feathered arrow points North whilst the Roman numerals increase to the eastward, then the brightest stars of the heavens near the meridian will be those stars in the plates whose right ascensions are indicated by figures below the name of the given month.

326. Orion and the stars it leads to.—Orion, the most beautiful constellation of the heavens, consists of a quadrilateral formed of three bright stars and one of lesser magnitude, the figure being longer in the North and South direction. The NE. star is α Orionis (Betelgeux); the NW., γ Orionis (Bellatrix); and the SW., β Orionis (Rigel).

Within the quadrilateral are three small stars, nearly equidistant, and in a line nearly NW. and SE., forming what is known as Orion's belt. Nearly midway between the two northern stars and a little further to the northward are three small stars forming a triangle in the imaginary head of Orion.

α Canis Majoris or Sirius.—This, the brightest star of the heavens, shines with a scintillating white light. The three

PLATE VII.

THE PRINCIPAL STARS OF DECLINATION LESS THAN 45°, N. AND S., AND OF R. A. 0—XII HOURS.

A map showing the right ascensions of stars that will be near the local meridian about 8.30 p. m. in the months of

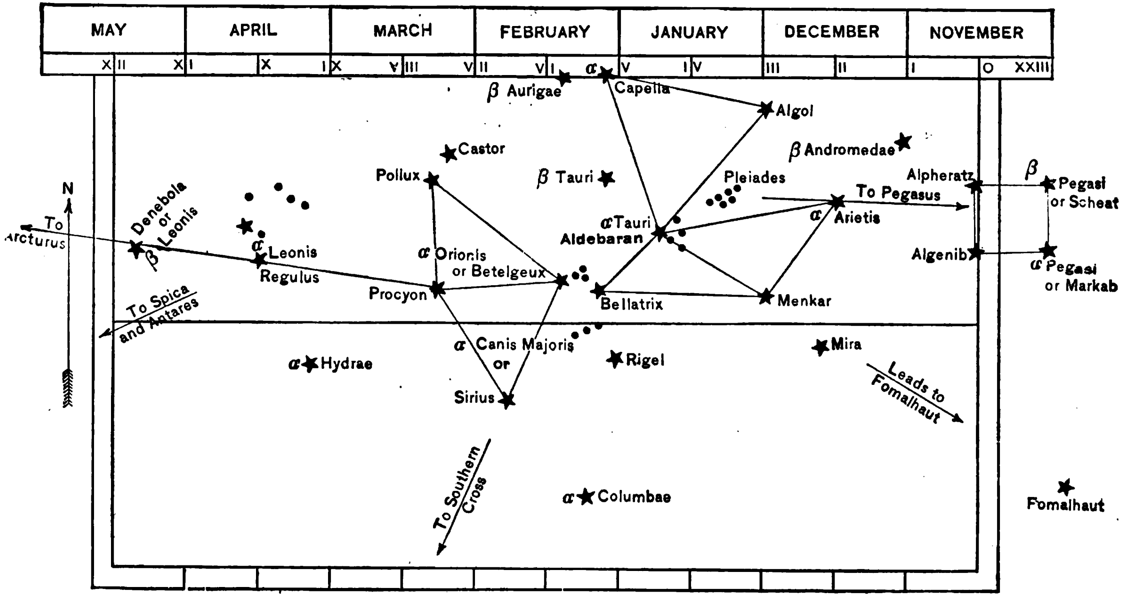
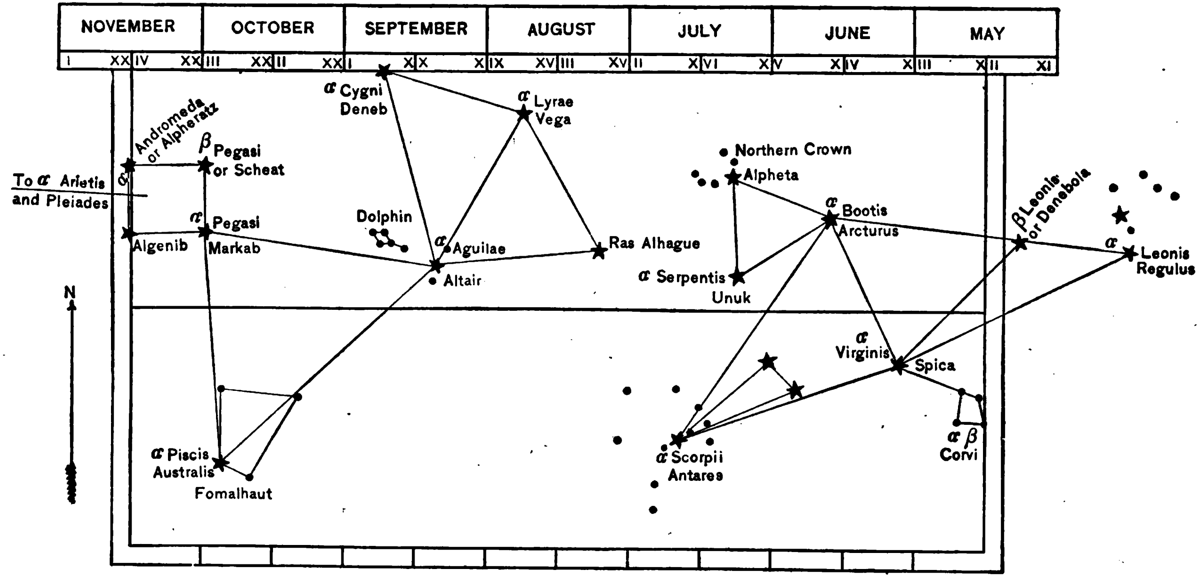


PLATE VIII.

THE PRINCIPAL STARS OF DECLINATION LESS THAN 45°, N. AND S., AND OF R. A. XII—XXIV HOURS.

A map showing the right ascensions of stars that will be near the local meridian about 8.30 p. m. in the months of



stars of Orion's belt point southeastward to Sirius, which forms an equilateral triangle with Betelgeux and Procyon. Sirius, Rigel, and the triangle in Orion's head form a triangle right angled at Rigel.

α Canis Minoris or Procyon.—A line from Bellatrix through Betelgeux, curving to the southward and eastward, leads to Procyon, a star of a yellowish tint. A line from Arcturus through Denebola and Regulus leads to Procyon.

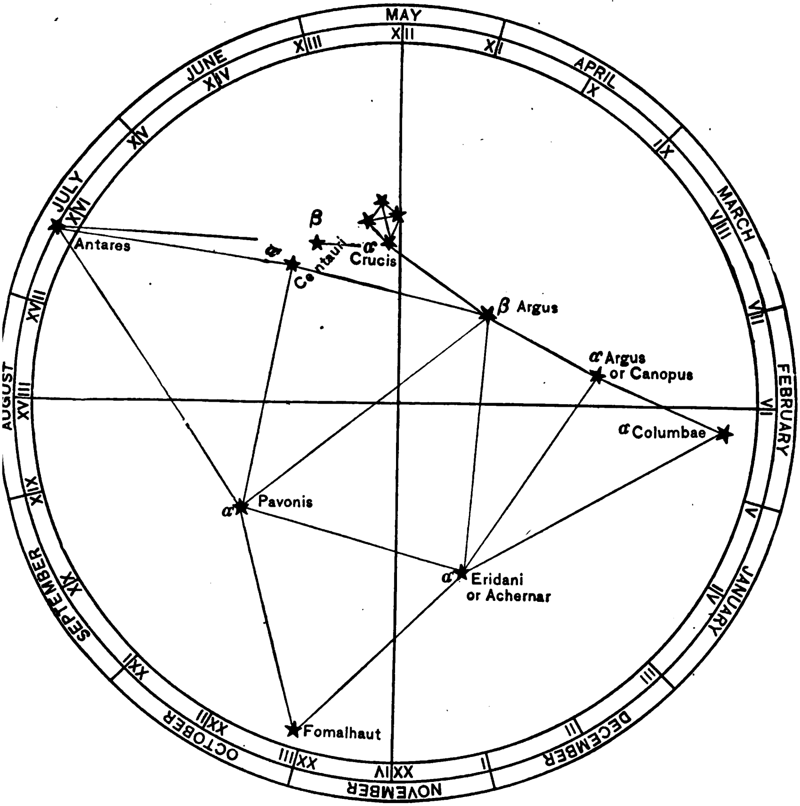
α Tauri or Aldebaran.—A line from Betelgeux through the three stars in Orion's head and extended to three times the distance leads to Aldebaran, which, shining with a decided reddish tint, is conspicuous as forming a V with four other stars.

α Arietis or Hamel.—A line drawn from Betelgeux through Aldebaran leads to Hamel, which may be known by two small stars southwestward of it. Hamel, Menkar, and α Tauri form a triangle nearly right angled at Menkar.

β Leonis or Denebola.—A line from Procyon through Regulus leads to Denebola at a little over half the distance. For Regulus, see Art. 324.

α Virginis or Spica.—About 35° SE. from Denebola is Spica, a bright white star, which forms with Arcturus and Denebola an equilateral triangle. Four stars of the constellation Corvus form the shape of a "spanker," the gaff pointing to Spica.

α Scorpii or Antares.—A line from the Dolphin through Altair leads to Antares which is a bright star of a decided reddish tinge, forming with adjacent stars the approximate figure of a hand glass, Antares at junction of glass and handle. It forms with α and β Libræ a long triangle, α Libræ being on a line between Spica and Antares. A line from Regulus through Spica extended to the same distance passes a little to the southward of Antares. A line from α^2 Crucis through β Centauri, produced three times its length, leads to Antares (Plate IX).



PRINCIPAL STARS AROUND THE SOUTH CELESTIAL POLE OF A DECLINATION GREATER THAN 30° S.

α **Piscis Australis or Fomalhaut.**—A line from Scheat through Markab extended about 45° leads to Fomalhaut, which forms with three other stars an irregular quadrilateral. It forms an equilateral triangle with α Pavonis and Achernar (Plate IX).

The Pleiades.—A line from midway between Algenib and Alpheratz through α Arietis, extended the same distance, leads to the Pleiades, a remarkable cluster, of which six stars are visible to the naked eye.

327. Description of Plate IX.—This plate shows the principal stars of the southern hemisphere whose declination exceeds 30°. What was said in Art. 324 about the Roman numerals and names of months around the margin of Plate VI, apply to the numerals and months of this plate with this exception, that the observer faces the South.

The Southern Cross.—This is the most conspicuous constellation of the southern hemisphere, and is outlined by four bright stars; when the cross is above the pole, α^2 Crucis is the southernmost, β Crucis the easternmost, γ Crucis the northernmost, and δ Crucis the westernmost star of the cross. As a line through α and β Centauri points directly to the cross, those two stars are known as the pointers.

α **Argus or Canopus.**—This star is next to Sirius in brilliancy, is midway between Rigel and the cross. Rigel, α Columbæ, Canopus, β Argus, and α^2 Crucis are at equal distances apart in a very slightly curved line.

α **Eridani or Achernar.**—Is about midway between Canopus and Fomalhaut, forms an equilateral triangle with α Pavonis and Fomalhaut, also with β Argus and α Columbæ.

328. In cloudy weather.—In case the surrounding heavens are clouded and it is desired to ascertain the name of a single star that may be out, its altitude and azimuth having been taken, the name may be found in the Nautical Almanac, from its right ascension and declination obtained from the Star Identification Tables, Hydrographic Office Publication No. 127, which give simultaneous values of the declination and hour angle corresponding to values of the latitude, altitude, and azimuth ranging from 0° to 88° in latitude and altitude and from 0° to 180° in azimuth; or from the Altitude and Azimuth Tables of Aquino in Hydrographic Office Publication No. 200. In the absence of these publications, the following method affords a means of identification:

(1) when the body is on the meridian, its right ascension is the L. S. T. of the instant of observation (Art. 173), and the declination may be found from the known latitude of the place and the measured true meridian altitude; (2) in case the body is not on the meridian, the approximate right ascension (the L. S. T. at transit) may be obtained from the local time of observation and the body's estimated hour angle, and the approximate declination from the known latitude and the estimated altitude at transit; (3) the star may be projected stereographically from its observed altitude and azimuth, and the right ascension and declination determined with sufficient accuracy to distinguish its name; (4) the coordinates may be obtained by the use of a celestial globe; (5) having determined by observation the true altitude and azimuth of a heavenly body, its right ascension may be found by applying to the L. S. T. of observation the hour angle taken from the azimuth tables, as explained in Appendix C, and the declination may then be found from the formula

$$\cos d = \sin Z \cos h \operatorname{cosec} t.^*$$

This formula will give the numerical value of the declination but will not determine the sign, about which, however, there should be no ambiguity except when the declination is very small. Having determined the numerical value of the declination by computation, enter the tables with L, t and $+d$ (that is of same name as latitude) and see if the azimuth found tabulated there agrees with that used in the computation. If agreement is found, the declination is properly marked; if not, the declination is negative and verification should be sought on that supposition. With the values of L, t and Z, an inspection of the tables in most cases will determine the value of the declination without the necessity of computation.

When inspecting the Nautical Almanac or a star table in an effort to identify an observed heavenly body, through an agreement of tabulated coordinates with those obtained by any of the methods referred to above, the navigator should always consider the possibility of having observed a planet instead of a fixed star.

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[*] The use of this formula in this connection was first proposed by Lt.-Comdr. G. W. Logan, U. S. N., in The Proceedings of U. S. Naval Institute, No. 104.

There are various graphic methods for determining the names of stars; those proposed by Admiral Sigsbee and Lt. Comdr. Rust, U. S. N., Mr. G. W. Littlehales of the Hydrographic Office, and Lt. Radler de Aquino of the Brazilian Navy, are among the best. For the details of Admiral Sigsbee's method the student is referred to H. O. chart No. 1560; for Comdr. Rust's method to the Proceedings of U. S. Naval Institute, Nos. 116, 123, and 124; for Mr. Littlehales' method to his admirable work "Altitude, Azimuth, and Geographical Position"; for Lt. Radler de Aquino's method to the nomogram explained in Appendix D.

It is well, when navigating, to note at twilight the approximate bearings and altitudes of prominent stars whose names are known, whether desired for observations or not; then under circumstances above referred to, a single bright star peeping out from the clouds at a time when an observation is desired might be recognized from its approximate position, noted about the same time a night or two before, when the weather conditions were such as to make identification without question.

Ex. 227.—At sea, January 19, 1918, a. m., in latitude by D. R. 50° 33′ N. and longitude by D. R. 40° 04′ W., weather cloudy, a bright star was observed, through a break in the clouds, on the meridian bearing South; star's sextant altitude 51° 54′ 10″; I. C. + 3′; height of eye 36 feet; W. T. of observation 2^h 11^m 10^s; C—W 2^h 39^m 55^s; chronometer slow on G. M. T. 1^m 10^s; what was the name of the star?

	h m s		° ′ ″		′ ″
W. T.	2 11 10	✳'s h_s	51 54 10 S	I. C.	+3 00
C—W	2 39 55	Corr.	− 3 38	Dip	−5 53
C. C.	+ 1 10			Ref.	−0 45
		✳'s h	51 50 32 S		
G. M. T. Jan. 18	16 52 15	✳'s z	38 09 28 N	Corr.	−3 38
		Latitude	50 33 00 N		
R. A. M. ⊙	19 48 19.9				
Corr. G. M. T.	+ 2 46.3	✳'s d	12 23 32 N		
G. S. T.	12 43 21.2				
Long. W.	2 40 16				
L. S. T. or ✳'s R. A.	10 03 05.2				

An inspection of the mean place tables of the Ephemeris and Nautical Almanac of 1918 shows the above star to have been *a* Leonis (Regulus) whose tabulated coordinates were: R. A. 10^h

04^m $00^s.419$, dec. N. 12° 22′ 06″.46. In practice, at sea, it will be unnecessary to correct the R. A. M. ⊙ for the G. M. T., an approximate R. A. being generally sufficient for the identification of the star.

Ex. 228.—April 5, 1918, about 7^h 11^m 18^s, p. m., of local mean time, in latitude by D. R. 20° 40′ S. and longitude by D. R. 90° 12′ E., weather cloudy, observed a bright star through a break in the clouds; star's sextant altitude 25° 58′ 40″; I. C.+1′; height of eye 19 feet; star's bearing (p. s. c.) or $Z_N=309°$, variation −10°, ship heading East (p. s. c.), deviation +2°. Required the hour angle and name of star.

	h m s
L. M. T.	7 11 18
Long. E.	6 00 48
G. M. T. April 5	1 10 30
R. A. M. ⊙	0 51 54.6
Corr. G. M. T.	11.6
G. S. T.	2 02 36.2
Long. E.	6 00 48
L. S. T.	8 03 24.2
✳'s *t*	+ 3 33 56
✳'s R. A.	4 29 28.2

	° ′ ″
✳'s h_s	25 58 40
Corr.	− 5 15
✳'s *h*	25 53 25
✳'s Z_N (p. s. c.)	309°
C. E.	− 8
✳'s Z_N (true)	301
✳'s *Z* (true)	S 121 W

✳'s *Z* (expressed in time) $=8^h$ $08^m.4$

	′ ″
I. C.	+1 00
Dip	−4 16
Ref.	−1 59
Corr.	−5 15

Entering the azimuth tables (H. O. Pub. No. 120) in the given latitude 20° 40′ S., with the altitude 25°.9 in the declination column, and *Z* (expressed as time) 8^m $03^m.4$ in the hour angle column, we find tabulated in the usual place of the azimuth the star's hour angle $t=53°$ 29′ W. or $+3^h$ 33^m 56^s. Converting the L. M. T. of observation into L. S. T. and applying the hour angle to this L. S. T., the star's right ascension is found to be 4^h 29^m $28^s.2$; then, by substitution in the formula $\cos d=\sin Z \cos h \operatorname{cosec} t$, we find $d=16°$ 22′. This is evidently North for, by using the above *L* and *t* with declination North 16° 22′, we will obtain by inspection of the tables the proper azimuth which would not be obtained on the supposition that the declination is South. An inspection of the Ephemeris and Nautical Almanac of 1918 will show this star to have been α Tauri (Aldebaran), its coordinates in the mean place tables being: R. A., 4^h 31^m $12^s.798$; and *d*, N. 16° 20′ 43″.69.

$Z=121°$	sin	9.93807
$h=$ 25° 53′ 25″	cos	9.95406
$t=$ 53° 29′	cosec	10.09491
$d=$ 16° 22′	cos	9.98204

CHAPTER XXIV.

GENERAL OBSERVATIONS.

329. Watchfulness over the compasses.—The navigator should see the compasses, especially the "standard," properly centered in their binnacles and so swung that the rims of the bowls will be horizontal; that the cards traverse freely in the bowls and that the bowls swing properly in their gimbals; that the lubber's lines are in the fore-and-aft line of the ship; that each compass has sufficient sensibility, magnetic moment, and steadiness; that there is no concealed iron in, or near, the binnacle that may probably disturb the compass; that the electric light wires in the vicinity of the compass are double, the direct and return wires close together and well insulated, and that there is no short circuit through the bridge or binnacle fittings; that quartermasters and others near the compass have no steel grommets, side arms, bayonets, nor other metal on their persons that may affect the compass; that all correctors are in place and the quadrantal correctors are free from magnetism.

When entirely free from magnetism, these spheres will have a magnetic axis in the direction of the line of force; North or red polarity being to northward of and below the center, South or blue polarity to southward of and above the center, however rapidly they may be rotated or their positions changed. The point of North polarity should attract the South end of a magnetic needle and repel the North end; if it does not, the particular sphere possesses magnetism which should be removed by heating it to a dark heat only, covering with ashes, and allowing it to slowly cool.

After compensating, care must be taken to see each magnet system secured in its proper direction, the clamp screw being set up when the lines traced on the frame and supporting plate of the carrier are in coincidence; also, that the key is removed from the mechanism of both carriers when they are at the proper height.

Attention is also called to the fact that the quadrantal correctors may be accidentally moved, even when the securing nuts are well set up.

When steaming under forced draft, all compasses, especially one near the smoke stack, should be carefully watched; there are cases on record of marked changes in compass deviations, due to changes in the magnetic character of the casing and smoke stack, caused by the intense heat of the escaping gases under such circumstances.

Unceasing vigilance the only safe-guard.—The fact cannot be too strongly impressed on the inexperienced navigator that however well a compass may be compensated at one moment, this compensation cannot be assumed as correct at a later time, whether in the same or a different locality, unless there is a frequent determination or verification of the deviations. It has been shown that the deviations will change on change of magnetic latitude, or of longitude, if it involves a change of magnetic dip or horizontal force, and from other causes; and the navigator must anticipate these changes and ascertain the deviations of his compasses daily, in port and at sea, on as many points as possible, at times when the dynamos are in operation under (1) normal, (2) full load.

Record of compass observations.—A systematic record should be kept of all observations for compass deviations in the Compass Journal provided for that purpose.

Special use of the standard.—A ship's course should always be directed by the standard compass, and all courses and bearings entered in the ship's log-book should be those shown by that compass alone; since all bearings, before being used, must be corrected by the **deviation due to the direction of the ship's head** at the time they were taken, it is apparent that the ship's heading (p. s. c.) should also be recorded whenever a bearing (p. s. c.) is entered in the log-book.

Comparison of compasses.—Whenever a ship is steadied on a course (p. s. c.), the headings by the steering, pilot house, and check compasses should be noted and recorded in the rough note-book, so that if, from any cause, the standard becomes deranged, the fact may be made immediately apparent. It would be a useful practice at sea to record as above the headings by all compasses at "eight bells." The officer-of-the-deck, during his watch, should frequently compare the readings of the standard and steering compasses, especially when the ship heels; and, whenever the course is changed, he should personally see the ship on her proper course per standard and that both the quartermaster and helmsman have noted the corresponding heading per steering compass.

Change in position of correctors.—Whilst it is desirable to keep the deviations of the standard a minimum, still no radical change should be made in the position of correctors except when it is evident a permanent change of deviations has taken place and in cases where an opportunity may be given to obtain residual deviations on at least 16 points before proceeding to sea, and then only by permission of the captain, as required by the Naval Regulations.

Effects of lightning.—Marked changes in compass deviations in iron or steel ships may occur if the ship is struck by lightning, even when the condition of ship and fittings is otherwise unaffected. A case is of record in which the magnetic polarity of a ship and, in consequence, the sign of its compass deviations were entirely reversed by a stroke of lightning.

Effects of target practice.—The magnetism of a ship and hence its compass deviations are liable to change during target practice, especially when heavy guns are fired and the ship is kept on one heading during the practice.

These effects may be minimized if, during the course of the practice, the ship is headed in all quadrants; or, at least, if half the practice occurs with the ship's head in one direction, and the other half with the ship's head in the opposite direction.

Immediately after the practice, the ship should be swung for deviations, though these may be expected to change, as the effects of the firing will dissipate after a few days. A record should be kept of the changes, and the ship again swung at the first opportunity after the ship's magnetism has become normal.

It is reasonable to expect a less change in the deviations, due to target practice, in an old ship than in one recently launched.

Effect of retentive magnetism or "The Gaussin Error."—When a ship is kept continuously on or near one compass heading for several days, whether at anchor, alongside a dock, or underway, and especially if underway and subjected to much vibratory motion, a temporary magnetic character is produced in the "soft" iron and iron of qualities intermediate between "soft" and "hard."

The time a ship is on a particular heading plays an important part in the intensity of the magnetism induced, and affects the rapidity of its disappearance on change of the ship's direction; the effects of retentive magnetism are particularly noticeable in vessels on a northerly or southerly course after having steered for some time on an easterly or westerly course, and more so in high than in low latitudes.

Having headed westerly for several days, if the course is changed to the southward, a deviation of + sign will be introduced for the new course; if the course is changed to the northward, a deviation of (—) sign will be introduced; all in addition to the tabulated value for that heading (p. c.). The reverse happens for a vessel which has been steaming for any length of time on easterly courses.

When changing course in the vicinity of dangers, the navigator must remember this tendency of the compass needle to deviate in the direction from which the course was changed, and, if possible, check the deviations by observations of the sun, planets, or stars; if the weather proves thick or foggy, he must proceed with extreme caution, lay his course with a large allowance for safety, and on soundings "use the machine" or "keep the lead going."

330. Sextant.—This subject has been exhaustively treated in Chapter IX, and it only remains now as a matter of emphasis again to caution the navigator always to handle his sextant with care, to guard it from all jars or shocks, and unnecessary exposure to the sun's rays, to heat, or dampness. It should be put away before target practice and not left, as valuable sextants have been left, on a chart table in the pilot house to be blown off by the blast of a bow gun.

After the instrument has been once properly adjusted, there should be no reason for its getting out of adjustment, if carefully handled; however, this adjustment must be frequently verified and the I. C. must be obtained every time an observation is taken.

The careful navigator will not only not lend his sextant to any one, but he will permit nobody but himself to handle it.

331. Chronometers.—The navigator should bear in mind the fact that chronometers are delicate instruments, worthy of all the care and attention recommended for them in Chapter X. Their error and rate should be frequently determined and always just before sailing.

When in cruising grounds, away from any place where chronometers may be rated, the errors of all chronometers should be checked up occasionally from the second differences, which may be done provided no two of the chronometers are running alike, that is, have rates of the same amount and sign, and provided further that the temperature curve, as determined on board, of one of the chronometers will give a close approximation to the rate of that chronometer for the mean temperature of the elapsed time; each chronometer may be tried for this purpose in the effort to satisfy the equations below.

The method embodies simply the solution of two equations with three unknown quantities, the value of one being assumed from the best data obtainable as a trial value in the determination of the others.

Let x, y, and z be the mean daily rates of chronometers A, B, and C, respectively, since last rating.

Let a be the mean value of the second differences of A and B,

b, the mean value of the second differences of A and C;

then the equations will be of the general form,

$$x + y = a,$$
$$x + z = b.$$

Having found x, y, and z, the G. M. T. corresponding to a given absolute instant of time as found from each chronometer should be the same.

It must not be forgotten that x, y, and z are the mean rates for the interval since previous rating, and each may differ from the real rate at the instant considered.

332. Caution as to charts.—As required by naval regulations, the navigator must familiarize himself with the sailing directions and charts of his cruising grounds, see that they are of recent date and give the latest obtainable information.

When shaping a course through localities where the variation may have a considerable change in an ordinary day's run, notably off the Newfoundland and North American coasts and in the westward approaches to the English Channel, it would be advisable to measure off roughly the probable day's run and note the change of variation in that time. If the number of degrees of change is say 6° W. in 24 hours, it is plain that a magnetic course laid down at the beginning of the day cannot be continued throughout it without involving a large error in position; therefore, it is advisable to alter the course 1° at regular intervals, and to allow for, say, an increase of 6° westerly variation, proceed thus:

Having found at the outset the correct true course, obtain the first compass course by applying to the true course the proper deviation and the *mean* variation from the chart for the first four hours' run; at the end of this and each succeeding four hours' run of the day, alter the course by 1°, and to the right since the variation is westerly.

However carefully the work may be done, absolute accuracy in a position plotted by run or cross bearings cannot be expected. The paper used is dampened in the making of the chart and distortion takes place on drying; this distortion varies with the quality and size of the paper. Besides, charts will become distorted from use in damp or foggy weather. For these reasons, the navigator should not rely entirely on positions by graphic methods, and those entered in the log-book should be by computation.

In the selection of charts for use in navigating, those of the largest scale available should be selected; they will surely possess greater detail, and perhaps may be from plates on which, in view of the scale of the chart, the latest corrections have been made.

Close attention should be given to the date of the survey from which a chart resulted, especially if the chart is of regions where the bottom is of shifting sand. For instance, on Nantucket shoals the bottom is of such a shifting nature that only charts of that locality of a recent date should be regarded as reliable.

Considering the fact that these shoals extend miles beyond the points at which, when surveying, signals may be carried, it is evident that many soundings, even in shoal water, have been located by D. R. alone.

Soundings so located, in such regions, where currents are strong and variable, should be regarded with suspicion, and the prudent navigator will always navigate those shoals with caution.

Close attention should also be given to the amount of detail on the charts used; when charts show soundings to be few and far between in the vicinity of shallow water or of an occasional reef, it is wise to presume that the locality has been only partially surveyed, and that perhaps there are many dangers near, even if uncharted.

If the soundings, even though few, show only deep water, it is fair to assume that the intervening spaces, in which no soundings are given, may be navigated with safety.

333. Before going to sea, or entering pilot waters.—Before reaching pilot waters, the navigator should see all charts of the locality corrected to date; should study these charts, the sailing directions, and the light and buoy lists of those waters; and should see the sounding machine in good order and ready for use.

Before going to sea, he should see the log lines properly marked, sand glasses timed, patent logs in good order, and should have obtained the deviations of the standard compass, the error and rate of all chronometers on board. Before entering or leaving port, he should personally see the lead lines well soaked in water, stretched, and properly marked. To facilitate the work of marking the log and lead lines, the required distances should be laid off in a suitable place on deck, or on the fore-and-aft bridge, and permanently marked with copper tacks.

334. Discrepancy in a. m. and p. m. sights.—Abnormal refraction may be looked for in the Red Sea, Persian Gulf, and in the regions of the Gulf stream, or wherever there is a marked difference between the temperature of the air and the water.

When such refraction exists, there will be an apparent discrepancy between the results obtained from forenoon and afternoon sights, because the error introduced by using the tabulated dip affects them the opposite way; therefore, under such circumstances, it would be better to reduce the longitude from both a. m. and p. m. sights to noon, and then to take the mean of these two resulting longitudes as the correct noon longitude.

Fortunately, however, a navigator, when navigating the aforementioned waters, is not restricted in taking sights to times when refraction is abnormal; star sights may most likely be obtained at more favorable times.

335. Error of a ship's position.—It must not be forgotten that a ship's position at sea is only approximate, even when determined by the most exact navigator, under the most favorable circumstances, and with the best instruments obtainable.

The extent of error depends on instrumental imperfections, errors of tabulated dip and refraction, error of time, errors of observation, and, in the case of double altitudes, the error of the intervening run; these may be increased by the circumstances of unfavorable location of the body observed, of bad weather, or rough seas.

Under average conditions, the ship's position may be assumed in doubt at least two miles, and as the sign of this probable error may be + or (—), the ship's position may be anywhere within a circle described from the determined position as a center with a radius of 2 miles, and courses should be shaped with this uncertainty in view.

When desiring to lay a course, from a position determined at sea, to pass a danger, it would be prudent to multiply the assumed average error by a number (2, 3, or 4, according to circumstances), called a "coefficient of safety," and, considering the result obtained as the limit of possible error, to describe a circle about the determined position with that limit as a radius; then to shape the course from a point on that side of the circle nearest the danger to be passed.

The general principle embodied in the use of a "coefficient of safety" was more correctly applied in connection with Sumner lines as explained in Arts. 293 and 294, wherein a ship's position was shown to be somewhere within a parallelogram formed by drawing parallels on each side of each line of position and at such distances as to include errors of altitude, time, etc., which might be assumed by the navigator as probable under existing conditions. By using the parallelogram, the navigator is better enabled to see in which direction his position is the most in doubt (see Figs. 126, 127, 128).

336. The advisability of keeping landmarks in sight.— Considering the uncertainty of positions at sea, it is prudent, when navigating coasts well charted, lighted, and buoyed, especially when there are outlying lightships, as in the case of the eastern coast of the United States, to make certain landmarks or lights in regular succession and at short intervals of time, being careful, however, to see that the ship is not set by currents into regions of possible danger.

If by so doing a vessel is not taken too much out of the direct course to destination, it is always advisable, whenever possible, to sight a mark from which to take a fresh departure, whether making ready to close in with the land for the purpose of entering port, or to put out to sea in view of approaching fog or bad weather.

In pilot waters, the ship's position should be located by observations of permanent landmarks if practicable, as buoys are frequently out of place and lightships are sometimes so, even when just replaced, on the station. In the vicinity of dangers, whether buoys are in sight or not, the use of the danger angle is advisable (see Arts. 118 and 119).

In running a channel, where there are no well-defined landmarks, and as to which there may be some doubt, the ship may be steered through it by zig-zagging occasionally from side to side, keeping the lead going, and thus showing on which side of the channel the vessel may be and in which direction the course must be changed to find deeper water, care being taken, however, not to run into dangerously shoal water.

In going in or out of port, try to pick up a range, ahead or astern as the case may be; and, as local currents may be uncertain, watch for any possible indications of their set and strength, such as the riding of buoys, the general heading of vessels at anchor, or the opening of the range on which the ship may be steering. A comparison of the courses and distances sailed by compass and those made good as indicated by bearings will give the set and drift of the current.

Before anchoring, the navigator should know not only the set of the tide but the maximum rise and fall to ensure having, at low tide, sufficient water under the bottom. When desiring to find an anchorage on two bearings, approach it upon that bearing which may be the most convenient one, reduce speed, and stop in sufficient time to let go the anchor when the ship is also upon the second bearing.

337. Disregarding the seconds of data when solving the astronomical triangle.—The ship's position being subject to the errors enumerated in Art. 335, and hence uncertain, even under the most favorable conditions, some navigators believe themselves justified in using their data only to the nearest minute of arc in solving the astronomical triangle. If the errors due to such procedure were known to offset other errors, such theories would be tenable; but as it is equally probable that they would augment them, it seems advisable, in the absence of any definite knowledge as to the effect of neglecting the seconds, to exercise great care in obtaining data and then to use the values obtained, when working sights for either latitude or longitude.

TABLE I.

Change in longitude due to a change of 1′ in latitude.

Bearing.	LATITUDE. 0°	1°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	Bearing.
°	′	′	′	′	′	′	′	′	′	′	.	′	′	′	′	′	°
1	57.29	57.30	57.32	57.43	57.61	57.85	58.17	58.57	59.04	59.60	60.24	60.97	61.79	62.71	63.74	64.88	1
2	28.64	28.64	28.65	28.71	28.79	28.92	29.08	29.28	29.51	29.79	30.11	30.47	30.89	31.35	31.86	32.43	2
3	19.08	19.08	19.09	19.13	19.19	19.27	19.38	19.51	19.67	19.85	20.06	20.31	20.58	20.89	21.23	21.61	3
4	14.30	14.30	14.31	14.34	14.38	14.44	14.52	14.62	14.74	14.88	15.04	15.22	15.42	15.65	15.91	16.20	4
5	11.43	11.43	11.44	11.46	11.49	11.54	11.61	11.69	11.78	11.89	12.02	12.16	12.33	12.51	12.72	12.95	5
6	9.51	9.52	9.52	9.54	9.57	9.61	9.66	9.73	9.81	9.90	10.00	10.12	10.26	10.41	10.59	10.78	6
7	8.14	8.15	8.15	8.16	8.19	8.22	8.27	8.33	8.39	8.47	8.56	8.67	8.78	8.91	9.06	9.22	7
8	7.12	7.12	7.12	7.13	7.15	7.18	7.22	7.27	7.33	7.40	7.48	7.57	7.67	7.79	7.92	8.06	8
10	5.67	5.67	5.68	5.69	5.70	5.73	5.76	5.80	5.85	5.90	5.96	6.03	6.12	6.21	6.31	6.42	10
12	4.71	4.71	4.71	4.72	4.73	4.75	4.78	4.81	4.85	4.89	4.95	5.01	5.07	5.15	5.23	5.33	12
14	4.01	4.01	4.01	4.02	4.03	4.05	4.07	4.10	4.13	4.17	4.22	4.27	4.33	4.39	4.46	4.54	14
16	3.49	3.49	3.49	3.50	3.51	3.52	3.54	3.56	3.59	3.63	3.67	3.71	3.76	3.82	3.88	3.95	16
18	3.08	3.08	3.08	3.08	3.10	3.11	3.13	3.15	3.17	3.20	3.24	3.28	3.32	3.37	3.42	3.49	18
20	2.75	2.75	2.75	2.75	2.76	2.77	2.79	2.81	2.83	2.86	2.89	2.92	2.96	3.01	3.06	3.11	20
22	2.47	2.47	2.48	2.48	2.49	2.50	2.51	2.53	2.55	2.58	2.60	2.63	2.67	2.71	2.75	2.80	22
24	2.25	2.25	2.25	2.25	2.26	2.27	2.28	2.30	2.32	2.34	2.36	2.39	2.42	2.46	2.50	2.54	24
26	2.05	2.05	2.05	2.05	2.06	2.07	2.08	2.10	2.11	2.13	2.16	2.18	2.21	2.24	2.28	2.32	26
28	1.88	1.88	1.88	1.88	1.89	1.90	1.91	1.92	1.94	1.96	1.98	2.00	2.03	2.06	2.09	2.13	28
30	1.73	1.73	1.73	1.74	1.74	1.75	1.76	1.77	1.78	1.80	1.82	1.84	1.87	1.90	1.93	1.96	30
32	1.60	1.60	1.60	1.60	1.61	1.62	1.63	1.64	1.65	1.66	1.68	1.70	1.73	1.75	1.78	1.81	32
34	1.48	1.48	1.48	1.49	1.49	1.50	1.50	1.52	1.53	1.54	1.56	1.58	1.60	1.62	1.65	1.68	34
36	1.38	1.38	1.38	1.38	1.38	1.39	1.40	1.41	1.42	1.43	1.45	1.47	1.48	1.51	1.53	1.56	36
38	1.28	1.28	1.28	1.28	1.29	1.29	1.30	1.31	1.32	1.33	1.35	1.36	1.38	1.40	1.42	1.45	38
40	1.19	1.19	1.19	1.19	1.20	1.20	1.21	1.22	1.23	1.24	1.25	1.27	1.28	1.30	1.33	1.35	40
42	1.11	1.11	1.11	1.11	1.12	1.12	1.13	1.14	1.14	1.15	1.17	1.18	1.20	1.22	1.24	1.26	42
44	1.04	1.04	1.04	1.04	1.04	1.05	1.05	1.06	1.07	1.08	1.09	1.10	1.12	1.13	1.15	1.17	44

46	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.06	1.07	1.09	46
48	0.90	0.90	0.90	0.90	0.90	0.91	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.99	1.00	1.02	48
50	0.84	0.84	0.84	0.84	0.84	0.85	0.85	0.86	0.87	0.87	0.88	0.89	0.91	0.92	0.93	0.95	50
52	0.78	0.78	0.78	0.78	0.79	0.79	0.79	0.80	0.80	0.81	0.82	0.83	0.84	0.85	0.87	0.88	52
54	0.73	0.73	0.73	0.73	0.73	0.73	0.74	0.74	0.75	0.76	0.76	0.77	0.78	0.79	0.81	0.82	54
56	0.67	0.67	0.67	0.68	0.68	0.68	0.68	0.69	0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.76	56
58	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.64	0.64	0.65	0.66	0.66	0.67	0.68	0.69	0.71	58
60	0.58	0.58	0.58	0.58	0.58	0.58	0.59	0.59	0.60	0.60	0.61	0.61	0.62	0.63	0.64	0.65	60
62	0.53	0.53	0.53	0.53	0.53	0.54	0.54	0.54	0.55	0.55	0.56	0.57	0.57	0.58	0.59	0.60	62
64	0.49	0.49	0.49	0.49	0.49	0.49	0.50	0.50	0.50	0.51	0.51	0.52	0.53	0.53	0.54	0.55	64
66	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.46	0.46	0.46	0.47	0.47	0.48	0.49	0.50	0.50	66
68	0.40	0.40	0.40	0.40	0.40	0.41	0.41	0.41	0.42	0.42	0.42	0.43	0.44	0.44	0.45	0.46	68
70	0.36	0.36	0.36	0.36	0.37	0.37	0.37	0.37	0.37	0.38	0.38	0.39	0.39	0.40	0.40	0.41	70
72	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.34	0.34	0.34	0.35	0.35	0.36	0.36	0.37	72
74	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.30	0.30	0.30	0.31	0.31	0.31	0.32	0.33	74
76	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.27	0.27	0.28	0.28	76
78	0.21	0.21	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.24	0.24	78
80	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.19	0.19	0.19	0.20	0.20	80
81	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.18	0.18	81
82	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.16	0.16	82
83	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.14	0.14	83
84	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	84
85	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	85
86	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	86
87	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	87
88	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	88
89	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	89
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

TABLE I.—Continued.

Bearing.	LATITUDE.																Bearing.
	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	60°	
°	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	°
1	66.15	67.56	69.10	70.81	72.70	74.79	77.09	79.64	82.47	85.62	89.13	93.05	97.47	102.5	108.1	114.6	1
2	33.07	33.77	34.54	35.40	36.34	37.38	38.53	39.81	41.22	42.80	44.55	46.51	48.72	51.21	54.04	57.27	2
3	22.03	22.50	23.02	23.59	24.21	24.91	25.68	26.53	27.47	28.52	29.68	30.99	32.46	34.12	36.01	38.16	3
4	16.51	16.86	17.25	17.68	18.15	18.67	19.24	19.88	20.59	21.37	22.25	23.23	24.33	25.57	26.99	28.60	4
5	13.20	13.48	13.79	14.13	14.50	14.92	15.38	15.89	16.45	17.08	17.78	18.57	19.45	20.44	21.57	22.86	5
6	10.99	11.22	11.48	11.76	12.07	12.42	12.80	13.23	13.70	14.22	14.80	15.45	16.19	17.01	17.95	19.03	6
7	9.40	9.60	9.82	10.07	10.34	10.63	10.96	11.32	11.72	12.17	12.67	13.23	13.86	14.56	15.37	16.29	7
8	8.22	8.39	8.58	8.79	9.03	9.29	9.57	9.89	10.24	10.63	11.07	11.56	12.11	12.72	13.43	14.23	8
10	6.55	6.69	6.84	7.01	7.20	7.40	7.63	7.88	8.16	8.48	8.82	9.21	9.65	10.14	10.70	11.34	10
12	5.43	5.55	5.67	5.81	5.97	6.14	6.33	6.54	6.77	7.03	7.32	7.64	8.00	8.41	8.88	9.41	12
14	4.63	4.73	4.84	4.96	5.09	5.24	5.40	5.58	5.77	5.99	6.24	6.51	6.82	7.17	7.57	8.02	14
16	4.03	4.11	4.21	4.31	4.43	4.55	4.69	4.85	5.02	5.21	5.42	5.66	5.93	6.24	6.58	6.97	16
18	3.55	3.63	3.71	3.80	3.91	4.02	4.14	4.28	4.43	4.60	4.79	5.00	5.24	5.50	5.81	6.15	18
20	3.17	3.24	3.31	3.40	3.49	3.59	3.70	3.82	3.95	4.11	4.27	4.46	4.67	4.91	5.19	5.49	20
22	2.86	2.92	2.98	3.06	3.14	3.23	3.33	3.44	3.56	3.70	3.85	4.02	4.21	4.43	4.67	4.95	22
24	2.59	2.65	2.71	2.78	2.85	2.93	3.02	3.12	3.23	3.36	3.49	3.65	3.82	4.02	4.24	4.49	24
26	2.37	2.42	2.47	2.53	2.60	2.68	2.76	2.85	2.95	3.06	3.19	3.33	3.49	3.66	3.87	4.10	26
28	2.17	2.22	2.27	2.32	2.39	2.45	2.53	2.61	2.71	2.81	2.93	3.05	3.20	3.36	3.55	3.76	28
30	2.00	2.04	2.09	2.14	2.20	2.26	2.33	2.41	2.49	2.59	2.69	2.81	2.95	3.10	3.27	3.46	30
32	1.85	1.89	1.93	1.98	2.03	2.09	2.15	2.22	2.30	2.39	2.49	2.60	2.72	2.86	3.02	3.20	32
34	1.71	1.75	1.79	1.83	1.88	1.93	1.99	2.06	2.13	2.22	2.31	2.41	2.52	2.65	2.80	2.96	34
36	1.59	1.62	1.66	1.70	1.75	1.80	1.85	1.91	1.98	2.06	2.14	2.24	2.34	2.46	2.60	2.75	36
38	1.48	1.51	1.54	1.58	1.62	1.67	1.72	1.78	1.84	1.91	1.99	2.08	2.18	2.29	2.41	2.56	38
40	1.38	1.41	1.44	1.47	1.51	1.56	1.60	1.66	1.71	1.78	1.85	1.94	2.03	2.13	2.25	2.38	40
42	1.28	1.31	1.34	1.37	1.41	1.45	1.49	1.54	1.60	1.66	1.73	1.80	1.89	1.99	2.09	2.22	42
44	1.20	1.22	1.25	1.28	1.31	1.35	1.39	1.44	1.49	1.55	1.61	1.68	1.76	1.85	1.95	2.07	44

46	1.11	1.14	1.16	1.19	1.23	1.26	1.30	1.34	1.39	1.44	1.50	1.57	1.64	1.73	1.82	1.93	46
48	1.04	1.06	1.09	1.11	1.14	1.17	1.21	1.25	1.30	1.35	1.40	1.46	1.53	1.61	1.70	1.80	48
50	0.97	0.99	1.01	1.04	1.06	1.09	1.13	1.17	1.21	1.25	1.31	1.36	1.43	1.50	1.58	1.68	50
52	0.90	0.92	0.94	0.97	0.99	1.02	1.05	1.09	1.12	1.17	1.22	1.27	1.33	1.40	1.47	1.56	52
54	0.84	0.86	0.88	0.90	0.92	0.95	0.98	1.01	1.05	1.09	1.13	1.18	1.23	1.30	1.37	1.45	54
56	0.78	0.79	0.81	0.83	0.86	0.88	0.91	0.94	0.97	1.01	1.05	1.10	1.15	1.21	1.27	1.35	56
58	0.72	0.74	0.75	0.77	0.79	0.82	0.84	0.87	0.90	0.93	0.97	1.01	1.06	1.12	1.18	1.25	58
60	0.67	0.68	0.70	0.71	0.73	0.75	0.78	0.80	0.83	0.86	0.90	0.94	0.98	1.03	1.09	1.15	60
62	0.61	0.63	0.64	0.66	0.67	0.69	0.72	0.74	0.77	0.79	0.83	0.86	0.90	0.95	1.00	1.06	62
64	0.56	0.57	0.59	0.60	0.62	0.64	0.66	0.68	0.70	0.73	0.76	0.79	0.83	0.87	0.92	0.97	64
66	0.51	0.52	0.54	0.55	0.56	0.58	0.60	0.62	0.64	0.66	0.69	0.72	0.76	0.79	0.84	0.89	66
68	0.47	0.48	0.49	0.50	0.51	0.53	0.54	0.56	0.58	0.60	0.63	0.65	0.69	0.72	0.76	0.81	68
70	0.42	0.43	0.44	0.45	0.46	0.47	0.49	0.51	0.52	0.54	0.57	0.59	0.62	0.65	0.68	0.73	70
72	0.37	0.38	0.39	0.40	0.41	0.42	0.44	0.45	0.47	0.49	0.51	0.53	0.55	0.58	0.61	0.65	72
74	0.33	0.34	0.35	0.35	0.36	0.37	0.39	0.40	0.41	0.43	0.45	0.46	0.49	0.51	0.54	0.57	74
76	0.29	0.29	0.30	0.31	0.32	0.32	0.34	0.35	0.36	0.37	0.39	0.40	0.42	0.45	0.47	0.50	76
78	0.24	0.25	0.26	0.26	0.27	0.28	0.29	0.29	0.31	0.32	0.33	0.34	0.36	0.38	0.40	0.42	78
80	0.20	0.21	0.21	0.22	0.22	0.23	0.24	0.24	0.25	0.26	0.27	0.29	0.30	0.31	0.33	0.35	80
81	0.18	0.19	0.19	0.20	0.20	0.21	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.30	0.32	81
82	0.16	0.17	0.17	0.17	0.18	0.18	0.19	0.19	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.28	82
83	0.14	0.14	0.15	0.15	0.16	0.16	0.16	0.17	0.18	0.18	0.19	0.20	0.21	0.22	0.23	0.25	83
84	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.18	0.19	0.20	0.21	84
85	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.16	0.16	0.17	85
86	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.14	86
87	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.09	0.09	0.10	0.10	87
88	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	88
89	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	89
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

TABLE II.

Correction to be added to the observed altitude of the sun's lower limb.

Correction of the Sun's Altitude for Semi-diameter Dip, Parallax and Refraction. S. D. taken as 16′.

HEIGHT OF THE EYE ABOVE THE SEA IN FEET.

Alt. of Sun.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
7	5 39	5 30	5 21	5 13	5 05	4 57	4 50	4 43	4 36	4 29	4 22	4 16	4 09	4 03	3 57	3 51	3 45
8	6 30	6 21	6 12	6 04	5 56	5 48	5 41	5 34	5 27	5 20	5 13	5 07	5 00	4 54	4 48	4 42	4 36
9	7 10	7 01	6 52	6 44	6 36	6 28	6 21	6 14	6 07	6 00	5 53	5 47	5 40	5 34	5 28	5 22	5 16
10	7 44	7 35	7 26	7 18	7 10	7 02	6 55	6 48	6 41	6 34	6 27	6 21	6 14	6 08	6 02	5 56	5 50
11	8 12	8 03	7 54	7 46	7 38	7 30	7 23	7 16	7 09	7 02	6 55	6 49	6 42	6 36	6 30	6 24	6 18
12	8 35	8 26	8 17	8 09	8 01	7 53	7 46	7 39	7 32	7 25	7 18	7 12	7 05	6 59	6 53	6 47	6 41
13	8 56	8 47	8 38	8 30	8 22	8 14	8 07	8 00	7 53	7 46	7 39	7 33	7 26	7 20	7 14	7 08	7 02
14	9 13	9 04	8 55	8 47	8 39	8 31	8 24	8 17	8 10	8 03	7 56	7 50	7 43	7 37	7 31	7 25	7 19
15	9 29	9 20	9 11	9 03	8 55	8 47	8 40	8 33	8 26	8 19	8 12	8 06	7 59	7 53	7 47	7 41	7 35
16	9 41	9 32	9 23	9 15	9 07	8 59	8 52	8 45	8 38	8 31	8 24	8 18	8 11	8 05	7 59	7 53	7 47
17	9 54	9 45	9 36	9 28	9 20	9 12	9 05	8 58	8 51	8 44	8 37	8 31	8 24	8 18	8 12	8 06	8 00
18	10 04	9 55	9 46	9 38	9 30	9 22	9 15	9 08	9 01	8 54	8 47	8 41	8 34	8 28	8 22	8 16	8 10
19	10 14	10 05	9 56	9 48	9 40	9 32	9 25	9 18	9 11	9 04	8 57	8 51	8 44	8 38	8 32	8 26	8 20
20	10 23	10 14	10 05	9 57	9 49	9 41	9 34	9 27	9 20	9 13	9 06	9 00	8 53	8 47	8 41	8 35	8 29
21	10 31	10 22	10 13	10 05	9 57	9 49	9 42	9 35	9 28	9 21	9 14	9 08	9 01	8 55	8 49	8 43	8 37
22	10 39	10 30	10 21	10 13	10 05	9 57	9 50	9 43	9 36	9 29	9 22	9 16	9 09	9 03	8 57	8 51	8 45
23	10 46	10 37	10 28	10 20	10 12	10 04	9 57	9 50	9 43	9 36	9 29	9 23	9 16	9 10	9 04	8 58	8 52
24	10 52	10 43	10 34	10 26	10 18	10 10	10 03	9 56	9 49	9 42	9 35	9 29	9 22	9 16	9 10	9 04	8 58
25	10 58	10 49	10 40	10 32	10 24	10 16	10 09	10 02	9 55	9 48	9 41	9 35	9 28	9 22	9 16	9 10	9 04
26	11 03	10 54	10 45	10 37	10 29	10 21	10 14	10 07	10 00	9 53	9 46	9 40	9 33	9 27	9 21	9 15	9 09
27	11 08	10 59	10 50	10 42	10 34	10 26	10 19	10 12	10 05	9 58	9 51	9 45	9 38	9 32	9 26	9 20	9 14
28	11 13	11 04	10 55	10 47	10 39	10 31	10 24	10 17	10 10	10 03	9 56	9 50	9 43	9 37	9 31	9 25	9 19
29	11 17	11 08	10 59	10 51	10 43	10 35	10 28	10 21	10 14	10 07	10 00	9 54	9 47	9 41	9 35	9 29	9 23
30	11 21	11 12	11 03	10 55	10 47	10 39	10 32	10 25	10 18	10 11	10 04	9 58	9 51	9 45	9 39	9 33	9 27
31	11 25	11 16	11 07	10 59	10 51	10 43	10 36	10 29	10 22	10 15	10 08	10 02	9 55	9 49	9 43	9 37	9 31
32	11 29	11 20	11 11	11 03	10 55	10 47	10 40	10 33	10 26	10 19	10 12	10 06	9 59	9 53	9 47	9 41	9 35
33	11 32	11 23	11 14	11 06	10 58	10 50	10 43	10 36	10 29	10 22	10 15	10 09	10 02	9 56	9 50	9 44	9 38
34	11 36	11 27	11 18	11 10	11 02	10 54	10 47	10 40	10 33	10 26	10 19	10 13	10 06	10 00	9 54	9 48	9 42

35	11 39	11 30	11 21	11 13	11 05	10 57	10 50	10 43	10 36	10 29	10 22	10 16	10 09	10 03	9 57	9 51	9 45
36	11 41	11 32	11 23	11 15	11 07	10 59	10 52	10 45	10 38	10 31	10 24	10 18	10 11	10 05	9 59	9 53	9 47
37	11 44	11 35	11 26	11 18	11 10	11 02	10 55	10 48	10 41	10 34	10 27	10 21	10 14	10 08	10 02	9 56	9 50
38	11 46	11 37	11 28	11 20	11 12	11 04	10 57	10 50	10 43	10 36	10 29	10 23	10 16	10 10	10 04	9 58	9 52
39	11 49	11 40	11 31	11 23	11 15	11 07	11 00	10 53	10 46	10 39	10 32	10 26	10 19	10 13	10 07	10 01	9 55
40	11 52	11 43	11 34	11 26	11 18	11 10	11 03	10 56	10 49	10 42	10 35	10 29	10 22	10 16	10 10	10 04	9 58
42	11 56	11 47	11 38	11 30	11 22	11 14	11 07	11 00	10 53	10 46	10 39	10 33	10 26	10 20	10 14	10 08	10 02
44	12 01	11 52	11 43	11 35	11 27	11 19	11 12	11 05	10 58	10 51	10 44	10 38	10 31	10 25	10 19	10 13	10 07
46	12 04	11 55	11 46	11 38	11 30	11 22	11 15	11 08	11 01	10 54	10 47	10 41	10 34	10 28	10 22	10 16	10 10
48	12 07	11 58	11 49	11 41	11 33	11 25	11 18	11 11	11 04	10 57	10 50	10 44	10 37	10 31	10 25	10 19	10 13
50	12 11	12 02	11 53	11 45	11 37	11 29	11 22	11 15	11 08	11 01	10 54	10 48	10 41	10 35	10 29	10 23	10 17
52	12 14	12 05	11 56	11 48	11 40	11 32	11 25	11 18	11 11	11 04	10 57	10 51	10 44	10 38	10 32	10 26	10 20
54	12 17	12 08	11 59	11 51	11 43	11 35	11 28	11 21	11 14	11 07	11 00	10 54	10 47	10 41	10 35	10 29	10 23
56	12 20	12 11	12 02	11 54	11 46	11 38	11 31	11 24	11 17	11 10	11 03	10 57	10 50	10 44	10 38	10 32	10 26
58	12 22	12 13	12 04	11 56	11 48	11 40	11 33	11 26	11 19	11 12	11 05	10 59	10 52	10 46	10 40	10 34	10 28
60	12 24	12 15	12 06	11 58	11 50	11 42	11 35	11 28	11 21	11 14	11 07	11 01	10 54	10 48	10 42	10 36	10 30
65	12 31	12 22	12 13	12 05	11 57	11 49	11 42	11 35	11 28	11 21	11 14	11 08	11 01	10 55	10 49	10 43	10 37
70	12 36	12 27	12 18	12 10	12 02	11 54	11 47	11 40	11 33	11 26	11 19	11 13	11 06	11 00	10 54	10 48	10 42
75	12 40	12 31	12 22	12 14	12 06	11 58	11 51	11 44	11 37	11 30	11 23	11 17	11 10	11 04	10 58	10 52	10 46
80	12 46	12 37	12 28	12 20	12 12	12 04	11 57	11 50	11 43	11 36	11 29	11 23	11 16	11 10	11 04	10 58	10 52
85	12 50	12 41	12 32	12 24	12 16	12 08	12 01	11 54	11 47	11 40	11 33	11 27	11 20	11 14	11 08	11 02	10 56
90	12 54	12 45	12 36	12 28	12 20	12 12	12 05	11 58	11 51	11 44	11 37	11 31	11 24	11 18	11 12	11 06	11 00

CORRECTION OF THE SEMI-DIAMETER.

Jan. 1 +18″ “ 10 +18″ “ 20 +17″	Feb. 1 +16″ “ 10 +14″ “ 20 +12″	Mar. 1 +10″ “ 10 + 8″ “ 20 + 5″	April 1 +2″ “ 10 −1″ “ 20 −3″	May 1 − 6″ “ 10 − 8″ “ 20 −10″	June 1 −12″ “ 10 −13″ “ 20 −14″
July 1 −14″ “ 10 −14″ “ 20 −14″	Aug. 1 −13″ “ 10 −11″ “ 20 −10″	Sept. 1 −7″ “ 10 −5″ “ 20 −2″	Oct. 1 +1″ “ 10 +3″ “ 20 +6″	Nov. 1 + 9″ “ 10 +11″ “ 20 +13″	Dec. 1 +15″ “ 10 +17″ “ 20 +17″

TABLE II.—Continued.

Correction of the Sun's Altitude for Semi-diameter Dip, Parallax and Refraction. S. D. taken as 16′.

Alt. of Sun.	HEIGHT OF THE EYE ABOVE THE SEA IN FEET.																
	27	28	29	30	31	32	33	34	35	36	37	38	39	40	45	50	55
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	″	′ ″	″	′ ″	′ ″	′ ″	′ ″
7	3 39	3 34	3 28	3 23	3 18	3 12	3 07	3 02	2 57	2 52	2 47	2 43	2 38	2 33	2 09	1 49	1 29
8	4 30	4 25	4 19	4 14	4 09	4 03	3 58	3 53	3 48	3 43	3 38	3 34	3 29	3 24	3 00	2 40	2 20
9	5 10	5 05	4 59	4 54	4 49	4 43	4 38	4 33	4 28	4 23	4 18	4 14	4 09	4 04	3 40	3 20	3 00
10	5 44	5 39	5 33	5 28	5 23	5 17	5 12	5 07	5 02	4 57	4 52	4 48	4 43	4 38	4 14	3 54	3 34
11	6 12	6 07	6 01	5 56	5 51	5 45	5 40	5 35	5 30	5 25	5 20	5 16	5 11	5 06	4 42	4 22	4 02
12	6 35	6 30	6 24	6 19	6 14	6 08	6 03	5 58	5 53	5 48	5 43	5 39	5 34	5 29	5 05	4 45	4 25
13	6 56	6 51	6 45	6 40	6 35	6 29	6 24	6 19	6 14	6 09	6 04	6 00	5 55	5 50	5 26	5 06	4 46
14	7 13	7 08	7 02	6 57	6 52	6 46	6 41	6 36	6 31	6 26	6 21	6 17	6 12	6 07	5 43	5 23	5 03
15	7 29	7 24	7 18	7 13	7 08	7 02	6 57	6 52	6 47	6 42	6 37	6 33	6 28	6 23	5 59	5 39	5 19
16	7 41	7 36	7 30	7 25	7 20	7 14	7 09	7 04	6 59	6 54	6 49	6 45	6 40	6 35	6 11	5 51	5 31
17	7 54	7 49	7 43	7 38	7 33	7 27	7 22	7 17	7 12	7 07	7 02	6 58	6 53	6 48	6 24	6 04	5 44
18	8 04	7 59	7 53	7 48	7 43	7 37	7 32	7 27	7 22	7 17	7 12	7 08	7 03	6 58	6 34	6 14	5 54
19	8 14	8 09	8 03	7 58	7 53	7 47	7 42	7 37	7 32	7 27	7 22	7 18	7 13	7 08	6 44	6 24	6 04
20	8 23	8 18	8 12	8 07	8 02	7 56	7 51	7 46	7 41	7 36	7 31	7 27	7 22	7 17	6 53	6 33	6 13
21	8 31	8 26	8 20	8 15	8 10	8 04	7 59	7 54	7 49	7 44	7 39	7 35	7 30	7 25	7 01	6 41	6 21
22	8 39	8 34	8 28	8 23	8 18	8 12	8 07	8 02	7 57	7 52	7 47	7 43	7 38	7 33	7 09	6 49	6 29
23	8 46	8 41	8 35	8 30	8 25	8 19	8 14	8 09	8 04	7 59	7 54	7 50	7 45	7 40	7 16	6 56	6 36
24	8 52	8 47	8 41	8 36	8 31	8 25	8 20	8 15	8 10	8 05	8 00	7 56	7 51	7 46	7 22	7 02	6 42
25	8 58	8 53	8 47	8 42	8 37	8 31	8 26	8 21	8 16	8 11	8 06	8 02	7 57	7 52	7 28	7 08	6 48
26	9 03	8 58	8 52	8 47	8 42	8 36	8 31	8 26	8 21	8 16	8 11	8 07	8 02	7 57	7 33	7 13	6 53
27	9 08	9 03	8 57	8 52	8 47	8 41	8 36	8 31	8 26	8 21	8 16	8 12	8 07	8 02	7 38	7 18	6 58
28	9 13	9 08	9 02	8 57	8 52	8 46	8 41	8 36	8 31	8 26	8 21	8 17	8 12	8 07	7 43	7 23	7 03
29	9 17	9 12	9 06	9 01	8 56	8 50	8 45	8 40	8 35	8 30	8 25	8 21	8 16	8 11	7 47	7 27	7 07
30	9 21	9 16	9 10	9 05	9 00	8 54	8 49	8 44	8 39	8 34	8 29	8 25	8 20	8 15	7 51	7 31	7 11
31	9 25	9 20	9 14	9 09	9 04	8 58	8 53	8 48	8 43	8 38	8 33	8 29	8 24	8 19	7 55	7 35	7 15
32	9 29	9 24	9 18	9 13	9 08	9 02	8 57	8 52	8 47	8 42	8 37	8 33	8 28	8 23	7 59	7 39	7 19
33	9 32	9 27	9 21	9 16	9 11	9 05	9 00	8 55	8 50	8 45	8 40	8 36	8 31	8 26	8 02	7 42	7 22
34	9 36	9 31	9 25	9 20	9 15	9 09	9 04	8 59	8 54	8 49	8 44	8 40	8 35	8 30	8 06	7 46	7 26

35	9 39	9 34	9 28	9 23	9 18	9 12	9 07	9 02	8 57	8 52	8 47	8 43	8 38	8 33	8 09	7 49	7 29
36	9 41	9 36	9 30	9 25	9 20	9 14	9 09	9 04	8 59	8 54	8 49	8 45	8 40	8 85	8 11	7 51	7 31
37	9 44	9 39	9 33	9 28	9 23	9 17	9 12	9 07	9 02	8 57	8 52	8 48	8 43	8 88	8 14	7 54	7 34
38	9 46	9 41	9 35	9 30	9 25	9 19	9 14	9 09	9 04	8 59	8 54	8 50	8 45	8 40	8 16	7 56	7 36
39	9 49	9 44	9 38	9 33	9 28	9 22	9 17	9 12	9 07	9 02	8 57	8 53	8 48	8 43	8 19	7 59	7 39
40	9 52	9 47	9 41	9 36	9 31	9 25	9 20	9 15	9 10	9 05	9 00	8 56	8 51	8 46	8 22	8 02	7 42
42	9 56	9 51	9 45	9 40	9 35	9 29	9 24	9 19	9 14	9 09	9 04	9 00	8 55	8 50	8 26	8 06	7 46
44	10 01	9 56	9 50	9 45	9 40	9 34	9 29	9 24	9 19	9 14	9 09	9 05	9 00	8 55	8 31	8 11	7 51
46	10 04	9 59	9 53	9 48	9 43	9 37	9 32	9 27	9 22	9 17	9 12	9 08	9 03	8 58	8 34	8 14	7 54
48	10 07	10 02	9 56	9 51	9 46	9 40	9 35	9 30	9 25	9 20	9 15	9 11	9 06	9 01	8 37	8 17	7 57
50	10 11	10 06	10 00	9 55	9 50	9 44	9 39	9 34	9 29	9 24	9 19	9 15	9 10	9 05	8 41	8 21	8 01
52	10 14	10 09	10 03	9 58	9 53	9 47	9 42	9 37	9 32	9 27	9 22	9 18	9 13	9 08	8 44	8 24	8 04
54	10 17	10 12	10 06	10 01	9 56	9 50	9 45	9 40	9 35	9 30	9 25	9 21	9 16	9 11	8 47	8 27	8 07
56	10 20	10 15	10 09	10 04	9 59	9 53	9 48	9 43	9 38	9 33	9 28	9 24	9 19	9 14	8 50	8 30	8 10
58	10 22	10 17	10 11	10 06	10 01	9 55	9 50	9 45	9 40	9 35	9 30	9 26	9 21	9 16	8 52	8 32	8 12
60	10 24	10 19	10 13	10 08	10 03	9 57	9 52	9 47	9 42	9 37	9 32	9 28	9 23	9 18	8 54	8 34	8 14
65	10 31	10 26	10 20	10 15	10 10	10 04	9 59	9 54	9 49	9 44	9 39	9 35	9 30	9 25	9 01	8 41	8 21
70	10 36	10 31	10 25	10 20	10 15	10 09	10 04	9 59	9 54	9 49	9 44	9 40	9 35	9 30	9 06	8 46	8 26
75	10 40	10 35	10 29	10 24	10 19	10 13	10 08	10 03	9 58	9 53	9 48	9 44	9 39	9 34	9 10	8 50	8 30
80	10 46	10 41	10 35	10 30	10 25	10 19	10 14	10 09	10 04	9 59	9 54	9 50	9 45	9 40	9 16	8 56	8 36
85	10 50	10 45	10 39	10 34	10 29	10 23	10 18	10 13	10 08	10 03	9 58	9 54	9 49	9 44	9 20	9 00	8 40
90	10 54	10 49	10 43	10 38	10 33	10 27	10 22	10 17	10 12	10 07	10 02	9 58	9 53	9 48	9 24	9 04	8 44

CORRECTION OF THE SEMI-DIAMETER.

Jan. 1 +18″ " 10 +18″ " 20 +17″	Feb. 1 +16″ " 10 +14″ " 20 +12″	Mar. 1 +10″ " 10 + 8″ " 20 + 5″	April 1 +2″ " 10 —1″ " 20 —3″	May 1 — 6″ " 10 — 8″ " 20 —10″	June 1 —12″ " 10 —13″ " 20 —14″
July 1 —14″ " 10 —14″ " 20 —14″	Aug. 1 —13″ " 10 —11″ " 20 —10″	Sept. 1 —7″ " 10 —5″ " 20 —2″	Oct. 1 +1″ " 10 +3″ " 20 +6″	Nov. 1 + 9″ " 10 +11″ " 20 +13″	Dec. 1 +15″ " 10 +17″ " 20 +17″

TABLE III.

Correction of a Star's Altitude for Dip and Refraction.

[Correction to be subtracted from observed altitude.]

HEIGHT OF THE EYE ABOVE THE SEA IN FEET.

Alt. of Star.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
7	10 30	10 39	10 48	10 56	11 04	11 12	11 19	11 26	11 33	11 40	11 47	11 53	12 00	12 06	12 12	12 18	12 24
8	9 39	9 48	9 57	10 05	10 13	10 21	10 28	10 35	10 42	10 49	10 56	11 02	11 09	11 15	11 21	11 27	11 33
9	8 59	9 08	9 17	9 25	9 33	9 41	9 48	9 55	10 02	10 09	10 16	10 22	10 29	10 35	10 41	10 47	10 53
10	8 25	8 34	8 43	8 51	8 59	9 07	9 14	9 21	9 28	9 35	9 42	9 48	9 55	10 01	10 07	10 13	10 19
11	7 57	8 06	8 15	8 23	8 31	8 39	8 46	8 53	9 00	9 07	9 14	9 20	9 27	9 33	9 39	9 45	9 51
12	7 34	7 43	7 52	8 00	8 08	8 16	8 23	8 30	8 37	8 44	8 51	8 57	9 04	9 10	9 16	9 22	9 28
13	7 13	7 22	7 31	7 39	7 47	7 55	8 02	8 09	8 16	8 23	8 30	8 36	8 43	8 49	8 55	9 01	9 07
14	6 56	7 05	7 14	7 22	7 30	7 38	7 45	7 52	7 59	8 06	8 13	8 19	8 26	8 32	8 38	8 44	8 50
15	6 40	6 49	6 58	7 06	7 14	7 22	7 29	7 36	7 43	7 50	7 57	8 03	8 10	8 16	8 22	8 28	8 34
16	6 27	6 36	6 45	6 53	7 01	7 09	7 16	7 23	7 30	7 37	7 44	7 50	7 57	8 03	8 09	8 15	8 21
17	6 14	6 23	6 32	6 40	6 48	6 56	7 03	7 10	7 17	7 24	7 31	7 37	7 44	7 50	7 56	8 02	8 08
18	6 04	6 13	6 22	6 30	6 38	6 46	6 53	7 00	7 07	7 14	7 21	7 27	7 34	7 40	7 46	7 52	7 58
19	5 54	6 03	6 12	6 20	6 28	6 36	6 43	6 50	6 57	7 04	7 11	7 17	7 24	7 30	7 36	7 42	7 48
20	5 45	5 54	6 03	6 11	6 19	6 27	6 34	6 41	6 48	6 55	7 02	7 08	7 15	7 21	7 27	7 33	7 39
21	5 37	5 46	5 55	6 03	6 11	6 19	6 26	6 33	6 40	6 47	6 54	7 00	7 07	7 13	7 19	7 25	7 31
22	5 29	5 38	5 47	5 55	6 03	6 11	6 18	6 25	6 32	6 39	6 46	6 52	6 59	7 05	7 11	7 17	7 23
23	5 22	5 31	5 40	5 48	5 56	6 04	6 11	6 18	6 25	6 32	6 39	6 45	6 52	6 58	7 04	7 10	7 16
24	5 16	5 25	5 34	5 42	5 50	5 58	6 05	6 12	6 19	6 26	6 33	6 39	6 46	6 52	6 58	7 04	7 10
25	5 10	5 19	5 28	5 36	5 44	5 52	5 59	6 06	6 13	6 20	6 27	6 33	6 40	6 46	6 52	6 58	7 04
26	5 05	5 14	5 23	5 31	5 39	5 47	5 54	6 01	6 08	6 15	6 22	6 28	6 35	6 41	6 47	6 53	6 59
27	5 00	5 09	5 18	5 26	5 34	5 42	5 49	5 56	6 03	6 10	6 17	6 23	6 30	6 36	6 42	6 48	6 54
28	4 55	5 04	5 13	5 21	5 29	5 37	5 44	5 51	5 58	6 05	6 12	6 18	6 25	6 31	6 37	6 43	6 49
29	4 51	5 00	5 09	5 17	5 25	5 33	5 40	5 47	5 54	6 01	6 08	6 14	6 21	6 27	6 33	6 39	6 45

30	4 47	4 56	5 05	5 13	5 21	5 29	5 36	5 43	5 50	5 57	6 04	6 10	6 17	6 23	6 29	6 35	6 41
31	4 43	4 52	5 01	5 09	5 17	5 25	5 32	5 39	5 46	5 53	6 00	6 06	6 13	6 19	6 25	6 31	6 37
32	4 39	4 48	4 57	5 05	5 13	5 21	5 28	5 35	5 42	5 49	5 56	6 02	6 09	6 15	6 21	6 27	6 33
33	4 36	4 45	4 54	5 02	5 10	5 18	5 25	5 32	5 39	5 46	5 53	5 59	6 06	6 12	6 18	6 24	6 30
34	4 32	4 41	4 50	4 58	5 06	5 14	5 21	5 28	5 35	5 42	5 49	5 55	6 02	6 08	6 14	6 20	6 26
35	4 29	4 38	4 47	4 55	5 03	5 11	5 18	5 25	5 32	5 39	5 46	5 52	5 59	6 05	6 11	6 17	6 23
36	4 26	4 35	4 44	4 52	5 00	5 08	5 15	5 22	5 29	5 36	5 43	5 49	5 56	6 02	6 08	6 14	6 20
37	4 23	4 32	4 41	4 49	4 57	5 05	5 12	5 19	5 26	5 33	5 40	5 46	5 53	5 59	6 05	6 11	6 17
38	4 21	4 30	4 39	4 47	4 55	5 03	5 10	5 17	5 24	5 31	5 38	5 44	5 51	5 57	6 03	6 09	6 15
39	4 18	4 27	4 36	4 44	4 52	5 00	5 07	5 14	5 21	5 28	5 35	5 41	5 48	5 54	6 00	6 06	6 12
40	4 15	4 24	4 33	4 41	4 49	4 57	5 04	5 11	5 18	5 25	5 32	5 38	5 45	5 51	5 57	6 03	6 09
42	4 11	4 20	4 29	4 37	4 45	4 53	5 00	5 07	5 14	5 21	5 28	5 34	5 41	5 47	5 53	5 59	6 05
44	4 06	4 15	4 24	4 32	4 40	4 48	4 55	5 02	5 09	5 16	5 23	5 29	5 36	5 42	5 48	5 54	6 00
46	4 02	4 11	4 20	4 28	4 36	4 44	4 51	4 58	5 05	5 12	5 19	5 25	5 32	5 38	5 44	5 50	5 56
48	3 59	4 08	4 17	4 25	4 33	4 41	4 48	4 55	5 02	5 09	5 16	5 22	5 29	5 35	5 41	5 47	5 53
50	3 55	4 04	4 13	4 21	4 29	4 37	4 44	4 51	4 58	5 05	5 12	5 18	5 25	5 31	5 37	5 43	5 49
52	3 52	4 01	4 10	4 18	4 26	4 34	4 41	4 48	4 55	5 02	5 09	5 15	5 22	5 28	5 34	5 40	5 46
54	3 48	3 57	4 06	4 14	4 22	4 30	4 37	4 44	4 51	4 58	5 05	5 11	5 18	5 24	5 30	5 36	5 42
56	3 45	3 54	4 03	4 11	4 19	4 27	4 34	4 41	4 48	4 55	5 02	5 08	5 15	5 21	5 27	5 33	5 39
58	3 42	3 51	4 00	4 08	4 16	4 24	4 31	4 38	4 45	4 52	4 59	5 05	5 12	5 18	5 24	5 30	5 36
60	3 40	3 49	3 58	4 06	4 14	4 22	4 29	4 36	4 43	4 50	4 57	5 03	5 10	5 16	5 22	5 28	5 34
65	3 33	3 42	3 51	3 50	4 07	4 15	4 22	4 29	4 36	4 43	4 50	4 56	5 03	5 09	5 15	5 21	5 27
70	3 27	3 36	3 45	3 53	4 01	4 09	4 16	4 23	4 30	4 37	4 44	4 50	4 57	5 03	5 09	5 15	5 21
75	3 22	3 31	3 40	3 48	3 56	4 04	4 11	4 18	4 25	4 32	4 39	4 45	4 52	4 58	5 04	5 10	5 16
80	3 16	3 25	3 34	3 42	3 50	3 58	4 05	4 12	4 19	4 26	4 33	4 39	4 46	4 52	4 58	5 04	5 10
85	3 11	3 20	3 29	3 37	3 45	3 53	4 00	4 07	4 14	4 21	4 28	4 34	4 41	4 47	4 53	4 59	5 05
90	3 06	3 15	3 24	3 32	3 40	3 48	3 55	4 02	4 09	4 16	4 23	4 29	4 36	4 42	4 48	4 54	5 00

TABLE III.—Continued.

Correction of a Star's Altitude for Dip and Refraction.

[Correction to be subtracted from observed altitude.]

HEIGHT OF THE EYE ABOVE THE SEA IN FEET.

Alt. of Star.	27	28	29	30	31	32	33	34	35	36	37	38	39	40	45	50	55
°	′ ″	′ ″	″	′ ′	′ ″	′ ′	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
7	12 30	12 35	12 41	12 46	12 51	12 57	13 02	13 07	13 12	13 17	13 22	13 26	13 31	13 36	14 00	14 20	14 40
8	11 39	11 44	11 50	11 55	12 00	12 06	12 11	12 16	12 21	12 26	12 31	12 35	12 40	12 45	13 09	13 29	13 49
9	10 59	11 04	11 10	11 15	11 20	11 26	11 31	11 36	11 41	11 46	11 51	11 55	12 00	12 05	12 29	12 49	13 09
10	10 25	10 30	10 36	10 41	10 46	10 52	10 57	11 02	11 07	11 12	11 17	11 21	11 26	11 31	11 55	12 15	12 35
11	9 57	10 02	10 08	10 13	10 18	10 24	10 29	10 34	10 39	10 44	10 49	10 53	10 58	11 03	11 27	11 47	12 07
12	9 34	9 39	9 45	9 50	9 55	10 01	10 06	10 11	10 16	10 21	10 26	10 30	10 35	10 40	11 04	11 24	11 44
13	9 13	9 18	9 24	9 29	9 34	9 40	9 45	9 50	9 55	10 00	10 05	10 09	10 14	10 19	10 43	11 03	11 23
14	8 56	9 01	9 07	9 12	9 17	9 23	9 28	9 33	9 38	9 43	9 48	9 52	9 57	10 02	10 26	10 46	11 06
15	8 40	8 45	8 51	8 56	9 01	9 07	9 12	9 17	9 22	9 27	9 32	9 36	9 41	9 46	10 10	10 30	10 50
16	8 27	8 32	8 38	8 43	8 48	8 54	8 59	9 04	9 09	9 14	9 19	9 23	9 28	9 33	9 57	10 17	10 37
17	8 14	8 19	8 25	8 30	8 35	8 41	8 46	8 51	8 56	9 01	9 06	9 10	9 15	9 20	9 44	10 04	10 24
18	8 04	8 09	8 15	8 20	8 25	8 31	8 36	8 41	8 46	8 51	8 56	9 00	9 05	9 10	9 34	9 54	10 14
19	7 54	7 59	8 05	8 10	8 15	8 21	8 26	8 31	8 36	8 41	8 46	8 50	8 55	9 00	9 24	9 44	10 04
20	7 45	7 50	7 56	8 01	8 06	8 12	8 17	8 22	8 27	8 32	8 37	8 41	8 46	8 51	9 15	9 35	9 55
21	7 37	7 42	7 48	7 53	7 58	8 04	8 09	8 14	8 19	8 24	8 29	8 33	8 38	8 43	9 07	9 27	9 47
22	7 29	7 34	7 40	7 45	7 50	7 56	8 01	8 06	8 11	8 16	8 21	8 25	8 30	8 35	8 59	9 19	9 39
23	7 22	7 27	7 33	7 38	7 43	7 49	7 54	7 59	8 04	8 09	8 14	8 18	8 23	8 28	8 52	9 12	9 32
24	7 16	7 21	7 27	7 32	7 37	7 43	7 48	7 53	7 58	8 03	8 08	8 12	8 17	8 22	8 46	9 06	9 26
25	7 10	7 15	7 21	7 26	7 31	7 37	7 42	7 47	7 52	7 57	8 02	8 06	8 11	8 16	8 40	9 00	9 20
26	7 05	7 10	7 16	7 21	7 26	7 32	7 37	7 42	7 47	7 52	7 57	8 01	8 06	8 11	8 35	8 55	9 15
27	7 00	7 05	7 11	7 16	7 21	7 27	7 32	7 37	7 42	7 47	7 52	7 56	8 01	8 06	8 30	8 50	9 10
28	6 55	7 00	7 06	7 11	7 16	7 22	7 27	7 32	7 37	7 42	7 47	7 51	7 56	8 01	8 25	8 45	9 05
29	6 51	6 56	7 02	7 07	7 12	7 18	7 23	7 28	7 33	7 38	7 43	7 47	7 52	7 57	8 21	8 41	9 01

30	6 47	6 52	6 58	7 03	7 08	7 14	7 19	7 24	7 29	7 34	7 39	7 43	7 48	7 53	8 17	8 37	8 57
31	6 43	6 48	6 54	6 59	7 04	7 10	7 15	7 20	7 25	7 30	7 35	7 39	7 44	7 49	8 13	8 33	8 53
32	6 39	6 44	6 50	6 55	7 00	7 06	7 11	7 16	7 21	7 26	7 31	7 35	7 40	7 45	8 09	8 29	8 49
33	6 36	6 41	6 47	6 52	6 57	7 03	7 08	7 13	7 18	7 23	7 28	7 32	7 37	7 42	8 06	8 26	8 46
34	6 32	6 37	6 43	6 48	6 53	6 59	7 04	7 09	7 14	7 19	7 24	7 28	7 33	7 38	8 02	8 22	8 42
35	6 29	6 34	6 40	6 45	6 50	6 56	7 01	7 06	7 11	7 16	7 21	7 25	7 30	7 35	7 59	8 19	8 39
36	6 26	6 31	6 37	6 42	6 47	6 53	6 58	7 03	7 08	7 13	7 18	7 22	7 27	7 32	7 56	8 16	8 36
37	6 23	6 28	6 34	6 39	6 44	6 50	6 55	7 00	7 05	7 10	7 15	7 19	7 24	7 29	7 53	8 13	8 33
38	6 21	6 26	6 32	6 37	6 42	6 48	6 53	6 58	7 03	7 08	7 13	7 17	7 22	7 27	7 51	8 11	8 31
39	6 18	6 23	6 29	6 34	6 39	6 45	6 50	6 55	7 00	7 05	7 10	7 14	7 19	7 24	7 48	8 08	8 28
40	6 15	6 20	6 26	6 31	6 36	6 42	6 47	6 52	6 57	7 02	7 07	7 11	7 16	7 21	7 45	8 05	8 25
42	6 11	6 16	6 22	6 27	6 32	6 38	6 43	6 48	6 53	6 58	7 03	7 07	7 12	7 17	7 41	8 01	8 21
44	6 06	6 11	6 17	6 22	6 27	6 33	6 38	6 43	6 48	6 53	6 58	7 02	7 07	7 12	7 36	7 56	8 16
46	6 02	6 07	6 13	6 18	6 23	6 29	6 34	6 39	6 44	6 49	6 54	6 58	7 03	7 08	7 32	7 52	8 12
48	5 59	6 04	6 10	6 15	6 20	6 26	6 31	6 36	6 41	6 46	6 51	6 55	7 00	7 05	7 29	7 49	8 09
50	5 55	6 00	6 06	6 11	6 16	6 22	6 27	6 32	6 37	6 42	6 47	6 51	6 56	7 01	7 25	7 45	8 05
52	5 52	5 57	6 03	6 08	6 13	6 19	6 24	6 29	6 34	6 39	6 44	6 48	6 53	6 58	7 22	7 42	8 02
54	5 48	5 53	5 59	6 04	6 09	6 15	6 20	6 25	6 30	6 35	6 40	6 44	6 49	6 54	7 18	7 38	7 58
56	5 45	5 50	5 56	6 01	6 06	6 12	6 17	6 22	6 27	6 32	6 37	6 41	6 46	6 51	7 15	7 35	7 55
58	5 42	5 47	5 53	5 58	6 03	6 09	6 14	6 19	6 24	6 29	6 34	6 38	6 43	6 48	7 12	7 32	7 52
60	5 40	5 45	5 51	5 56	6 01	6 07	6 12	6 17	6 22	6 27	6 32	6 36	6 41	6 46	7 10	7 30	7 50
65	5 33	5 38	5 44	5 49	5 54	6 00	6 05	6 10	6 15	6 20	6 25	6 29	6 34	6 39	7 03	7 23	7 43
70	5 27	5 32	5 38	5 43	5 48	5 54	5 59	6 04	6 09	6 14	6 19	6 23	6 28	6 33	6 57	7 17	7 37
75	5 22	5 27	5 33	5 38	5 43	5 49	5 54	5 59	6 04	6 09	6 14	6 18	6 23	6 28	6 52	7 12	7 32
80	5 16	5 21	5 27	5 32	5 37	5 43	5 48	5 53	5 58	6 03	6 08	6 12	6 17	6 22	6 46	7 06	7 26
85	5 11	5 16	5 22	5 27	5 32	5 38	5 43	5 48	5 53	5 58	6 03	6 07	6 12	6 17	6 41	7 01	7 21
90	5 06	5 11	5 17	5 22	5 27	5 33	5 38	5 43	5 48	5 53	5 58	6 02	6 07	6 12	6 36	6 56	7 16

TABLE IV.

USED FOR CALCULATION OF COEFFICIENTS B, C, D, AND E.

PRODUCTS OF ARCS MULTIPLIED BY THE SINES OF 15° RHUMBS.

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
0 0	0 0	0 0	0 0	0 0	0 0	0 0
0 10	0 3	0 5	0 7	0 9	0 10	0 10
0 20	0 5	0 10	0 14	0 17	0 19	0 20
0 30	0 8	0 15	0 21	0 26	0 29	0 30
0 40	0 10	0 20	0 28	0 35	0 39	0 40
0 50	0 13	0 25	0 35	0 43	0 48	0 50
1 0	0 16	0 30	0 42	0 52	0 58	1 0
1 10	0 18	0 35	0 49	1 1	1 8	1 10
1 20	0 21	0 40	0 57	1 9	1 17	1 20
1 30	0 23	0 45	1 4	1 18	1 27	1 30
1 40	0 26	0 50	1 11	1 27	1 37	1 40
1 50	0 28	0 55	1 18	1 35	1 46	1 50
2 0	0 31	1 0	1 25	1 44	1 56	2 0
2 10	0 34	1 5	1 32	1 53	2 6	2 10
2 20	0 36	1 10	1 39	2 1	2 15	2 20
2 30	0 39	1 15	1 46	2 10	2 25	2 30
2 40	0 41	1 20	1 53	2 19	2 35	2 40
2 50	0 44	1 25	2 0	2 27	2 44	2 50
3 0	0 47	1 30	2 7	2 36	2 54	3 0
3 10	0 49	1 35	2 14	2 45	3 04	3 10
3 20	0 52	1 40	2 21	2 53	3 13	3 20
3 30	0 54	1 45	2 29	3 2	3 23	3 30
3 40	0 57	1 50	2 36	3 11	3 33	3 40
3 50	1 0	1 55	2 43	3 19	3 42	3 50
4 0	1 2	2 0	2 50	3 28	3 52	4 0
4 10	1 5	2 5	2 57	3 37	4 1	4 10
4 20	1 7	2 10	3 4	3 45	4 11	4 20
4 30	1 10	2 15	3 11	3 54	4 21	4 30
4 40	1 12	2 20	3 18	4 2	4 30	4 40
4 50	1 15	2 25	3 25	4 11	4 40	4 50

TABLE IV.—Products of Arcs Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
5 0	1 18	2 30	3 32	4 20	4 50	**5 0**
5 10	1 20	2 35	3 39	4 28	4 59	**5 10**
5 20	1 23	2 40	3 46	4 37	5 9	**5 20**
5 30	1 25	2 45	3 53	4 46	5 19	**5 30**
5 40	1 28	2 50	4 0	4 54	5 28	**5 40**
5 50	1 31	2 55	4 7	5 3	5 38	**5 50**
6 0	1 33	3 0	4 15	5 12	5 48	**6 0**
6 10	1 36	3 5	4 22	5 20	5 57	**6 10**
6 20	1 38	3 10	4 29	5 29	6 7	**6 20**
6 30	1 41	3 15	4 36	5 38	6 17	**6 30**
6 40	1 44	3 20	4 43	5 46	6 26	**6 40**
6 50	1 46	3 25	4 50	5 55	6 36	**6 50**
7 0	1 49	3 30	4 57	6 4	6 46	**7 0**
7 10	1 51	3 35	5 4	6 12	6 55	**7 10**
7 20	1 54	3 40	5 11	6 21	7 5	**7 20**
7 30	1 56	3 45	5 18	6 30	7 15	**7 30**
7 40	1 59	3 50	5 25	6 38	7 24	**7 40**
7 50	2 2	3 55	5 32	6 47	7 34	**7 50**
8 0	2 4	4 0	5 39	6 56	7 44	**8 0**
8 10	2 7	4 5	5 46	7 4	7 53	**8 10**
8 20	2 9	4 10	5 54	7 13	8 3	**8 20**
8 30	2 12	4 15	6 1	7 22	8 13	**8 30**
8 40	2 15	4 20	6 8	7 30	8 22	**8 40**
8 50	2 17	4 25	6 15	7 39	8 32	**8 50**
9 0	2 20	4 30	6 22	7 48	8 42	**9 0**
9 10	2 22	4 35	6 29	7 56	8 51	**9 10**
9 20	2 25	4 40	6 36	8 5	9 1	**9 20**
9 30	2 28	4 45	6 43	8 14	9 11	**9 30**
9 40	2 30	4 50	6 50	8 22	9 20	**9 40**
9 50	2 33	4 55	6 57	8 31	9 30	**9 50**

TABLE IV.—Products of Arcs
Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
10 0	2 35	5 0	7 4	8 40	9 40	10 0
10 10	2 38	5 5	7 11	8 48	9 49	10 10
10 20	2 40	5 10	7 18	8 57	9 59	10 20
10 30	2 43	5 15	7 25	9 6	10 9	10 30
10 40	2 46	5 20	7 33	9 14	10 18	10 40
10 50	2 48	5 25	7 40	9 23	10 28	10 50
11 0	2 51	5 30	7 47	9 32	10 38	11 0
11 10	2 53	5 35	7 54	9 40	10 47	11 10
11 20	2 56	5 40	8 1	9 49	10 57	11 20
11 30	2 59	5 45	8 8	9 58	11 6	11 30
11 40	3 1	5 50	8 15	10 6	11 16	11 40
11 50	3 4	5 55	8 22	10 15	11 26	11 50
12 0	3 6	6 0	8 29	10 24	11 35	12 0
12 10	3 9	6 5	8 36	10 32	11 45	12 10
12 20	3 12	6 10	8 43	10 41	11 55	12 20
12 30	3 14	6 15	8 50	10 50	12 4	12 30
12 40	3 17	6 20	8 57	10 58	12 14	12 40
12 50	3 19	6 25	9 4	11 7	12 24	12 50
13 0	3 22	6 30	9 12	11 16	12 33	13 0
13 10	3 24	6 35	9 19	11 24	12 43	13 10
13 20	3 27	6 40	9 26	11 33	12 53	13 20
13 30	3 30	6 45	9 33	11 41	13 2	13 30
13 40	3 32	6 50	9 40	11 50	13 12	13 40
13 50	3 35	6 55	9 47	11 59	13 22	13 50
14 0	3 37	7 0	9 54	12 7	13 31	14 0
14 10	3 40	7 5	10 1	12 16	13 41	14 10
14 20	3 43	7 10	10 8	12 25	13 51	14 20
14 30	3 45	7 15	10 15	12 33	14 0	14 30
14 40	3 48	7 20	10 22	12 42	14 10	14 40
14 50	3 50	7 25	10 29	12 51	14 20	14 50

TABLE IV.—Products of Arcs
Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
15 0	3 53	7 30	10 36	12 59	14 29	15 0
15 10	3 56	7 35	10 43	13 8	14 39	15 10
15 20	3 58	7 40	10 51	13 17	14 49	15 20
15 30	4 1	7 45	10 58	13 25	14 58	15 30
15 40	4 3	7 50	11 5	13 34	15 8	15 40
15 50	4 6	7 55	11 12	13 43	15 18	15 50
16 0	4 8	8 0	11 19	13 51	15 27	16 0
16 10	4 11	8 5	11 26	14 0	15 37	16 10
16 20	4 14	8 10	11 33	14 9	15 47	16 20
16 30	4 16	8 15	11 40	14 17	15 56	16 30
16 40	4 19	8 20	11 47	14 26	16 6	16 40
16 50	4 21	8 25	11 54	14 35	16 16	16 50
17 0	4 24	8 30	12 1	14 43	16 25	17 0
17 10	4 27	8 35	12 8	14 52	16 35	17 10
17 20	4 29	8 40	12 15	15 1	16 45	17 20
17 30	4 32	8 45	12 22	15 9	16 54	17 30
17 40	4 34	8 50	12 30	15 18	17 4	17 40
17 50	4 37	8 55	12 37	15 27	17 14	17 50
18 0	4 40	9 0	12 44	15 35	17 23	18 0
18 10	4 42	9 5	12 51	15 44	17 33	18 10
18 20	4 45	9 10	12 58	15 53	17 43	18 20
18 30	4 47	9 15	13 5	16 1	17 52	18 30
18 40	4 50	9 20	13 12	16 10	18 2	18 40
18 50	4 52	9 25	13 19	16 19	18 12	18 50
19 0	4 55	9 30	13 26	16 27	18 21	19 0
19 10	4 58	9 35	13 33	16 36	18 31	19 10
19 20	5 0	9 40	13 40	16 45	18 40	19 20
19 30	5 3	9 45	13 47	16 53	18 50	19 30
19 40	5 5	9 50	13 54	17 2	19 0	19 40
19 50	5 8	9 55	14 1	17 11	19 9	19 50

TABLE IV.—Products of Arcs Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
20 0	5 11	10 0	14 9	17 19	19 19	20 0
20 10	5 13	10 5	14 16	17 28	19 29	20 10
20 20	5 16	10 10	14 23	17 37	19 38	20 20
20 30	5 18	10 15	14 30	17 45	19 48	20 30
20 40	5 21	10 20	14 37	17 54	19 58	20 40
20 50	5 24	10 25	14 44	18 3	20 7	20 50
21 0	5 26	10 30	14 51	18 11	20 17	21 0
21 10	5 29	10 35	14 58	18 20	20 27	21 10
21 20	5 31	10 40	15 5	18 29	20 36	21 20
21 30	5 34	10 45	15 12	18 37	20 46	21 30
21 40	5 36	10 50	15 19	18 46	20 56	21 40
21 50	5 39	10 55	15 26	18 54	21 5	21 50
22 0	5 42	11 0	15 33	19 3	21 15	22 0
22 10	5 44	11 5	15 40	19 12	21 25	22 10
22 20	5 47	11 10	15 48	19 20	21 34	22 20
22 30	5 49	11 15	15 55	19 29	21 44	22 30
22 40	5 52	11 20	16 2	19 38	21 54	22 40
22 50	5 55	11 25	16 9	19 46	22 3	22 50
23 0	5 57	11 30	16 16	19 55	22 13	23 0
23 10	6 0	11 35	16 23	20 4	22 23	23 10
23 20	6 2	11 40	16 30	20 12	22 32	23 20
23 30	6 5	11 45	16 37	20 21	22 42	23 30
23 40	6 8	11 50	16 44	20 30	22 52	23 40
23 50	6 10	11 55	16 51	20 38	23 1	23 50
24 0	6 13	12 0	16 58	20 47	23 11	24 0
24 10	6 15	12 5	17 5	20 56	23 21	24 10
24 20	6 18	12 10	17 12	21 4	23 30	24 20
24 30	6 20	12 15	17 19	21 13	23 40	24 30
24 40	6 23	12 20	17 27	21 22	23 50	24 40
24 50	6 26	12 25	17 34	21 30	23 59	24 50

TABLE IV.—Products of Arcs Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
25 0	6 28	12 30	17 41	21 39	24 9	**25 0**
25 10	6 31	12 35	17 48	21 48	24 18	**25 10**
25 20	6 33	12 40	17 55	21 56	24 28	**25 20**
25 30	6 36	12 45	18 2	22 5	24 38	**25 30**
25 40	6 39	12 50	18 9	22 14	24 48	**25 40**
25 50	6 41	12 55	18 16	22 22	24 57	**25 50**
26 0	6 44	13 0	18 23	22 31	25 7	**26 0**
26 10	6 46	13 5	18 30	22 40	25 17	**26 10**
26 20	6 49	13 10	18 37	22 48	25 26	**26 20**
26 30	6 52	13 15	18 44	22 57	25 36	**26 30**
26 40	6 54	13 20	18 51	23 6	25 45	**26 40**
26 50	6 57	13 25	18 58	23 14	25 55	**26 50**
27 0	6 59	13 30	19 6	23 23	26 5	**27 0**
27 10	7 2	13 35	19 13	23 32	26 14	**27 10**
27 20	7 4	13 40	19 20	23 40	26 24	**27 20**
27 30	7 7	13 45	19 27	23 49	26 34	**27 30**
27 40	7 10	13 50	19 34	23 58	26 43	**27 40**
27 50	7 12	13 55	19 41	24 6	26 53	**27 50**
28 0	7 15	14 0	19 48	24 15	27 3	**28 0**
28 10	7 17	14 5	19 55	24 24	27 12	**28 10**
28 20	7 20	14 10	20 2	24 32	27 22	**28 20**
28 30	7 23	14 15	20 9	24 41	27 32	**28 30**
28 40	7 25	14 20	20 16	24 50	27 41	**28 40**
28 50	7 28	14 25	20 23	24 58	27 51	**28 50**
29 0	7 30	14 30	20 30	25 7	28 1	**29 0**
29 10	7 33	14 35	20 37	25 16	28 10	**29 10**
29 20	7 36	14 40	20 45	25 24	28 20	**29 20**
29 30	7 38	14 45	20 52	25 33	28 30	**29 30**
29 40	7 41	14 50	20 59	25 42	28 39	**29 40**
29 50	7 43	14 55	21 6	25 50	28 49	**29 50**

TABLE IV.—Products of Arcs

Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
30 0	7 46	15 0	21 13	25 59	28 59	30 0
30 10	7 48	15 5	21 20	26 8	29 8	30 10
30 20	7 51	15 10	21 27	26 16	29 18	30 20
30 30	7 54	15 15	21 34	26 25	29 28	30 30
30 40	7 56	15 20	21 41	26 33	29 37	30 40
30 50	7 59	15 25	21 48	26 42	29 47	30 50
31 0	8 1	15 30	21 55	26 51	29 57	31 0
31 10	8 4	15 35	22 2	26 59	30 6	31 10
31 20	8 7	15 40	22 9	27 8	30 16	31 20
31 30	8 9	15 45	22 16	27 17	30 26	31 30
31 40	8 12	15 50	22 24	27 25	30 35	31 40
31 50	8 14	15 55	22 31	27 34	30 45	31 50
32 0	8 17	16 0	22 38	27 43	30 55	32 0
32 10	8 20	16 5	22 45	27 51	31 4	32 10
32 20	8 22	16 10	22 52	28 0	31 14	32 20
32 30	8 25	16 15	22 59	28 9	31 24	32 30
32 40	8 27	16 20	23 6	28 17	31 33	32 40
32 50	8 30	16 25	23 13	28 26	31 43	32 50
33 0	8 32	16 30	23 20	28 35	31 53	33 0
33 10	8 35	16 35	23 27	28 43	32 2	33 10
33 20	8 38	16 40	23 34	28 52	32 12	33 20
33 30	8 40	16 45	23 41	29 1	32 22	33 30
33 40	8 43	16 50	23 48	29 9	32 31	33 40
33 50	8 45	16 55	23 55	29 18	32 41	33 50
34 0	8 48	17 0	24 3	29 27	32 50	34 0
34 10	8 51	17 5	24 10	29 35	33 0	34 10
34 20	8 53	17 10	24 17	29 44	33 10	34 20
34 30	8 56	17 15	24 24	29 53	33 19	34 30
34 40	8 58	17 20	24 31	30 1	33 29	34 40
34 50	9 1	17 25	24 38	30 10	33 39	34 50

TABLE IV.—Products of Arcs Multiplied by the Sines of 15° Rhumbs.—(*continued*).

ARCS.	S_1 Sin. 15°	S_2 Sin. 30°	S_3 Sin. 45°	S_4 Sin. 60°	S_5 Sin. 75°	ARCS.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
35 0	9 4	17 30	24 45	30 19	33 48	35 0
35 10	9 6	17 35	24 52	30 27	33 58	35 10
35 20	9 9	17 40	24 59	30 36	34 8	35 20
35 30	9 11	17 45	25 6	30 45	34 17	35 30
35 40	9 14	17 50	25 13	30 53	34 27	35 40
35 50	9 16	17 55	25 20	31 2	34 37	35 50
36 0	9 19	18 0	25 27	31 11	34 46	36 0
36 10	9 22	18 5	25 34	31 19	34 56	36 10
36 20	9 24	18 10	25 41	31 28	35 6	36 20
36 30	9 27	18 15	25 49	31 37	35 15	36 30
36 40	9 29	18 20	25 56	31 45	35 25	36 40
36 50	9 32	18 25	26 3	31 54	35 35	36 50
37 0	9 35	18 30	26 10	32 3	35 44	37 0
37 10	9 37	18 35	26 17	32 11	35 54	37 10
37 20	9 40	18 40	26 24	32 20	36 4	37 20
37 30	9 42	18 45	26 31	32 29	36 13	37 30
37 40	9 45	18 50	26 38	32 37	36 23	37 40
37 50	9 48	18 55	26 45	32 46	36 33	37 50
38 0	9 50	19 0	26 52	32 55	36 42	38 0
38 10	9 53	19 5	26 59	33 3	36 52	38 10
38 20	9 55	19 10	27 6	33 12	37 2	38 20
38 30	9 58	19 15	27 13	33 21	37 11	38 30
38 40	10 0	19 20	27 20	33 29	37 21	38 40
38 50	10 3	19 25	27 28	33 38	37 31	38 50

TABLE V.

DISTANCES FROM COMPASS AT WHICH QUADRANTAL CORRECTORS SHOULD BE PLACED FOR VALUE OF D.

FOR BINNACLES OF TYPE VI.

Distance from Compass on Graduated Quadrantal Arms.	Value of D.					
	For 7-Inch Spheres.		For 9-Inch Spheres.		For Filled Chain Boxes.	
Inches.	°	′	°	′	°	′
11	12	00	21	15		
11.5	9	15	19	00	10	30
12	7	45	17	00	8	45
12.5	6	45	14	15	7	30
13	5	45	12	00	6	15
13.5	4	45	10	00	5	15
14	4	00	8	45	4	20
14.5	3	30	7	30	4	00
15	3	00				

To find D when $\mathfrak{D}$ is given, use formula : $D = \mathfrak{D} \times 57^\circ.3$.

EXTRACTS FROM N. A., 1918.

SUN, 1918.

Day of Month.	Right Ascension of the Mean Sun at Greenwich Mean Noon.			
	January.	February.	March.	April.
	h m s	h m s	h m s	h m s
1	18 41 18.4	....	22 33 55.2	0 36 8.4
2	18 45 15.0	....	....	0 40 4.9
3	18 49 11.5	....	....	0 44 1.5
4	18 53 8.1	....	....	0 47 58.0
5	18 57 4.6	....	....	0 51 54.6
7	19 4 57.8	....	....	0 59 47.7
8	19 8 54.3	....	....	1 3 44.2
9	19 12 50.9	....	....	1 7 40.8
10	19 16 47.4	....	....	1 11 37.3
11	19 20 44.0	....	....	1 15 33.9
12	19 24 40.6	....	....	1 19 30.4
14	19 32 33.7	....	....	1 27 23.5
15	19 36 30.2	21 38 43.5	....	1 31 20.1
18	19 48 19.9	....	....	1 43 9.8
20	19 56 13.0	....	....	1 51 2.9
21	20 0 9.6	....	....	1 54 59.4
22	20 4 6.1	....	....	1 58 56.0
25	20 15 55.8	....	....	2 10 45.6
26	20 19 52.4	....	....	2 14 42.2

SUN, JANUARY, 1918.

G. M. T.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.
	Tuesday 1.		Monday 7.		Tuesday 15.		Friday 25.	
h	° ′	m s	° ′	m s	° ′	m s	° ′	m s
0	−23 3.1	−3 26.3	−22 26.6	−6 10.7	−21 13.3	−9 22.8	−19 5.4	−12 21.7
2	23 2.7	3 28.6	22 26.0	6 12.8	21 12.4	9 24.6	19 4.2	12 22.9
4	23 2.3	3 31.0	22 25.3	6 15.0	21 11.4	9 26.4	19 3.0	12 24.1
6	23 1.9	3 33.4	22 24.7	6 17.2	21 10.5	9 28.2	19 1.8	12 25.2
8	23 1.5	3 35.7	22 24.1	6 19.4	21 9.6	9 30.0	19 0.5	12 26.4
10	23 1.1	3 38.1	22 23.4	6 21.5	21 8.7	9 31.8	18 59.3	12 27.5
12	23 0.7	3 40.4	22 22.8	6 23.7	21 7.8	9 33.5	18 58.1	12 28.6
14	23 0.3	3 42.8	22 22.2	6 25.8	21 6.9	9 35.3	18 56.8	12 29.8
16	22 59.8	3 45.2	22 21.5	6 28.0	21 5.9	9 37.1	18 55.6	12 30.9
18	22 59.4	3 47.5	22 20.9	6 30.2	21 5.0	9 38.8	18 54.4	12 32.0
20	22 59.0	3 49.9	22 20.2	6 32.3	21 4.1	9 40.6	18 53.1	12 33.1
22	22 58.6	3 52.2	22 19.6	6 34.5	21 3.1	9 42.3	18 51.9	12 34.2
H. D.	0.2	1.2	0.3	1.1	0.5	0.9	0.6	0.6
	Wednesday 2.		Tuesday 8.		Friday 18.		Sunday 27.	
0	−22 58.1	−3 54.6	−22 18.9	−6 36.6	−20 38.9	−10 24.4	−18 35.5	−12 48.1
2	22 57.7	3 56.9	22 18.3	6 38.7	20 37.9	10 26.0	18 34.2	12 49.2
4	22 57.3	3 59.2	22 17.6	6 40.9	20 36.9	10 27.6	18 32.9	12 50.2
6	22 56.8	4 1.6	22 16.9	6 43.0	20 35.9	10 29.3	18 31.6	12 51.2
8	22 56.4	4 3.9	22 16.3	6 45.1	20 34.9	10 30.9	18 30.4	12 52.2
10	22 55.9	4 6.3	22 15.6	6 47.3	20 33.9	10 32.5	18 29.1	12 53.2
12	22 55.5	4 8.6	22 14.9	6 49.4	20 32.8	10 34.0	18 27.8	12 54.2
14	22 55.1	4 10.9	22 14.2	6 51.5	20 31.8	10 35.6	18 26.5	12 55.2
16	22 54.6	4 13.2	22 13.6	6 53.6	20 30.8	10 37.2	18 25.2	12 56.2
18	22 54.1	4 15.6	22 12.9	6 55.7	20 29.8	10 38.8	18 23.9	12 57.2
20	22 53.7	4 17.9	22 12.2	6 57.8	20 28.7	10 40.4	18 22.6	12 58.2
22	22 53.2	4 20.2	−22 11.5	−6 59.9	20 27.7	10 41.9	18 21.3	12 59.2
H. D.	0.2	1.2	0.3	1.1	0.5	0.8	0.6	0.5
	Thursday 3.		Thursday 10.		Sunday 20.		Wednesday 30.	
0	−22 52.7	−4 22.5	−22 2.3	−7 27.0	−20 14.1	−11 1.8	−17 48.1	−13 21.7
2	22 52.3	4 24.9	22 1.6	7 29.0	20 13.0	11 3.3	17 46.7	13 22.5
4	22 51.8	4 27.2	22 0.8	7 31.1	20 11.9	11 4.8	17 45.3	13 23.3
6	22 51.3	4 29.5	22 0.1	7 33.1	20 10.9	11 6.3	17 44.0	13 24.1
8	22 50.8	4 31.8	21 59.3	7 35.2	20 9.8	11 7.7	17 42.6	13 25.0
10	22 50.3	4 34.1	21 58.6	7 37.2	20 8.7	11 9.2	17 41.3	13 25.8
12	22 49.9	4 36.4	21 57.9	7 39.2	20 7.6	11 10.7	17 39.9	13 26.6
14	22 49.4	4 38.7	21 57.1	7 41.3	20 6.5	11 12.1	17 38.5	13 27.4
16	22 48.9	4 41.0	21 56.4	7 43.3	20 5.4	11 13.6	17 37.1	13 28.1
18	22 48.4	4 43.3	21 55.6	7 45.3	20 4.4	11 15.0	17 35.8	13 28.9
20	22 47.9	4 45.6	21 54 8	7 47.3	20 3.3	11 16.5	17 34.4	13 29.7
22	22 47.4	4 47.9	21 54.1	7 49.4	−20 2.2	−11 17.9	17 33.0	13 30.5
H. D.	0.2	1.2	0.4	1.0	0.5	0.7	0.7	0.4

SEMIDIAMETER.

Jan. 1.	Jan. 11.	Jan. 21.	Jan. 31.
′	′	′	′
16.30	16.29	16.28	16.26

NOTE.—The equation of time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, APRIL, 1918.

G. M. T.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.	Sun's Declination.	Equation of Time.
	Tuesday 2.		Friday 5.		Tuesday 9.		Thursday 25.	
	° ′	m s	° ′	m s	° ′	m s	° ′	m s
0	+4 43.2	−3 48.3	+5 52.1	−2 54.8	+7 22.6	−1 46.3	+13 0.8	+1 58.9
2	4 45.1	3 46.8	5 54.0	2 53.3	7 24.4	1 44.9	13 2.4	1 59.8
4	4 47.0	3 45.3	5 55.9	2 51.9	7 26.3	1 43.5	13 4.1	2 0.7
6	4 48.9	3 43.8	5 57.8	2 50.4	7 28.1	1 42.1	13 5.7	2 1.6
8	4 50.9	3 42.3	5 59.7	2 49.0	7 30.0	1 40.7	13 7.3	2 2.5
10	4 52.8	3 40.8	6 1.6	2 47.5	7 31.9	1 39.4	13 9.0	2 3.4
12	4 54.7	3 39.3	6 3.5	2 46.1	7 33.7	1 38.0	13 10.6	2 4.3
14	4 56.6	3 37.8	6 5.4	2 44.6	7 35.6	1 36.6	13 12.2	2 5.2
16	4 58.5	3 36.3	6 7.3	2 43.1	7 37.4	1 35.2	13 13.8	2 6.1
18	5 0.5	3 34.8	6 9.2	2 41.7	7 39.3	1 33.9	13 15.5	2 6.9
20	5 2.4	3 33.3	6 11.1	2 40.2	7 41.2	1 32.5	13 17.1	2 7.8
22	5 4.3	3 31.8	6 13.0	2 38.8	7 43.0	1 31.1	13 18.7	2 8.7
H. D.	1.0	0.7	0.9	0.7	0.9	0.7	0.8	0.4
	Wednesday 3.		Saturday 6.		Friday 19.		Friday 26.	
0	+5 6.2	−3 30.3	+6 14.9	−2 37.3	+10 59.2	+0 45.5	+13 20.3	+2 9.6
2	5 8.1	3 28.8	6 16.8	2 35.9	11 0.9	0 46.6	13 22.0	2 10.4
4	5 10.1	3 27.3	6 18.6	2 34.4	11 2.6	0 47.7	13 23.6	2 11.3
6	5 12.0	3 25.8	6 20.5	2 33.0	11 4.4	0 48.9	13 25.2	2 12.2
8	5 13.9	3 24.3	6 22.4	2 31.6	11 6.1	0 50.0	13 26.8	2 13.0
10	5 15.8	3 22.8	6 24.3	2 30.1	11 7.8	0 51.1	13 28.4	2 13.9
12	5 17.7	3 21.3	6 26.2	2 28.7	11 9.6	0 52.2	13 30.0	2 14.7
14	5 19.6	3 19.9	6 28.1	2 27.2	11 11.3	0 53.3	13 31.6	2 15.6
16	5 21.6	3 18.4	6 30.0	2 25.8	11 13.0	0 54.4	13 33.2	2 16.4
18	5 23.5	3 16.9	6 31.9	2 24.4	11 14.8	0 55.5	13 34.8	2 17.2
20	5 25.4	3 15.4	6 33.8	2 22.9	11 16.5	0 56.6	13 36.5	2 18.1
22	5 27.3	3 13.9	6 35.7	2 21.5	11 18.2	0 57.7	13 38.1	2 18.9
H. D.	1.0	0.7	0.9	0.7	0.9	0.6	0.8	0.4
	Thursday 4.		Sunday 7.		Tuesday 23.		Saturday 27.	
0	+5 29.2	−3 12.5	+6 37.5	−2 20.1	+12 21.0	+1 36.2	+13 39.7	+2 19.7
2	5 31.1	3 11.0	6 39.4	2 18.7	12 22.7	1 37.2	13 41.3	2 20.6
4	5 33.0	3 9.5	6 41.3	2 17.2	12 24.4	1 38.1	13 42.9	2 21.4
6	5 34.9	3 8.0	6 43.2	2 15.8	12 26.1	1 39.1	13 44.5	2 22.2
8	5 36.8	3 6.6	6 45.1	2 14.4	12 27.7	1 40.1	13 46.1	2 23.0
10	5 38.7	3 5.1	6 47.0	2 13.0	12 29.4	1 41.1	13 47.6	2 23.8
12	5 40.7	3 3.6	6 48.8	2 11.5	12 31.1	1 42.0	13 49.2	2 24.6
14	5 42.6	3 2.1	6 50.7	2 10.1	12 32.7	1 43.0	13 50.8	2 25.4
16	5 44.5	3 0.7	6 52.6	2 8.7	12 34.4	1 44.0	13 52.4	2 26.2
18	5 46.4	2 59.2	6 54.5	2 7.3	12 36.0	1 44.9	13 54.0	2 27.0
20	5 48.3	2 57.7	6 56.3	2 5.9	12 37.7	1 45.9	13 55.6	2 27.8
22	+5 50.2	−2 56.3	6 58.2	2 4.5	12 39.4	1 46.8	13 57.2	2 28.6
H. D.	1.0	0.7	0.9	0.7	0.8	0.5	0.8	0.4

G. M. T.	Sun's Declination.	Equation of Time.	G. M. T.	Sun's Declination.	Equation of Time.	G. M. T.	Sun's Declination.	Equation of Time.
	Monday. 29.			Monday 29.			Monday 29.	
	° ′	m s		° ′	m s		° ′	m s
0	+14 17.7	+2 38.6	8	+14 23.9	+2 41.5	16	+14 30.1	+2 44.4
2	14 19.2	2 39.3	10	14 25.4	2 42.2	18	14 31.7	2 45.1
4	14 20.8	2 40.0	12	14 27.0	2 42.9	20	14 33.2	2 45.8
6	14 22.3	2 40.8	14	14 28.6	2 43.7	22	14 34.8	2 46.5
						H. D.	0 8	0.4

SEMIDIAMETER: April 1, 16′.03; April 11, 15′.99; April 21, 15′.94; May 1, 15′.90.

NOTE.—The equation of time is to be applied to the G. M. T. in accordance with the sign as given.

MOON, 1918.

G. M. T.	Right Ascension.		Declination.		S. D.	H. P.
			January 8.			
h	h m s		° ′		′	′
6	11 28 14	216	− 2 15.4	240	14.8	54.2
8	11 31 50	216	2 30.4	240	14.8	54.2
10	11 35 26	216	3 3.4	240	14.8	54.2
			January 19.			
16	2 10 32	277	+17 42.5	194	15.9	58.4
18	2 15 09		18 1.9		15.9	58.3
			January 25.			
18	7 48 11	253	+18 57.2	164	15.1	55.3
20	7 52 24	252	18 40.8	168	15.1	55.2
22	7 56 36	250	18 24.0	171	15.1	55.2

G. M. T.	Right Ascension.		Declination.		S. D.	H. P.
			March 30.			
h	h m s		° ′		′	′
4	14 35 4	248	18 47.6	149	14.9	54.6
6	14 39 12	249	19 2.5	146	14.9	54.7
			April 10.			
10	0 52 48	289	+10 44.4	270	16.7	61.2
12	0 57 37	289	11 11.4	268	16.7	61.2
14	1 2 26	291	11 38.2	264	16.7	61.2
			April 22.			
10	11 31 11	218	− 2 26.1	233	14.7	54.0
12	11 34 49	217	2 49.4	233	14.7	54.0
			April 26.			
8	14 31 2	249	−18 28.3	152	14.9	54.7
10	14 35 11	251	18 43.5	148	15.0	54.8

MOON, 1918.

TIME OF TRANSIT, MERIDIAN OF GREENWICH.

Date.	Greenwich Mean Time.		Date.	Greenwich Mean Time.		Date.	Greenwich Mean Time.		Date.	Greenwich Mean Time.	
	h m			h m			h m			h m	
Jan. 1	15 35	41	Jan. 22	8 38	54	Mar. 28	12 55	44	Apr. 11	0 12	58
2	16 16	40	25	11 16	48	29	13 39	47	12	1 10	59
18	5 2	52	26	12 4	45	30	14 26	50	13	2 9	60
19	5 54	54	27	12 49	42	31	15 16	51	14	3 9	58
20	6 48	54	28	13 31	41						
21	7 42	56									

VENUS, 1918.

GREENWICH MEAN TIME.

Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.	Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.
	Noon.		Noon.				Noon.		Noon.		
	h m s		° ′		h m		h m s		° ′		h m
Jan. 14	21 56 38	30	−9 51.9	168	2 21	Mar. 1	20 52 24	8	−9 50.8	84	22 15
15	21 57 8	20	9 35.1	164	2 20	2	20 52 27	13	9 59.2	81	22 11

MARS, 1918.

GREENWICH MEAN TIME.

Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.	Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.
	Noon.		Noon.				Noon.		Noon.		
	h m s		° ′		h m		h m s		° ′		h m
Apr. 10	11 10 36	47	+8 28.2	31	9 57	May 26	11 21 7	65	+5 23.6	90	7 7
11	11 9 49	44	8 30.8	17	9 52	27	11 22 12	67	5 14.6	91	7 4

Hor. Parallax: Jan. 1, 0′.18; Feb. 1, 0′.17; Mar. 1, 0′.21; Apr. 1, 0′.22; May 1, 0′.18; June 1, 0′.14; July 1, 0′.11.

JUPITER, 1918.

GREENWICH MEAN TIME.

Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.	Date.	Apparent Right Ascension.		Apparent Declination.		Transit, Meridian of Greenwich.
	Noon.		Noon.				Noon.		Noon.		
	h m s		° ′		h m		h m s		° ′		h m
Jan. 3	4 1 49	19	+19 52.9	7	9 11	Feb. 22	4 2 55	22	+20 8.2	13	5 56
4	4 1 30	18	19 52.2	6	9 7	23	4 3 17	22	20 9.5	12	5 52
5	4 1 12		19 51.6		9 2	Mar. 1	4 5 43	27	20 17.5	14	5 31
Feb. 15	4 0 42	17	+20 0.4	10	6 21	2	4 6 10	27	20 18.9	15	5 28
16	4 0 59		20 1.4		6 17						

Polar Semidiameter: Jan. 1, 0′.37; Feb. 1, 0′.34; Mar. 1, 0′.31; Apr. 1, 0′.28; May 1, 0′.27; June 1, 0′.26; July 1, 0′.26. Hor. Parallax: Jan. 1, 0′.03; Feb. 1, 0′.03; Mar. 1, 0′.03; Apr. 1, 0′.03; May 1, 0′.02; June 1, 0′.02; July 1, 0′.02.

APPARENT PLACES OF STARS, 1918.

FOR THE UPPER TRANSIT AT GREENWICH.

No.	Constellation Name.	Right Ascension.													
			Jan. 1.	Feb. 1.	Mar. 1.	Apr. 1.	May 1.	June 1.	July 1.	Aug. 1.	Sept. 1.	Oct. 1.	Nov. 1.	Dec. 1.	Dec. 32.
		h m	s	s	s	s	s	s	s	s	s	s	s	s	s
5	α Urs. Min.	1 29	129.[illegible]	96.1	70.6	56.8	62.2	84.1	115.8	151.5	181.7	200.4	205.1	194.2	168.5
6	α Eridani	1 34	41.1	40.0	39.2	38.7	38.7	39.5	40.7	42.2	43.5	44.2	44.3	43.9	43.0
10	α Tauri	4 31	15.2	15.0	14.6	14.1	13.8	14.0	14.5	15.4	16.3	17.2	17.9	18.5	18.6
11	β Orionis	5 10	38.1	37.9	37.5	36.9	36.6	36.5	37.0	37.6	38.5	39.3	40.1	40.7	40.9
12	α Aurigæ	5 10	41.1	40.9	40.3	39.6	39.1	39.1	39.7	40.7	41.9	43.2	44.3	45.1	45.5
16	α Argus	6 22	10.6	10.3	9.6	8.5	7.5	6.9	6.8	7.3	8.3	9.5	10.7	11.6	12.0
17	α Can. Maj.	6 41	34.3	34.4	34.0	33.5	32.9	32.7	32.8	33.3	34.0	34.8	35.7	36.4	36.9
18	ε Can. Maj.	6 55	26.5	26.6	26.2	25.6	25.0	24.6	24.6	25.0	25.7	26.6	27.6	28.4	28.9
19	α Can. Min.	7 35	3.0	3.3	3.1	2.7	2.2	1.9	1.9	2.3	2.9	3.7	4.6	5.5	6.1
25	α Leonis	10 4	2.4	3.1	3.4	3.3	2.9	2.5	2.3	2.2	2.4	2.8	3.6	4.6	5.5
27	β Leonis	11 44	54.1	55.0	55.6	55.8	55.6	55.3	55.0	54.7	54.6	54.7	55.2	56.1	57.1
33	α Virginis	13 20	53.1	54.2	54.9	55.3	55.5	55.5	55.2	54.9	54.6	54.4	54.7	55.3	56.3
35	α Boötis	14 11	55.7	56.7	57.5	58.1	58.4	58.4	58.2	57.7	57.3	57.0	57.0	57.4	58.3
40	α Scorpii	16 24	22.7	23.7	24.6	25.5	26.3	26.8	27.0	26.8	26.3	25.8	25.5	25.7	26.3
49	α Aquilæ	19 46	46.7	47.0	47.5	48.3	49.2	50.1	50.7	51.0	50.9	50.5	49.9	49.6	49.6

No.	Declination.														Special Name.	Mag.
		Jan. 1.	Feb. 1.	Mar. 1.	Apr. 1.	May 1.	June 1.	July 1.	Aug. 1.	Sept. 1.	Oct. 1.	Nov. 1.	Dec. 1.	Dec. 32.		
	°	′	′	′	′	′	′	′	′	′	′	′	′	′		
5	+88	52.5	52.5	52.4	52.3	52.1	52.0	52.0	52.0	52.1	52.3	52.5	52.6	52.8	Polaris	2.1
6	−57	39.4	39.4	39.3	39.1	39.0	38.8	38.6	38.6	38.6	38.7	38.9	39.0	39.1	Achernar	0.6
10	+16	20.8	20.8	20.8	20.7	20.7	20.7	20.8	20.8	20.9	20.9	20.9	20.9	20.9	Aldebaran	1.1
11	− 8	17.7	17.8	17.9	17.9	17.8	17.8	17.7	17.6	17.5	17.5	17.6	17.6	17.7	Rigel	0.3
12	+45	55.1	55.1	55.1	55.1	55.1	55.0	54.9	54.9	54.9	54.9	54.9	55.0	55.1	Capella	0.2
16	−52	39.1	39.1	39.3	39.4	39.3	39.2	39.1	38.9	38.8	38.8	38.8	39.0	39.2	Canopus	−0.9
17	−16	36.2	36.4	36.4	36.4	36.4	36.4	36.3	36.2	36.1	36.1	36.1	36.2	36.4	Sirius	−1.6
18	−28	51.6	51.8	51.9	51.9	51.9	51.8	51.7	51.6	51.5	51.4	51.5	51.6	51.8	Adhara	1.6
19	+ 5	26.1	26.0	26.0	26.0	26.0	26.0	26.0	26.1	26.1	26.1	26.0	26.0	25.9	Procyon	0.5
25	+12	21.9	21.9	21.8	21.9	21.9	21.9	21.9	21.9	21.9	21.9	21.8	21.7	21.6	Regulus	1.3
27	+15	1.6	1.5	1.5	1.5	1.6	1.6	1.7	1.7	1.6	1.6	1.5	1.4	1.3	Denebola	2.2
33	−10	44.1	44.2	44.3	44.3	44.3	44.3	44.3	44.3	44.3	44.2	44.2	44.3	44.4	Spica	1.2
35	+19	36.3	36.2	36.2	36.2	36.3	36.3	36.4	36.4	36.4	36.4	36.3	36.2	36.0	Arcturus	0.2
40	−26	15.1	15.1	15.1	15.2	15.2	15.2	15.2	15.2	15.2	15.2	15.2	15.1	15.1	Antares	1.2
49	+ 8	39.1	39.0	39.0	39.0	39.0	39.1	39.2	39.3	39.4	39.4	39.4	39.4	39.3	Altair	0.9

EXTRACTS, AMERICAN EPHEMERIS AND N. A. 1918.

MOON, 1918.

GREENWICH MEAN TIME.

Hour.	Right Ascension.	Var. per Min.	Declination.	Var. per Min.	Hour.	Right Ascension.	Var. per Min.	Declination.	Var. per Min.
	January 1.					January 3.			
	h m s	s	° ′ ″	″		h m s	s	° ′ ″	″
11	10 10 28.17	1.8368	+6 21 16.9	−11.811	11	11 37 13.94	1.8029	−3 15 24.8	−11.978
12	10 12 18.31	1.8346	6 9 27.7	11.829	12	11 39 2.14	1.8038	3 27 23.1	11.966

MARS, 1918.

GREENWICH MEAN TIME.

Date.	Apparent Right Ascension.	Var. per Hour.	Apparent Declination.	Var. per Hour.	Logarithm of Distance from Earth.	Var. per Hour.	Semi-diameter.	Hor. Parallax.	Transit, Meridian of Greenwich.
	Noon.	Noon.	Noon.	Noon.	Noon.	Noon.	Noon.	Noon.	
	h m s	s	° ′ ″	″			″	″	h m
Feb. 13	12 14 58.72	−1.056	+2 23 56.2	+ 9.98	9.889 3281	−1500.8	6.51	11.35	14 41.5
14	12 14 31.90	1.179	2 28 4.0	10.67	9.885 7520	1479.0	6.57	11.45	14 37.1
Apr. 3	11 17 20.18	2.774	+8 4 55.1	+11.01	9.837 6615	+850.2	7.34	12.79	10 31.1

APPARENT PLACES OF STARS, 1918.

FOR THE UPPER TRANSIT AT WASHINGTON.

Washington Mean Time.	**α Canis Minoris.** (*Procyon.*) Mag. 0.5		Washington Mean Time.	**α Leonis.** (*Regulus.*) Mag. 1.3		Washington Mean Time.	**α Boötis.** (*Arcturus.*)	
	Right Ascension.	Declination.		Right Ascension.	Declination.		Right Ascension.	Declination.
	h m 7 35	° ′ +5 25		h m 10 4	° ′ +12 21		h m 14 11	° ′ +19 36
	s	″		s	″		s	″
Mar. 31.3	2.653 170	58.33 9	Jan. 10.6	2.686	53.86	Jan. 10.8	55.973 332	15.85 199
Apr. 10.3	2.483	58.42	Mar. 31.4	3.266 101	51.06 47	20.8	56.305	13.86
			Apr. 10.4	3.165	51.53			
Mean Place	0.618	69.26	Mean Place	0.419	66.46	Mean Place	55.236	31.57

APPARENT PLACES OF STARS, 1918.

CIRCUMPOLAR STARS.

FOR THE UPPER TRANSIT AT WASHINGTON.

α Ursæ Minoris.
(*Polaris.*)
Mag. 2.1

Wash. Mean Time.	Right Ascension.	Declination.
	h m	° ′
April	1 29	+88 52
	s	″
9.0	55.87	13.63
10.0	55.91	13.27

50.70 +50.69
1^{h} 30^{m} $42^{s}.307$
$+88^{\circ}$ $52'$ $2''.06$

SUN, 1918.

FOR WASHINGTON APPARENT NOON.

Date.	Apparent Right Ascension.	Var. per Hour.	Apparent Declination.	Var. per Hour.	Equation of Time. Mean—App.	Var. per Hour.	Semi-diam.	S. T. of Sem. Pass. Merid.	Siderea Time o Mean No
	h m s	s	° ′ ″	″	m s	s	′ ″	m s	h m
Jan. 8	19 16 28.26	10.926	−22 17 10.9	+19.94	+6 42.20	+1.067	16 17.75	1 10.66	19 9 44
9	19 20 50.24	10.904	22 8 59.2	21.03	7 7.55	1.045	16 17.71	1 10.58	19 13 41
11	19 29 32.56	10.858	21 51 17.9	23.18	7 56.62	0.998	16 17.62	1 10.43	19 21 34
12	19 33 52.86	10.833	21 41 48.8	24.23	8 20.31	0.974	16 17.57	1 10.85	19 25 31
Apr. 3	0 48 19.07	9.110	+ 5 11 12.4	+57.49	+3 26.40	−0.745	16 1.33	1 4.51	0 44 52
4	0 51 57.79	9.117	5 34 9.5	57.26	3 8.62	0.738	16 1.04	1 4.53	0 48 48
5	0 55 36.68	9.125	5 57 0.9	57.02	2 51.00	0.730	16 0.76	1 4.56	0 52 45
9	1 10 14.40	9.163	7 27 22.0	55.88	1 42.70	0.692	15 59.65	1 4.69	1 8 31
10	1 13 54.43	9.174	7 49 39.3	55.56	1 26.22	0.681	15 59.37	1 4.73	1 12 27
26	2 13 20.69	9.425	13 24 27.8	+48.48	−2 11.78	−0.430	15 55.25	1 5.63	2 15 32
27	2 17 7.16	9.446	13 43 44.8	47.93	2 21.84	0.409	15 55.00	1 5.70	2 19 29

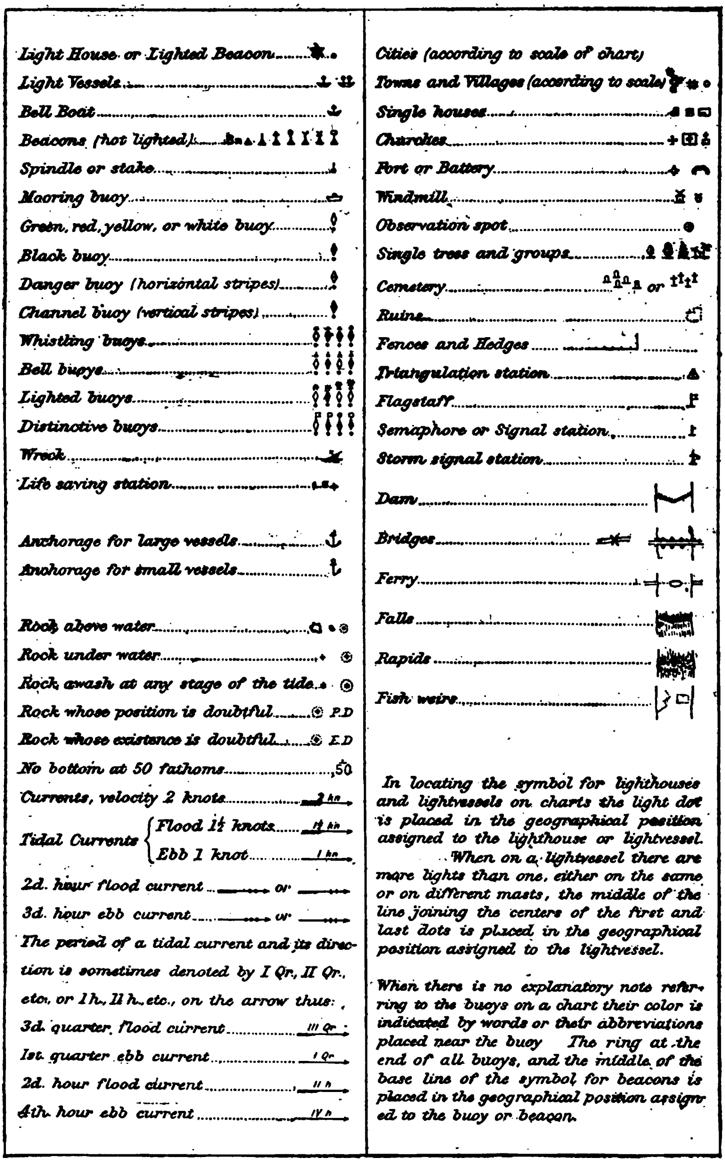
SUN, 1918.

FOR GREENWICH MEAN NOON.

Date.	Day of the Week.	Apparent Right Ascension.	Var. per Hour.	Apparent Declination.	Var. per Hour.	Semi-diam.	Hor. Par.	Equation of Time. App.—Mean.	Var. per Hour.	Sidereal Ti or Right Asc sion of Me Sun.
		h m s	s	° ′ ″	″	′ ″	″	m s	s	h m s
Jan. 1	Tu	18 44 44.66	11.042	−23 3 5.4	+11.80	16 17.88	8.95	− 3 26.25	−1.185	18 41 18.4
5	Sa	19 2 22.05	10.988	22 40 33.1	16.35	16 17.84	8.95	5 17.41	1.127	18 57 4.6
7	Mo	19 11 8.42	10.947	22 26 34.3	18.59	16 17.79	8.95	6 10.65	1.090	19 4 57.7
8	Tu	19 15 30.91	10.927	22 18 54.9	19.69	16 17.76	8.95	6 36.59	1.071	19 8 54.3
9	We	19 19 52.92	10.906	22 10 49.1	20.79	16 17.72	8.95	7 2.04	1.050	19 12 50.8
11	Fr	19 28 35.34	10.860	21 53 19.4	22.94	16 17.63	8.95	7 51.35	1.004	19 20 43.9
12	Sa	19 32 55.70	10.836	21 43 56.0	24.00	16 17.58	8.95	8 15.15	0.979	19 24 40.5
20	Su	20 7 14.82	10.604	20 14 4.1	32.01	16 17.06	8.94	11 1.81	0.747	19 56 13.0
25	Fr	20 28 17.58	10.440	−19 5 25.7	+36.56	16 16.58	8.94	−12 21.73	−0.583	20 15 55.7
Feb. 13	We	21 45 13.38	9.811	13 31 49.0	50.20	16 13.59	8.91	14 23.03	0.046	21 30 50.3
Apr. 2	Tu	0 43 53.17	9.104	+ 4 43 9.7	57.76	16 1.67	8.80	3 48.26	+0.752	0 40 4.9
3	We	0 47 31.74	9.110	5 6 13.6	57.55	16 1.39	8.80	3 30.28	0.746	0 44 1.4
9	Tu	1 9 27.05	9.162	7 22 33.0	55.96	15 59.71	8.78	1 46.28	0.694	1 7 40.7
11	Th	1 16 47.87	9.185	8 7 3.7	55.21	15 59.16	8.78	1 13.49	0.672	1 15 33.8
19	Fr	1 46 20.80	9.295	10 59 10.6	52.11	15 57.06	8.76	0 45.51	0.562	1 47 6.3
22	Mo	1 57 31.82	9.346	12 0 52.2	50.69	15 56.30	8.75	1 24.15	0.511	1 58 55.9
24	We	2 5 1.30	9.383	12 41 1.1	49.67	15 55.80	8.75	1 47.78	0.473	2 6 49.0
29	Mo	2 23 53.29	9.488	14 17 39.5	46.91	15 54.55	8.74	2 38.56	0.370	2 26 31.8
30	Tu	2 27 41.22	9.508	14 36 18.3	46.82	15 54.31	8.73	2 47.19	0.348	2 30 28.4

U. S. Hydrographic Office Charts—Conventional Signs and Symbols

PLATE X.



The period of a tidal current and its direction is sometimes denoted by I Qr., II Qr., etc., or I h., II h., etc., on the arrow thus:

In locating the symbol for lighthouses and lightvessels on charts the light dot is placed in the geographical position assigned to the lighthouse or lightvessel.

When on a lightvessel there are more lights than one, either on the same or on different masts, the middle of the line joining the centers of the first and last dots is placed in the geographical position assigned to the lightvessel.

When there is no explanatory note referring to the buoys on a chart their color is indicated by words or their abbreviations placed near the buoy. The ring at the end of all buoys, and the middle of the base line of the symbol for beacons is placed in the geographical position assigned to the buoy or beacon.

Lighthouse .... ✱

Lighthouse on small scale chart • *Old light tower* .... ●

Beacon, lighted .... ★ *Beacon, not lighted* .... ▲

Spindle (or stake) .... ⫰ *add word Spindle if space allows.*

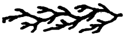
Lightship .... ✹ *Wreck* .... ⊹

Anchorage .... ⚓ *Covering and uncovering rock* .... ●

Rock awash at low water .... ✳ *Sunken rock* .... +

Life Saving Station .... ✚ L.S.S. (T) .... *signifies connection with telegraphic system.*

Kelp ....



No bottom at 20 fathoms .... $\overset{\circ}{\underline{20}}$ *etc.*

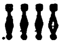
Red buoy ♁ *or add word white or yellow as required.*

Black buoy .... ♦

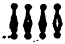
Horizontally striped buoy .... ♦

Perpendicularly striped buoy .... ♦

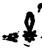
Buoys with perch and square ....



Buoys with perch and ball ....



Lighted buoy .... ✱ , *in place of* • , *as* ....



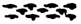
etc.

Mooring buoy .... ⊖

Landmark, as Cupola, Standpipe, etc. .... ○

Whirlpool .... ⬤

Tide rip ....



Current, not tidal, drift in knots as .... ⤜—— 2.0 ——→

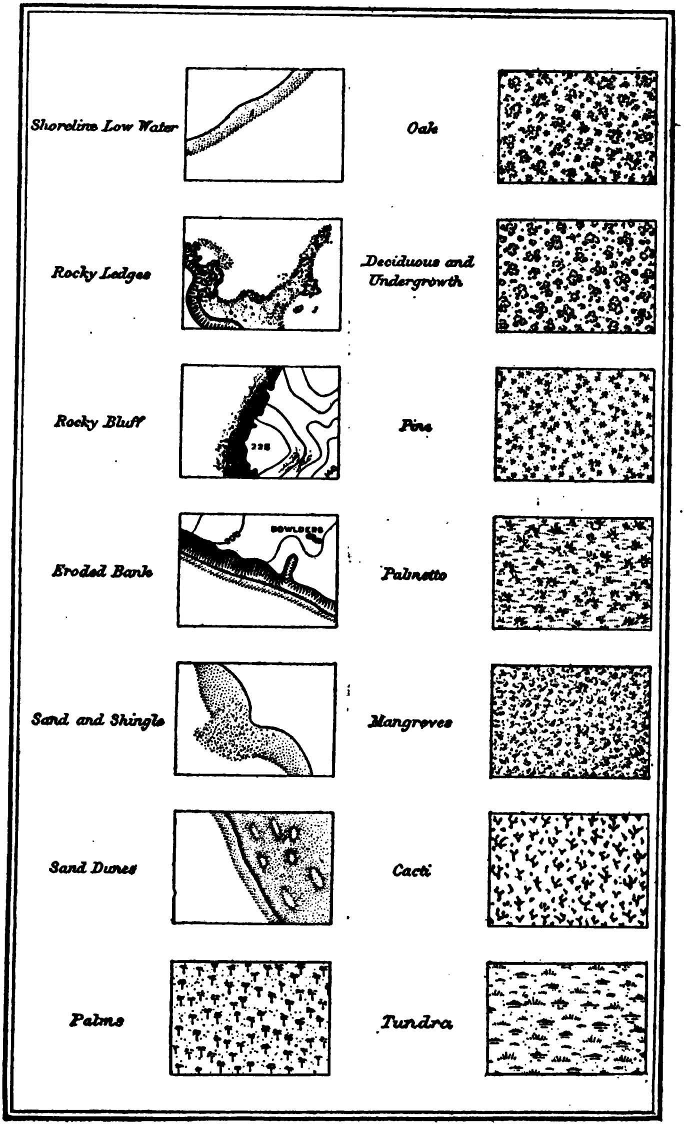
" *, flood, first quarter, drift in knots, as* .... ——— 0.4 ——↣

" " " *, second* " " " " " .... ——— 1.0 ——⇸

" " " *, third* " " " " " .... ——— 0.3 ——⇻

" *ebb,* ....———————→ *otherwise like flood.*

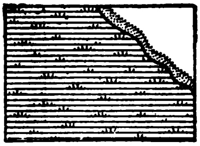
U S. Coast and Geodetic Survey Charts—Topographic Signs, PLATE XII.



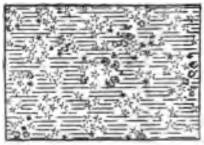
U. S. Coast and Geodetic Survey Charts—Topographic Signs.

PLATE XIII.

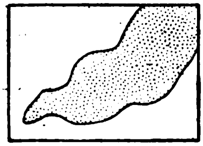
Salt Marsh



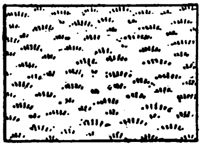
Cypress Swamp



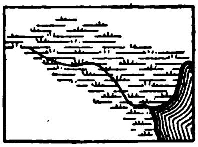
Salt Pond



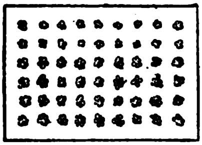
Grass.



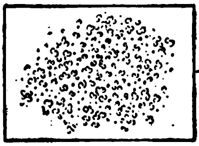
Fresh Marsh and Fresh Pond



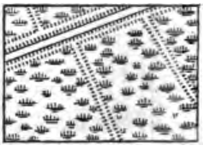
Orchard



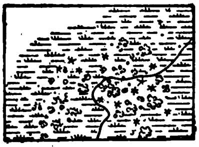
Oyster Bed



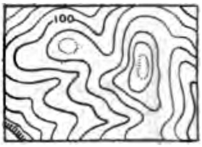
Rice Dikes & Ditches



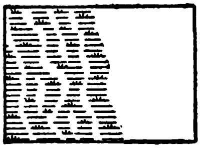
Wooded Marsh



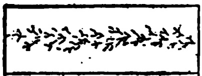
Curves of equal elevation and intermediate curves.



Submerged Marsh



Eel Grass



Kelp

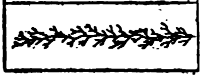


PLATE XIV.

GENERAL ABBREVIATIONS ON HYDROGRAPHIC OFFICE CHARTS AND U. S. COAST AND GEODETIC SURVEY CHARTS.

ABBREVIATIONS FOR KINDS OF BOTTOM.

M.	Mud.	bk.	Black.	hrd.	Hard.
S	Sand.	wh	White.	sft	Soft.
G.	Gravel.	rd	Red.	fne	Fine.
Sh	Shells.	yl	Yellow.	crs	Coarse.
P	Pebbles.	gy	Gray.	brk	Broken.
Sp	Specks.	bu	Blue.	lrg	Large.
Cl	Clay.	dk	Dark.	sml	Small.
St	Stones.	lt	Light.	rky	Rocky.
Co	Coral.	gn	Green.	stk	Sticky.
Oz	Ooze.	br	Brown.	stf	Stiff.

ABBREVIATIONS NEAR BUOYS.

U. S. C. and G. S. Charts.		Hydrographic Office Charts.			
C.	Can.	B, bk	Black.	Y, yl	Yellow.
N.	Nun.	W, wh	White.	Ch, chec	Checkered.
S.	Spar.	R, rd	Red.	H.S.	Horizontal stripes.
		G, gn	Green.	V.S.	Vertical stripes.

ABBREVIATIONS FOR LIGHTS.

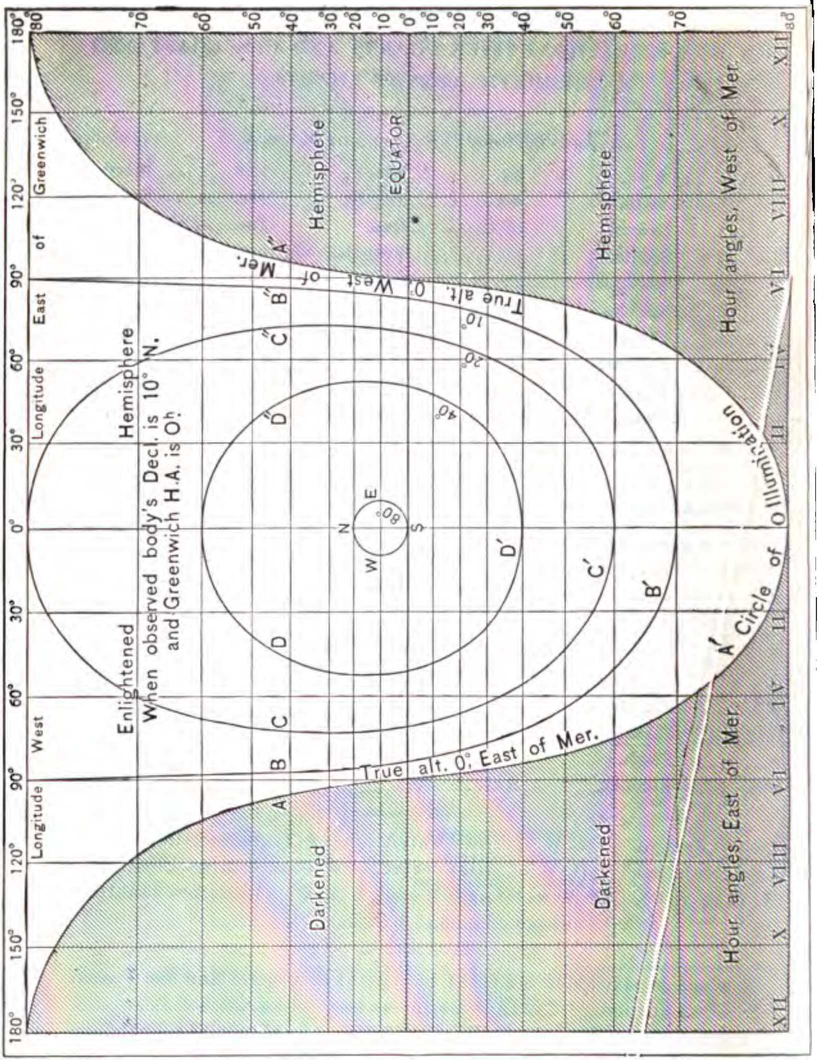
F	Fixed.	Rev	Revolving.	V	Varied by.
Flg	Flashing.	E.	Eclipses.	Sec	Sector.
Fl.	Flash.	W	White.		
Fls.	Flashes.	R.	Red.		

Bn.	Beacon.	L. W.	Low water.	L. S. S.	Life-saving station.
kn.	Knots.	H. W.	High water.	P. D.	Position doubtful.
H. W. F. & C.	High water at full and change of moon.			E. D.	Existence doubtful.

A wireless station is indicated by a point in a circle and the legend "Wireless Station." The information as to a submarine bell is covered by the legend "Submarine Bell" at the spot, and, in case of a lightship, in the table.

PLATE XV.

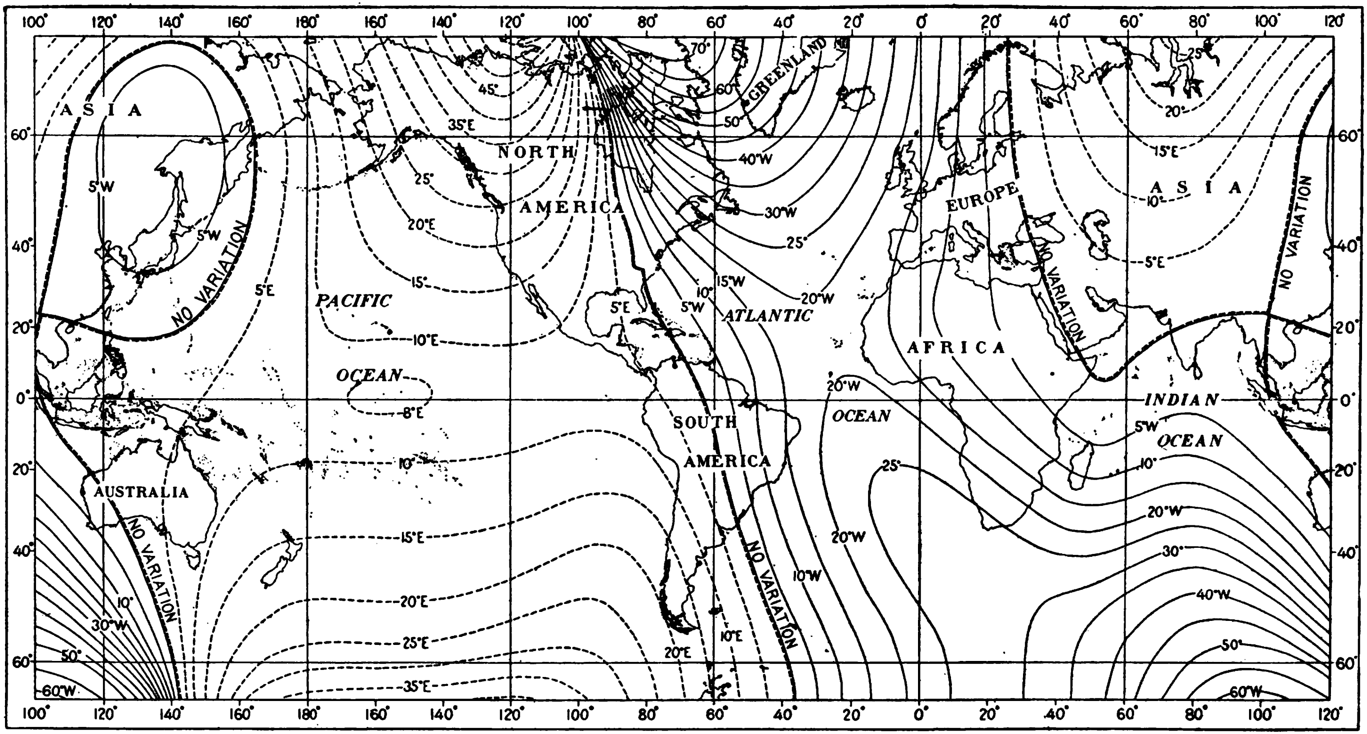
CIRCLES OF EQUAL ALTITUDE ON THE MERCATOR CHART



(1) Pole without the circle, curve is closed; as *CC'C"*, *DD'D"*.
(2) Pole within the circle, curve is sinusoidal; as *AA'A"*.
(3) Pole on the circle, curve is open, branches meeting at infinity, asymptotes parallel to meridians; as *BB'B"*.

PLATE XVI.

THE VARIATION OF THE COMPASS
FOR THE YEAR 1918



APPENDIX A.

DESCRIPTION OF THE SUBMARINE-BELL SYSTEM.

The equipment furnished by the submarine-bell company to a vessel or station depends on the purposes for which intended, and may therefore be considered under the two general heads given below.

The sending apparatus, consisting of the bell and accessories, varies according to the use made of the system. When installed on light ships and tenders, the outfit consists of a bell mounted on a case containing the striking mechanism which is operated by compressed air supplied, through a hose, from air tanks; a davit, with chain and windlass, for raising or lowering the bell over the ship's side; and a code ringer for so controlling the strokes as to make automatically the code number of the light ship.

For use near certain dangers or turning points, the bell is hung on a tripod standing on the bottom, and the clapper is actuated by powerful magnets energized by a current sent from a shore power-house, through an armor-protected submarine cable.

When suspended from buoys the bell has a mechanism consisting of a combination of ratchets and pawls through whose agency a spring is compressed to a certain point by the wave-action on the buoy, and then automatically released, causing the clapper to strike the bell.

The receiving apparatus installed in vessels consists of two small tanks placed in the forward part of the vessel, well below the water-line, one against the starboard side, one against the port side. In each tank are two microphones immersed in liquid which receive the sounds, when the sound-waves strike the ship's side. These sounds are transmitted to the pilot-house, or other location of the direction-indicator. Wires are run from the tanks to the battery box which supplies the power, thence to the direction-indicator which is a small round metallic case fastened to the wall with telephone receivers hung on each side, and bearing on its face a switch for connecting either starboard or port microphones with the receivers; a dial indicates the one connected.

APPENDIX B.

COMPENSATION OF COMPASSES AT A SHIP-YARD BEFORE PROCEEDING TO SEA.*

Before a vessel is sent to sea for the first time from a dock-yard or a navy-yard, the navigator should, by a preliminary compensation, so reduce the deviations and equalize the directive force of the compasses that they may be used to steer by until he shall be able to compensate them regularly and obtain a residual curve.

This preliminary compensation should always be made, when possible, from data obtained from observations and vibrations on two headings, assuming $\mathfrak{A}$ and $\mathfrak{E}$ as zero, and determining $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ by the method of Art. 95 if computation is made, as it should be whenever construction would give acute angles of intersection, or by the method of Art. 113 if the dygogram is used.

Then compensation should be made as explained in Art. 110; or, by using the indications of the dygogram, neutralizing first the quadrantal force and then in the proper order as shown by the dygogram the semicircular forces. As each corrector is placed, the deviation should be reduced to the amount indicated by a dygogram of the remaining force or forces only.

Provided the compasses are uninfluenced by the presence of other vessels, structures or masses of steel or iron, the required observations may be obtained:

(1) When the vessel is in drydock and also, at an earlier or later time, alongside a dock or sea-wall.

(2) When moored alongside a dock or sea-wall and the ship can be either winded or sprung out to a suitable heading (see Arts. 95, 110, and 113).

Whilst λ, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ may all be determined when observations on two headings are possible, it may sometimes happen that these can be made on only one heading. Under such circumstances λ and $\mathfrak{D}$ must be assumed, and, for reasons given in Art. 95, they may be assumed as those of a similarly situated compass on a similar ship.

Make the necessary observations and vibrations referred to in Art. 94 before the quadrantal spheres are placed and determine $\mathfrak{B}$ and $\mathfrak{C}$ by computation, using equations (69a) and (70a); or by construction as explained in Art. 113a. Then having $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$, proceed to compensate as directed in Art. 110. If these values are found by construction, Art. 113a, the dygogram may be used when compensating to indicate the deviation and the changes in deviation as each force is successively neutralized.

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* See "The First Compensation of a Vessel's Compasses," issued by Bureau of Equipment, Navy Department, 1906.

SPECIAL PROCEDURE WHEN THE SHIP IS ON CERTAIN HEADINGS.

Special procedure for compensation of the compass, ship heading on a cardinal point magnetic (assuming $\mathfrak{A}$ and $\mathfrak{E}$ as zero).

Having obtained the coefficients $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ by any of the methods explained in Chapter IV, it may happen, when compensation takes place on one heading, as contemplated in Art. 94, that the vessel is unavoidably heading with the keel-line north and south magnetic or east and west magnetic. Again, the vessel may be on the stocks or alongside a dock with the keel-line as indicated above and compass coefficients unknown, when it becomes necessary to assume λ and $\mathfrak{D}$ as those of a compass similarly situated on a sister ship, to determine $\mathfrak{B}$ and $\mathfrak{C}$, and then to compensate the compass (see Arts. 94 and 113a). In such cases compensation for two of the forces according to Art. 110 may appear indeterminate and special procedure becomes necessary.

(1) **Let the heading be assumed as magnetic north.**—On this heading neither $\mathfrak{B}$ nor $\mathfrak{D}$ produces any deviation but they both influence the amount produced by $\mathfrak{C}$. With the given or assumed $\mathfrak{D}$, find $D = \mathfrak{D} \times 57^\circ.3$; for this value of D take from Table V the distance at which the quadrantal spheres should be placed to neutralize the quadrantal force and so place them on the arms. This eliminates $\mathfrak{D}$. With $\mathfrak{D}$ neutralized, it is evident that the deviation produced by $\mathfrak{C}$ is still influenced by the force $\mathfrak{B}$ acting in the fore and aft line. Were it not for the influence of $\mathfrak{B}$, $\mathfrak{C}$ would produce a deviation equal to $\tan^{-1} \mathfrak{C}$, and if the forces acting in the fore and aft line should be so altered that a deviation of $\tan^{-1} \mathfrak{C}$ should be shown by the compass, the force $\mathfrak{B}$ would then be compensated.

Therefore, run the fore and aft carrier down, fill tubes with magnets, red ends forward if $\mathfrak{B}$ is +, aft if $\mathfrak{B}$ is (—); raise carrier slowly till the required deviation, $\tan^{-1} \mathfrak{C}$ (angle POC'' in Figs. 149 and 150), is indicated, and clamp the carrier; the force $\mathfrak{B}$ is thus neutralized.

Run the athwartship carrier down; fill tubes with magnets and place them red ends to starboard if $\mathfrak{C}$ is +, to port if $\mathfrak{C}$ is (—); raise carrier slowly till compass points north magnetic, which is the ship's heading; the force $\mathfrak{C}$ is thus neutralized and the compass is compensated.

The steps above taken may be illustrated by the dygogram (Figs. 149 and 150). Let $OP =$ unity, $PD' = \mathfrak{D}$, $D'B = \mathfrak{B}$, $BC = \mathfrak{C}$;

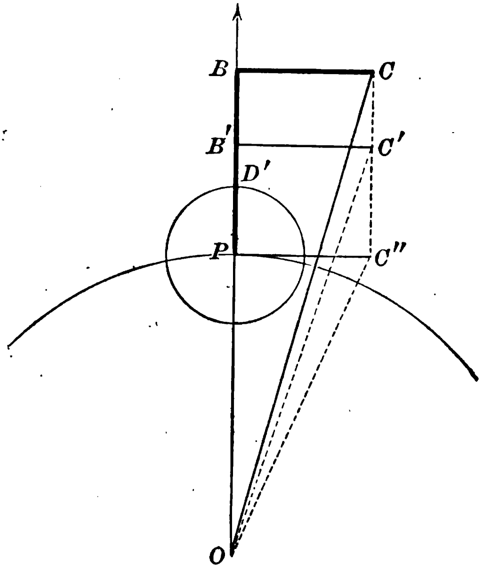

Fig. 149.

both $\mathfrak{B}$ and $\mathfrak{C}$ being + in Fig. 149 and — in Fig. 150. Though $\mathfrak{B}$ and $\mathfrak{D}$ produce no deviation on this heading, magnetic north, they influence the amount produced by $\mathfrak{C}$; the deviation under the various influences being *POC* and the direction of the needle *OC*.

When $\mathfrak{D}$ is compensated, *PD'* shortens to zero, *D'B* and *BC* respectively assume the positions *PB'* and *B'C'* and the needle takes the position *OC'*, a position it should assume when under the influences only of $\mathfrak{B}$ and $\mathfrak{C}$. If $\mathfrak{B}$ were then compensated, the line *PB'* would shorten to zero and *B'C'* would assume the position PC'', the needle would take the direction OC'', and the deviation would become POC'' (which angle equals $\tan^{-1}\mathfrak{C}$) and be due to $\mathfrak{C}$ alone uninfluenced by any other of the ship's forces; therefore, after $\mathfrak{D}$ has been eliminated by placing the spheres according to Table V, and only the semicircular forces remain acting, ship heading north magnetic, place the fore-and-aft corrector-magnets so as to alter the deviation from POC' to

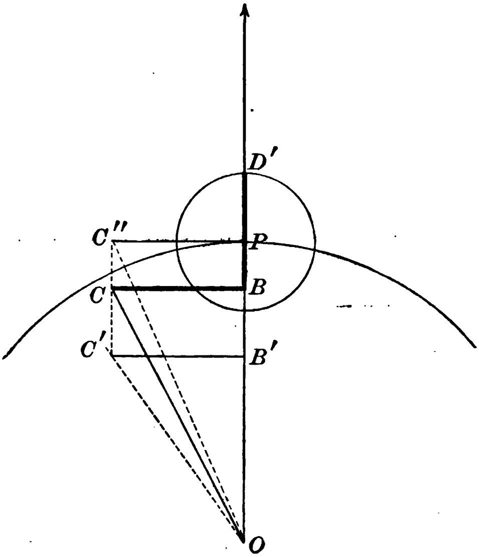

FIG. 150.

POC'', then place the athwartship corrector-magnets to reduce the deviation from POC'' to zero; the needle will take the direction of OP and the compass will be compensated.

(2) **Let the heading be assumed as magnetic east.**—In this case, $\mathfrak{B}$ produces the deviation, the amount of which, however, is influenced by the forces $\mathfrak{C}$ and $\mathfrak{D}$ acting athwartship and in the magnetic meridian; therefore, the procedure should be as follows: Place quadrantal spheres as indicated above, neutralizing $\mathfrak{D}$; by means of the athwartship magnets so alter the athwartship forces that the compass will indicate a deviation of $\tan^{-1}\mathfrak{B}$ (angle

POB′, Figs. 151 and 152), thus neutralizing ℭ; then, by means of magnets properly placed in the fore-and-aft carrier, make the compass indicate the heading east magnetic, eliminating the force 𝔅 and completing the compensation of the compass.

The steps above taken when the ship headed east magnetic may be illustrated by the dygograms (Figs. 151 and 152); both 𝔅 and ℭ being + in Fig. 151 and — in Fig. 152.

When 𝔇 has been neutralized *PD* shortens to zero, *DB* and *BC* respectively assume the positions *PB′* and *B′C′*, the deviation becomes *POC′*, and the needle takes the direction *OC′*. When ℭ has been neutralized, *B′C′* shortens to zero, the needle lies in direction *OB′*, the deviation becomes *POB′* (which angle equals $\tan^{-1}$ 𝔅) and is due to 𝔅 alone uninfluenced by any other of the ship's forces. If this value is reduced to zero by fore-and-aft corrector-magnets properly placed, the compass needle should take the direction *OP* and the compensation be effected.

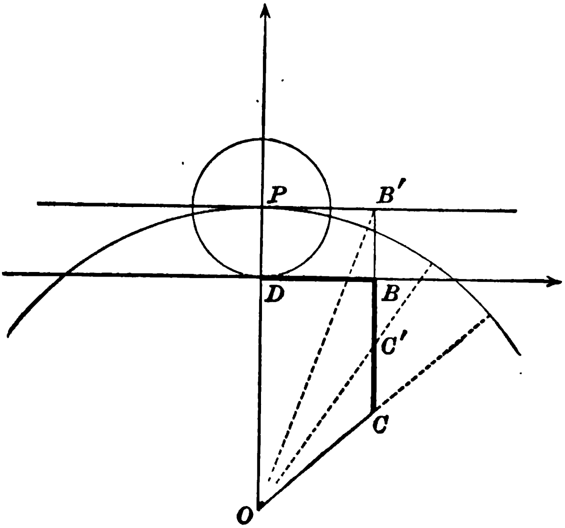

FIG. 151.

(3) With 𝔇 eliminated and ship heading east or west per compass.—If the heading is such that after the quadrantal force has been eliminated, the ship should be heading east per compass when the deviation is easterly, or west per compass when the deviation is westerly, the athwartship force ℭ will lie in the vertical plane through the compass needle, and, as shown by the dygograms (Figs. 153 and 154), the corresponding corrector-magnets, if used first, would have no apparent effect when compensating. However, knowing the deviation and compass heading, we may easily find the magnetic heading and the amounts of deviation produced respectively by the forces 𝔅 and ℭ on that heading; after which, the compensation by the method of Art. 110 is very simple, for the force 𝔅, being more nearly at right angles with the needle, should be eliminated first and then the force ℭ.

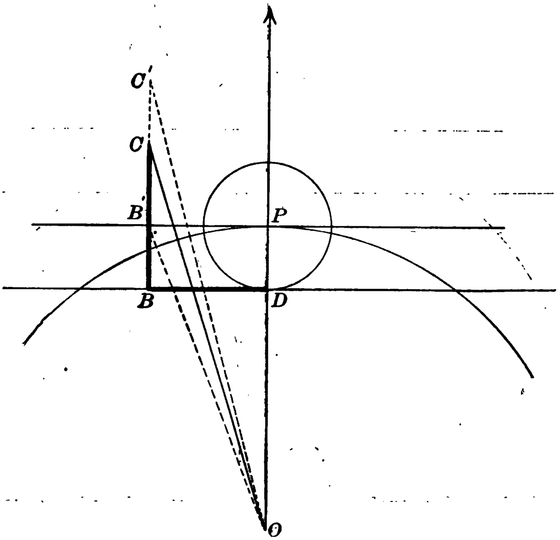

FIG. 152.

If not wishing to compute the deviation due to these forces, nor to apply the method of Art. 110, we may construct a dygogram and use it in compensating. The dygograms (Figs. 153 and 154) show that if the force 𝔅 should be compensated, *PB* would shorten to zero, the needle would assume the position *OC'*, the deviation would become *POC'*, and the remaining force ℭ would be left at a favorable angle with the direction of the needle for its elimination. Therefore, place the fore-and-aft

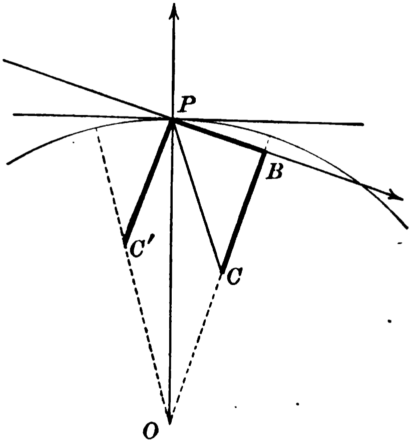

FIG. 153.

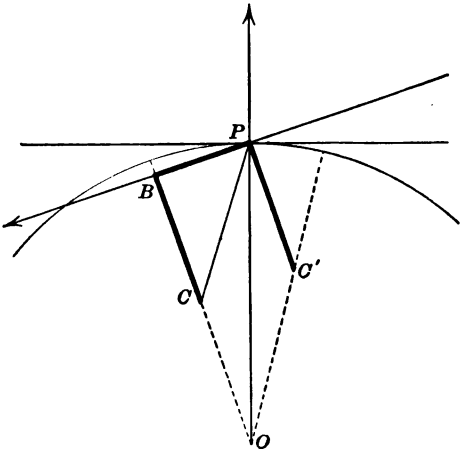

FIG. 154.

corrector-magnets at such a height as to change the deviation from *POB* to *POC′*, compensating the force $\mathfrak{B}$; then place the athwartship corrector-magnets at such a height as to reduce the deviation from *POC′* to zero, compensating the force 𝕮, and thereby effecting the complete compensation of the compass.

(4) **On a heading of no semicircular deviation.**—If the heading of the ship is such that, after the quadrantal force has been eliminated, no deviation is shown when 𝕭 and 𝕮 are known to have appreciable values, then it is evident that the semicircular forces are neutralizing each other on that particular heading. This state of affairs is indicated by the dygogram (Fig. 155), the compass needle there lying in the meridian *OP*.

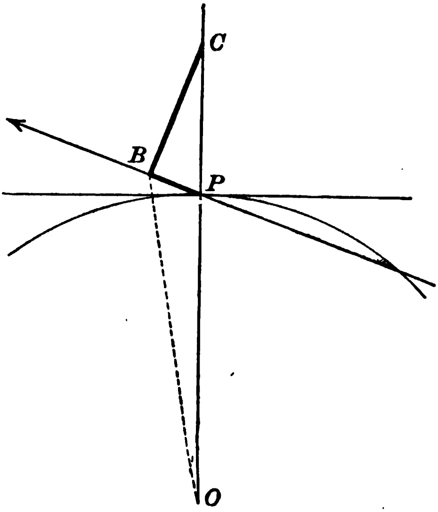

FIG. 155.

The method employed in Art. 110 for the elimination of 𝕭 and 𝕮 may be followed in this case and the compensation of the compass effected without any difficulty.

However, if not wishing to compute the deviations due to 𝕭 and 𝕮, the dygogram may be used in compensating as indicated below; at all events, it will serve the useful purpose, as it does in all cases when used, of indicating which force it is preferable to eliminate first.

For the particular values of 𝕭 and 𝕮 indicated by Fig. 155, it is evident that it would be better to eliminate first the force $\mathfrak{C}$, shortening *BC* to zero, causing the needle to lie in the direction *OB*, and leaving the remaining force $\mathfrak{B}$ at an angle with the direction of the needle more favorable for its elimination than would have been the angle for the elimination of $\mathfrak{C}$ had the force $\mathfrak{B}$ been neutralized first.

Therefore, place the athwartship magnet-correctors at such a height as to produce a deviation equal to the angle *POB*, compensating the force $\mathfrak{C}$; then place the fore-and-aft corrector-magnets so as to reduce the deviation *POB* to zero, eliminating the force $\mathfrak{B}$, and completing the compensation of the compass.

APPENDIX C.

GENERAL USE OF AZIMUTH TABLES.

By the azimuth tables issued to the navy the azimuth (*Z*) is found when the hour angle (*t*), the declination (*d*), and the latitude (L) are given; in other words, one angle of a spherical triangle may be found when two sides and the included angle are given. Therefore, these tables may be used to find the position angle (*M*) which may be desired for use in Littlehales' method of equal altitudes (Art. 270), the hour angle (*t*) of an unidentified heavenly body whose true altitude and true azimuth are known (Art. 328), and the great circle course from one given place to another given place (Art. 135).

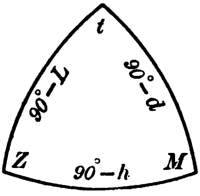

FIG. 156.

Let Fig. 156 represent the astronomical triangle lettered as shown; then, having given *t*, *L*, and *d*, to find *Z*, we have from Napier's analogies

$$\left.\begin{aligned}\text{Tan}\ \tfrac{1}{2}\,(Z-M) &= \cot \tfrac{1}{2}\,t \sin \tfrac{1}{2}\,(L-d) \sec \tfrac{1}{2}\,(L+d)\\ \text{Tan}\ \tfrac{1}{2}\,(Z+M) &= \cot \tfrac{1}{2}\,t \cos \tfrac{1}{2}\,(L-d) \operatorname{cosec} \tfrac{1}{2}\,(L+d)\end{aligned}\right\} \quad (a)$$

these being the formulæ by which the azimuth tables, now issued to the navy by the navy department, were computed.

(A) Having given *L*, *d*, and *t*, to find *Z*, we have the following rule: Enter the azimuth tables, in the given latitude; at the intersection of the horizontal line through the given hour angle and the vertical column under the given declination will be found the required true azimuth, estimated from the elevated pole and reckoned from 0° to 180° towards the east or west as the body is east or west of the meridian (see Art. 221).

H. O. Publication No. 71, in which the latitude runs from 0° to 61° and declination from 0° to 23°, each at an interval of 1°, has a different table for north and south latitudes. H. O. Pub. No. 120, in which the latitude runs from 0° to 70° and declination from 24° to 70°, each at an interval of 1°, has but one table for both cases; but, when the latitude and declination are of a different name, the tables are to be entered with the supplement of the hour angle and the supplement of the tabulated azimuth is to be taken for the required true azimuth.

(B) Given *t*, *L*, and *d*, to find the position angle *M*.—In this case we will have from Napier's analogies

$$\left.\begin{aligned}\operatorname{Tan}\tfrac{1}{2}(M-Z) &= \cot\tfrac{1}{2}t\sin\tfrac{1}{2}(d-L)\sec\tfrac{1}{2}(d+L)\\ \operatorname{Tan}\tfrac{1}{2}(M+Z) &= \cot\tfrac{1}{2}t\cos\tfrac{1}{2}(d-L)\operatorname{cosec}\tfrac{1}{2}(d+L)\end{aligned}\right\}\text{(b).}$$

Comparing formulæ (b) with formulæ (a) there is noted an interchange of *M* and *Z*, and of *L* and *d*, and it is evident that the tables may be used to find *M*, having given *t*, *L*, and *d;* but it must be remembered that by following a rule similar to that for taking out the azimuth in the cases when *L* and *d* are of a different name, we shall obtain the supplement of the required *M* instead of *M* itself. It must also be remembered, *in the case of an observed heavenly body,* that the position angle *M* will be greater than 90° only when the declination is greater than the latitude and of the same name and when the body is observed between the point of maximum azimuth and the upper meridian, and that it will not be greater than 90° when *L* and *d* are of a different name. Therefore, the following rules should be followed: (1) When *L* and *d* are of the same name, enter the azimuth tables with *t* in the hour angle column, using the given declination as latitude and the given latitude as declination, and take out the value of *M* from the tabulated azimuths. (2) When using H. O. Pub. No. 71, *L* and *d* being of a different name, follow the above rule and *M* will be the supplement of the angle taken from the tabulated azimuths. (3) When using H. O. Pub. No. 120, *L* and *d* being of a different name, enter the tables with $12^h - t$ in the hour angle column, using the declination as latitude and latitude as declination, and the angle taken from the tabulated azimuths will itself be the position angle *M*.

The t referred to here is the body's hour angle from the upper meridian; when finding M for use in the solution of equal altitudes, it will be sufficiently accurate to consider the hour angle as half the elapsed time when the first observation is east of the meridian, and as the supplement of half the elapsed time when the first observation is west of the meridian (see solution of Ex. 207, Art. 270).

(C) **Given Z, L, and h, to find t.**—In this case we will have from Napier's analogies

$$\left.\begin{aligned}\operatorname{Tan}\tfrac{1}{2}(t-M) &= \cot\tfrac{1}{2}Z\sin\tfrac{1}{2}(L-h)\sec\tfrac{1}{2}(L+h)\\ \operatorname{Tan}\tfrac{1}{2}(t+M) &= \cot\tfrac{1}{2}Z\cos\tfrac{1}{2}(L-h)\operatorname{cosec}\tfrac{1}{2}(L+h)\end{aligned}\right\}(c).$$

Comparing formulæ (c) with formulæ (a) there is noted an interchange of Z and t and a substitution of h for d; so to find a heavenly body's hour angle (t), having given the latitude (L) of the ship, the unknown body's true altitude (h) and its true azimuth (Z) estimated from the elevated pole, we have the following rule: Convert the azimuth into time and consider it as an hour angle; enter the azimuth tables in the given latitude, with the azimuth used as an hour angle and the altitude used as declination, and take from the tabulated azimuths the body's hour angle expressed in arc (see Art. 328 and Ex. 228).

(D) **To find the great circle course between two places.**—Enter the tables in the given latitude of the place of departure, with the difference of longitude between the two places expressed as time in the hour angle column, and the latitude of destination in the declination column, and take from the tabulated azimuths the great circle course named from the elevated pole, towards east or west as the place of destination is to eastward or westward of place of departure (see page 276).

When the difference of longitude between the two places is greater than 6 hours and the value is not found tabulated, as it may not be in H. O. Pub. No. 71, enter the tables in the given latitude of departure, with the supplement of the difference of longitude expressed as time in the hour angle column, and the latitude of destination with name changed in the declination column, then the supplement of the tabulated azimuth will be the great circle course to be marked as before directed.

APPENDIX D.

SOLUTION OF THE ASTRONOMICAL TRIANGLE BY NOMOGRAPHY.*

The nomogram constructed by Dr. Pesci of the Royal Italian Naval School is a diagram for the graphic solution of equations of the form tan a tan b = sin c by the ingenious method of aligned numbered points. It has been adapted by Lt. Radler de Aquino, Brazilian Navy, to the solution of the astronomical triangle through similar equations given below in group (a), which are of the form cot a cot b = cos c.

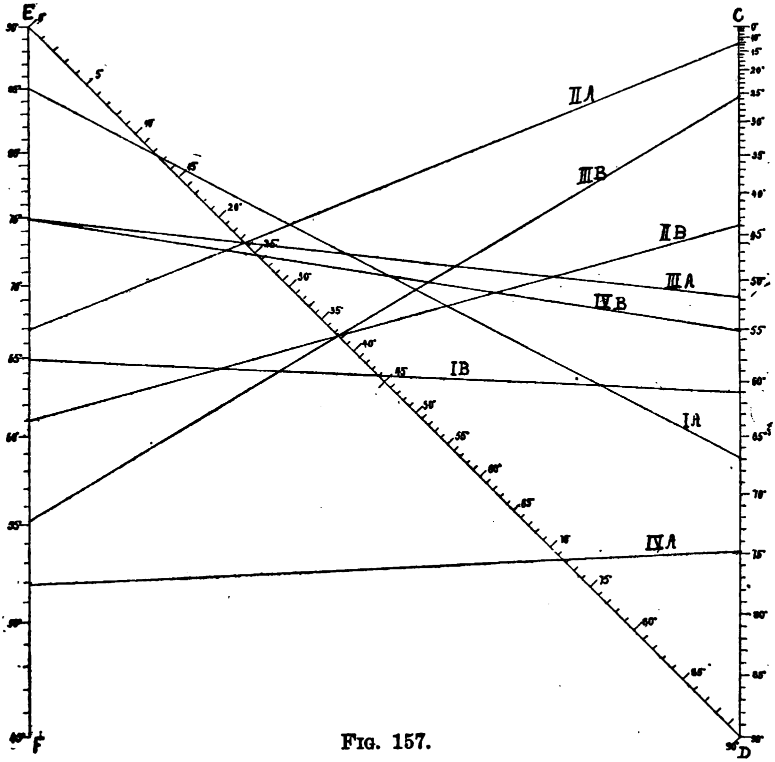

FIG. 157.

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* See "Proceedings of the U. S. Naval Institute," No. 126, page 633.

This nomogram consists of two parallel scales of equal length separated by a distance equal to that length and so joined by a diagonal scale as to form a figure similar to the letter *N* (Fig. 157). As adapted by Lt. Radler de Aquino, *CD* is a scale of consines, *EF* of cotangents (limited to angles greater than 45°), and *ED* is so graduated that when the numbered points corresponding to any two given quantities of the equation cot *a* cot *b* = cos *c*, each taken on its proper scale, are aligned, the numbered point where the line intersects the remaining scale will correspond to the required quantity.

The points corresponding to cosines, the right-hand functions of group (a), must always be taken on the right-hand scale *CD;* the points corresponding to cotangents, the left-hand functions of group (a), may be taken from the other two scales (the left and diagonal) indiscriminately, provided that the left scale is used only for cotangents of angles greater than 45°. This restriction on the use of the left-hand scale arises from the nature of the equation cot *a* cot *b* = cos *c*, which shows that both *a* and *b* cannot be less than 45° at once; and, therefore, when either *a* or *b* is less than 45° it must be read on the diagonal scale, the remaining one being read on the left scale; and when both *a* and *b* are greater than 45°, either may be read on either the left or diagonal scale. Any two points of the nomogram, not on the right-hand scale, aligned with 0° of the right-hand scale, are marked by complimentary numbers and for this reason the graduation is easily checked.

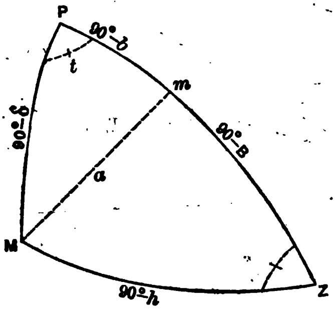

FIG. 158.

Let Fig. 158 be the astronomical triangle; *P*, the elevated pole; *Z*, the zenth of the observer; and *M*, the observed body whose altitude is *h* and declination is *d;* then *PZ* is the co-latitude. If the triangle is divided into two right triangles by a perpendicular *Mm* and the parts be lettered as shown, a complete solution may be effected, through Napier's rules, by the following equations:

$$\left.\begin{aligned} \cot b \cot(90^\circ - d) &= \cos t \\ \cot B \cot(90^\circ - h) &= \cos Z \\ \cot t \cot(90^\circ - a) &= \cos b \\ \cot Z \cot(90^\circ - a) &= \cos B \end{aligned}\right\}(\mathrm{a}).$$

The following precepts are given by Lt. Radler de Aquino for the determination, without resort to signs, of the values of B and the quadrant of Z in the various cases dependent on the relative values of L and d:

$$d \text{ and } L \text{ of same name} \begin{cases} t<90^\circ \begin{cases} b>L: B=(90^\circ+L)-b;\ Z<90^\circ \\ b<L: B=(90^\circ+b)-L;\ Z>90^\circ \end{cases} \\ t>90^\circ \qquad\qquad B=(L+b)-90^\circ;\ Z<90^\circ \end{cases}$$

$$d \text{ and } L \text{ of different name} \qquad B=90^\circ-(L+b);\ Z>90^\circ$$

In other words, in the first two cases when d and L are of the same name and $t < 6^h$, 90° must be added to the smaller of the two quantities b and L and from the sum the greater should be subtracted. In the third and fourth cases b and L are always added together; if the sum is greater than 90°, subtract 90° from it; if less than 90°, it is subtracted from 90°.

The precepts given above for determining the value of B and the quadrant of Z might be replaced by the much simpler and more covenient precepts of Art. 249, provided the equations in group (a) should be expressed in the nomenclature of that article, a nomenclature with which the American naval service is familiar. The student is referred to Art. 249 and to Figs. 115 and 116 therein representing the various positions of the heavenly body dependent on the values of L and d for a full explanation of the reasons given for the precepts suggested below.

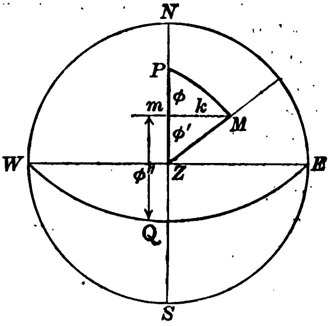

FIG. 159.

Let PMZ, Fig. 159, be the astronomical triangle projected on the plane of the horizon; then

m is the foot of the perpendicular k dropped from the position angle M on PZ.

$\phi = Pm$ is the polar distance of m; and if $\phi'' = 90^\circ - \phi$,

$\phi'' = Qm$ is the declination of m; and

$\phi' = mZ$ is the zenith distance of m.

Applying Napier's rules we have

$$\left.\begin{aligned} \cot \phi'' \cot (90^\circ - d) &= \cos t \\ \cot t \ \ \cot (90^\circ - k) &= \cos \phi'' \\ \cot Z \ \ \cot (90^\circ - k) &= \cos (90^\circ - \phi') \\ \cot (90^\circ - h) \cot (90^\circ - \phi') &= \cos Z \\ L &= \phi'' + \phi'. \end{aligned}\right\}\text{(b).}$$

From the above equations and Art. 249, we have the following precepts:

(1) ϕ'', the declination of m, is taken out in the same quadrant as t and is marked N. or S. like the declination.

(2) ϕ', the zenith distance of m, is marked N. if the body bears southerly, or S. if the body bears northerly.

(3) Z is taken from the nomogram in the first quadrant and is marked N. if ϕ' is marked S., or S. if ϕ' is marked N.; also E. or W. as the body is east or west of the meridian. This is a corollary of (2).

(4) ϕ'' and ϕ' are combined algebraically to give the latitude L, and, when any two of the three are known, the third may be found.

There are various purposes for which the nomogram may be used and the manner of its use will be apparent; but its principal use to the navigator will be when he is seeking (1) h and Z, having given L, d, and t; (2) t and d, having given h and Z of a body observed at a given place. It may also be used in finding the great circle course and distance between two places, but these may more easily and conveniently be found from a great circle chart.

The following is the key when seeking h and Z:

ϕ'' is determined by $90^\circ - d$ and t.
ϕ' is determined by formula $L = \phi'' + \phi'$.
$90^\circ - k$ is determined by t and ϕ''.
Z is determined by $90^\circ - k$ and $90^\circ - \phi'$.
$90^\circ - h$ (or h) is determined by $90^\circ - \phi'$ and Z.

Ex. 229.—April 3, 1918, a. m., in latitude 25° 40′ S. and longitude 104° 05′ 30″ E., the L. A. T. was $7^h\ 32^m\ 30^s$ a. m. (or $t = -66^\circ\ 52'\ 30''$) and the declination was 4° 55′.4 N., required the true h and Z.

Solution (Fig. 157): With $90^\circ - d$ and t, find $\phi'' = 12^\circ\ 30'$ N. (marked N. as d is N.), line IA; and as $L = \phi'' + \phi'$, $\phi' = 38^\circ\ 10'$ S. and $90^\circ - \phi' = 51^\circ\ 50'$.

With t and ϕ'', find $90^\circ - k = 23^\circ\ 30'$ (line IIA).

With $90^\circ - k$ and $90^\circ - \phi'$, find $Z = 74^\circ.9$ (line IIIA); as ϕ' is S. and t is (—), $Z =$ N. $74^\circ.9$ E.

With Z and $90^\circ - \phi'$, find $90^\circ - h = 71^\circ\ 38'$ (line IVA) and $h = 18^\circ\ 24'$.

The following is the key when seeking t and d:

$90^\circ - \phi'$ is determined by $90^\circ - h$ and Z.
ϕ'' is determined from the formula $L = \phi'' + \phi'$.
$90^\circ - k$ is determined by Z and $90^\circ - \phi'$.
t is determined by $90^\circ - k$ and ϕ''.
$90^\circ - d$ (or d) is determined by ϕ'' and t.

Ex. 230.—April 5, 1918, p. m., in latitude 20° 38′ S. and longitude 90° 10′ E., weather cloudy, a bright star was observed, through a break in the clouds, bearing N. 61° E. (true); ✳'s h 25° 10′; W. T. of obs. $7^h\ 20^m\ 40^s$; C—W $5^h\ 51^m\ 30^s$, chronometer fast of G. M. T. $1^m\ 06^s.3$; required the name of the star.

Solution (Fig. 157): With $90^\circ - h$ and Z, find $90^\circ - \phi' = 44^\circ$ (line IB) and $\phi' = 46^\circ$ S. (marked S. as the body bears N.). From $L = \phi'' + \phi'$ we have $\phi'' = 25^\circ\ 22'$ N. With Z and $90^\circ - \phi'$, find $90^\circ - k = 37^\circ\ 40'$ (line IIB). With $90^\circ - k$ and ϕ'', find $t = 55^\circ\ 10' = +3^h\ 40^m\ 40^s$, body being east of the meridian (line IIIB). With ϕ'' and t, find $90^\circ - d = 74^\circ\ 50'$ (line IVB) and $d = 15^\circ\ 10'$ N., d being of the same name as ϕ''. From the other data we have L. S. T. $= 8^h\ 03^m\ 50^s$ and therefore the ✳'s R. A. is $11^h\ 44^m\ 30^s$; this with the declination of 15° 10′ N. identifies the star as β leonis.

APPENDIX E.—TABLE OF COMPASS POINTS AND DEGREES FROM N. (TO THE RIGHT).

N to E	°	′	″	E to S	°	′	″	S to W	°	′	″	W to N	°	′	″
North	0	00	00	East	90	00	00	South	180	00	00	West	270	00	00
N¼E	2	48	45	E¼S	92	48	45	S¼W	182	48	45	W¼N	272	48	45
N½E	5	37	30	E½S	95	37	30	S½W	185	37	30	W½N	275	37	30
N¾E	8	26	15	E¾S	98	26	15	S¾W	188	26	15	W¾N	278	26	15
N by E	11	15	00	E by S	101	15	00	S by W	191	15	00	W by N	281	15	00
N by E¼E	14	03	45	ESE¾E	104	03	45	S by W¼W	194	03	45	WNW¾W	284	03	45
N by E½E	16	52	30	ESE½E	106	52	30	S by W½W	196	52	30	WNW½W	286	52	30
N by E¾E	19	41	15	ESE¼E	109	41	15	S by W¾W	199	41	15	WNW¼W	289	41	15
NNE	22	30	00	ESE	112	30	00	SSW	202	30	00	WNW	292	30	00
NNE¼E	25	18	45	SE by E¾E	115	18	45	SSW¼W	205	18	45	NW by W¾W	295	18	45
NNE½E	28	07	30	SE by E½E	118	07	30	SSW½W	208	07	30	NW by W½W	298	07	30
NNE¾E	30	56	15	SE by E¼E	120	56	15	SSW¾W	210	56	15	NW by W¼W	300	56	15
NE by N	33	45	00	SE by E	123	45	00	SW by S	213	45	00	NW by W	303	45	00
NE¾N	36	33	45	SE¾E	126	33	45	SW¾S	216	33	45	NW¾W	306	33	45
NE½N	39	22	30	SE½E	129	22	30	SW½S	219	22	30	NW½W	309	22	30
NE¼N	42	11	15	SE¼E	132	11	15	SW¼S	222	11	15	NW¼W	312	11	15
NE	45	00	00	SE	135	00	00	SW	225	00	00	NW	315	00	00
NE¼E	47	48	45	SE¼S	137	48	45	SW¼W	227	48	45	NW¼N	317	48	45
NE½E	50	37	30	SE½S	140	37	30	SW½W	230	37	30	NW½N	320	37	30
NE¾E	53	26	15	SE¾S	143	26	15	SW¾W	233	26	15	NW¾N	323	26	15
NE by E	56	15	00	SE by S	146	15	00	SW by W	236	15	00	NW by N	326	15	00
NE by E¼E	59	03	45	SSE¾E	149	03	45	SW by W¼W	239	03	45	NNW¾W	329	03	45
NE by E½E	61	52	30	SSE½E	151	52	30	SW by W½W	241	52	30	NNW½W	331	52	30
NF by E¾E	64	41	15	SSE¼E	154	41	15	SW by W¾W	244	41	15	NNW¼W	334	41	15
ENE	67	30	00	SSE	157	30	00	WSW	247	30	00	NNW	337	30	00
ENE¼E	70	18	45	S by E¾E	160	18	45	WSW¼W	250	18	45	N by W¾W	340	18	45
ENE½E	73	07	30	S by E½E	163	07	30	WSW½W	253	07	30	N by W½W	343	07	30
ENE¾E	75	56	15	S by E¼E	165	56	15	WSW¾W	255	56	15	N by W¼W	345	56	15
E by N	78	45	00	S by E	168	45	00	W by S	258	45	00	N by W	348	45	00
E¾N	81	33	45	S¾E	171	33	45	W¾S	261	33	45	N¾W	351	33	45
E½N	84	22	30	S½E	174	22	30	W½S	264	22	30	N½W	354	22	30
E¼N	87	11	15	S¼E	177	11	15	W¼S	267	11	15	N¼W	357	11	15

APPENDIX F.

THE SPERRY GYRO-COMPASS.

The construction of one of the standard types of this compass

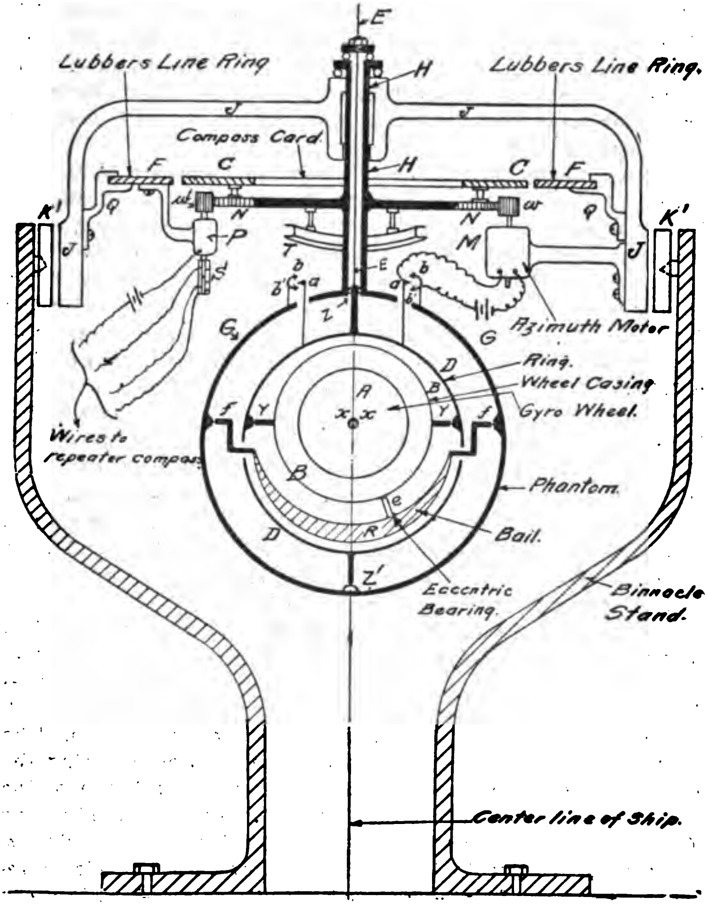

FIG. 160.—Partial Diagram of Section in an East and West Line Looking North, Illustrating Mounting and the Follow-up System.

is shown in the accompanying illustration, Fig. 160. The gyroscope wheel *A* is mounted to spin on a horizontal axis, *xx*, within the casing *B*, which is pivoted on the horizontal axis *YY* through its center of gravity and carried by the frame or vertical ring *D*. The ring *D* is suspended by the torsionless strand *E* and guided by bearings *ZZ'* to allow a free oscillation of limited amount about its vertical axis *ZZ'* within the frame or phantom *G*.

The phantom *G* has a hollow stem *H* to which the strand *E* is attached at its upper end, and the stem forms a journal for rotation in azimuth with respect to the supporting base frame *J*. The frame is mounted in gimbal rings *K'K'* on the binnacle in the same or similar manner as the ordinary magnetic compass is mounted in its binnacle stand.

Secured rigidly to the stem *H* of the phantom is a large gear wheel, *NN*, having 360 teeth, one tooth for each degree of azimuth. This gear wheel can only move with the phantom and consequently when the gear wheel, *NN*, is moved the phantom must move.

Secured rigidly to the frame *J*, and thus fixed relatively to the ship is a motor, *M*, whose small spur wheel *w* engages with the teeth of *NN*. Mounted rigidly on *NN* is the compass card *CC*, graduated to 360°. Flush with the surface of the compass card is a flat ring, *FF*, on which is engraved the lubber's line. *FF* is supported by brackets, *QQ*, on the frame *J*. Secured rigidly to the lubber's ring *FF* is a transmitter *P* whose function is to transmit electrically to the repeater compass at the helmsman, or to the pelorus repeaters, any movement made by the compass card *CC*.

The wheel *A*, together with the wheel casing *B* and the ring *D*, is called the sensitive element. As a matter of fact, and most important, the sensitive element is the real gyroscopic compass, and all the other mechanism is installed simply to reproduce the exact movement, headings, or readings of the sensitive element in azimuth without interfering with it. Attached to the sensitive element, on the vertical posts, *a*, *a*, as shown, are two electrical trolley contacts, *a*, *a*, which make a light electrical contact with double stationary contacts *bb'*, *bb'* carried by the phantom. The object of this mechanism is to make the phantom carrying the compass card follow exactly every movement in azimuth of the axis of gyro-wheel and thus register in degrees either the heading (course) of the ship or the direction in which the gyro-axis, *xx*, is pointing relative to the meridian. Furthermore, this movement, by means of the repeating transmitter, *P*, is sent to every steering compass, bridge pelorus, and repeater compass in the ship. This work is performed without any interference with the freedom of action of the sensitive element except that of the very light touch of the electrical contacts *aa* and *bb′*, *bb′*.

In order to introduce the horizontal couple necessary to suppress the oscillations in azimuth which would otherwise constantly attend the operation of this mechanism, the pivot *e*, connecting the heavy bail *R* to the rotor casing, *B*, is placed slightly eccentrically, at a distance from the vertical axis of the rotor of about ⅜ inch. As the bail is pivoted at the points *ff* which are above its center of gravity, its natural tendency is to hang vertically downwards. There are slots in *D* which allow *R* to do this. The tendency of the rotor *A* to keep its position in space as the earth rotates under it causes the axis *xx* to tilt, and the bail to be raised from its normal position, and the eccentricity of the pin *e*, by which the bail is raised, causes the gravity couple on *B* to have a horizontal component. This component provides the necessary vertical precession to produce a spiral motion of the axis by which it reaches its rest point. This point is always to the east of north by a small amount which varies as the tangent of the geographical latitude, and it has a slight altitude from the horizon, excepting when the instrument is located on the equator. This error, together with that arising through the speed and course of the ship, is semi-automatically allowed for; and the errors produced by rolling and pitching are also mechanically corrected by the application of stabilizing gyroscopes and counter-weights.

The two-gyro-compass.—To obviate the loss of directive force occasioned by the introduction of the one or more auxiliary gyroscopes that are employed to prevent disturbing forces, due to the movements of the ship upon which the compass is mounted, from reaching the compass and causing deviations from the meridian in its pointing, a new type is coming into use, called the two-gyro-compass, in which two exactly similar gyroscopes are mounted and controlled in such a way that the total gyroscopic power of both gyroscopes is used in causing the compass to seek and hold to the meridian, while the gyroscopes mutually oppose and neutralize deviations due to the disturbing forces.

The two-gyro-compass, Fig. 161, consists of two rotors *A* mounted in cases *B*, which cases are mounted on horizontal bearings *C*, in vertical rings *D*. These rings are supported on vertical torsion wires from frame *E*, and guided by vertical guide bearings *F*. Frame *E* is supported on vertical bearings *G* in spider *H* and is controlled by means of an azimuth motor not shown. This motor is controlled by contacts *I* between one of the vertical rings and frame *E* and impresses forces upon the two gyroscopes through spring *J*, which is attached to the two rotor casings by means of arms *K*. A transmitter is operated from the azimuth gear and serves to transmit the readings of the master compass to the repeater compasses. This transmitter and the repeater compasses operate upon a 22-volt direct-current supply, but all other parts of the compass, including the azimuth motor, operate upon a three-phase, 40-volt, 230-cycle supply. The horizontal bearings in which the rotor casings are mounted in the vertical rings are so placed that the east element (element is here used to include the rotor, stator and casing) is twice as pendulous as the west element. The control system for the compass has been designed so that the west rotor must spin clockwise and the east rotor counter-clockwise when looking at the south seeking end of the compass, that is, both rotors spin down in the center.

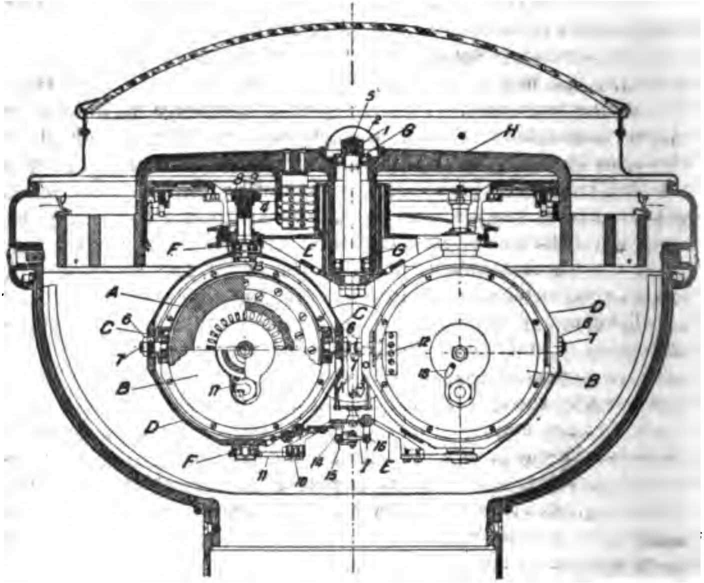

FIG. 161.

Let us assume the north seeking end of the compass to be pointing towards the west, then as the earth rotates, the south seeking end of the wheels (see figure, which shows a south elevation of the compass) will rise, and, as the rotors are pendulous, precession in opposite directions will take place about the vertical axes of the gyros, the east gyro precessing in a counter-clockwise direction and the west gyro in clockwise direction when looking down upon the compass. As the azimuth motor which controls the movements of the main frame is controlled by contacts placed between the west vertical ring and the main frame, the frame will follow all movements of this ring. This causes added displacement of the east ring with respect to the frame and a torque will be produced about the vertical axis of the east gyro, in such direction as to cause a lowering of the south end of its axis; that is, a relative movement between the vertical ring and the casing in a direction opposite to that produced by the rotation of the earth. The spring connecting the two casing will produce a torque about the vertical axis of the west gyro in such direction as to cause an elevation of the south end of its axis; that is, a relative movement between its casing and vertical ring in the same direction as that produced by the rotation of the earth. These movements will continue until the shaft of the east gyro passes the horizontal and its south end becomes depressed half as much as the south end of the shaft of the west gyro is elevated. As the east element is twice as pendulous as the west and the elements are tilted in opposite directions, it will result that they will have the same rate and same direction of precession about their vertical axes; in other words, both elements are now seeking the meridian.

The directive force of the compass is equal to the sum of the directive forces of the two gyros, because the east gyro is controlled by the azimuth motor in such a way that it will always be inclined to the horizontal in a direction opposite to and an amount equal to that which would be produced by the rotation of the earth and its torque is therefore added to that of the west gyro. Also the west gyro, which is half as pendulous as the east, will have the same torque as the east gyro about its vertical axis, because the azimuth motor acting through the connecting spring causes it to take up a position with its horizontal axis inclined twice as much as it would be if affected only by the rotation of the earth.

If the compass starts with its north seeking end pointing toward the west, it will swing past the meridian to a position a few degrees east of the meridian, then back to the west, continuing these oscillations with the angle decreasing very rapidly until it settles exactly upon the meridian. The period of this oscillation is about 80 or 90 minutes.

In the two-gyro-compass, no attempt is made to stabilize the elements as is done in some of the single-gyro types of compass or to use a separate pendulum and stabilize its point of attachment to the rotor casing as is done in others. On this account, any rolling of the ship when on an intercardinal heading will result in a torque about the vertical axis of each element, the same as in an unstabilized compass of the single-gyro type. This torque will be twice as large in the east element as in the west on account of the east element being made twice as pendulous, and would result in movement about the horizontal case supporting bearings at twice the rate of the west element, if it were not for the effect of the suspension of the east vertical ring. As the two gyros will precess in opposite directions about their horizontal axes, the azimuth motor will be brought into action and will be made to supply a torque exactly equal to the sum of the torques about the vertical axes of the two elements and opposite in direction. This torque will be divided between the east and west elements in the ratio of 2:1, due to the fact that only the connecting spring applies the torque to the west element, while both the connecting spring and suspension apply torque to the east element.

By properly choosing the pendulous factors of the two elements, any acceleration, retardation or turning of the ship upon which the compass is mounted may be made to cause a change in the azimuth of the compass approximately equal to the change in the settling point due to the new speed and course of the ship. This, of course, will be strictly correct for only one latitude, the same as in the single-gyro type of compass, because the correction for speed and heading of the ship varies with the latitude.

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